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Fluid Fuel Point Kinetics Reformulation

2025 European MELCOR Users' Group Meeting

April 7th-11th, 2025

SAND2025-04008PE

Overview

Brief history

- Fluid fuel point kinetics (FFPK) model developed in recent years
- Good performance in public demonstrations
- External users observed suspicious reactivity and power response under certain conditions
- Reviewed and slighty modified the model formulation, solution methodology, and results output

Mathematical model review

- Delayed neutron precursors and standard point reactor kinetics model
- Fluid fuel point reactor kinetics model
  - System of equations
  - Steady-state initialization
  - Reactivity
  - "Perfect" control system model
  - Ancillary fluid flow quantities

Validation – Zero power MSRE (ORNL) coast-down and ramp-up

Summary

Delayed Neutrons and Reactor Kinetics

Time-dependent neutron population (kinetics) plus system feedback mechanisms (dynamics)

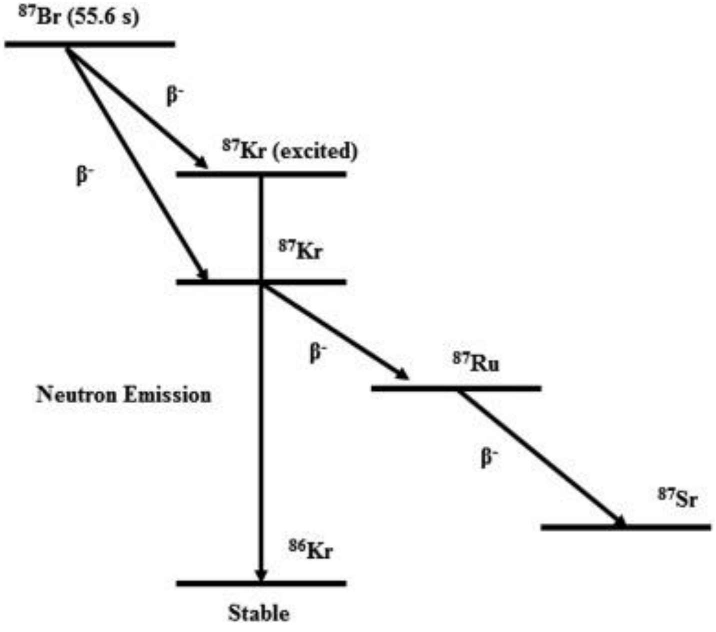
Delayed neutron (DN) emission from DN precursor (DNP) decay governs dynamic response

- Solid fuel –DNP's stay and hence DN's contribute to economy
- Fluid fuel – DNP's move (ex-core) and lost DN's impact economy

DNP grouping helps with analyses (group decay, abundance)

Process of DNP advection with flowing fuel is DNP "drift"

Cannot neglect the kinetic/dynamic implications of DNP "drift"



Standard Point Reactor Kinetics Model

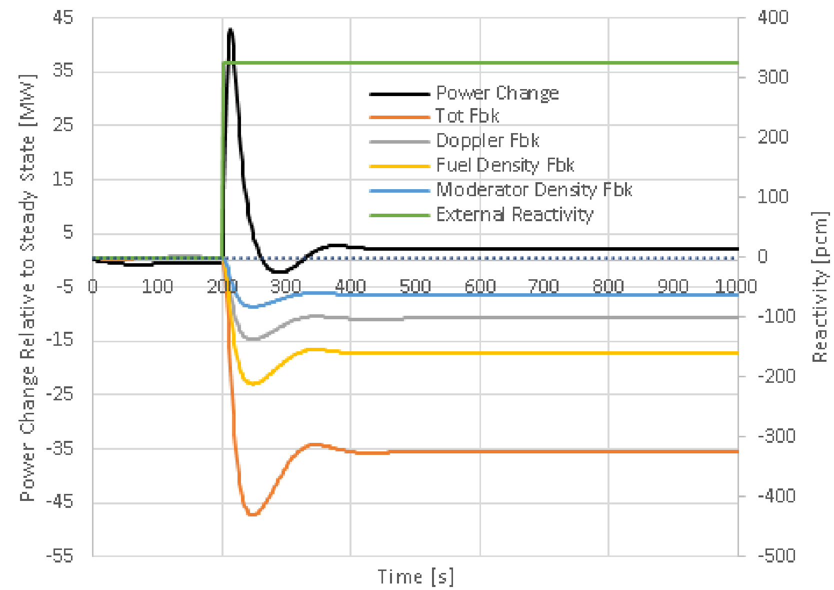
6 DNP group PRKE's

$$\frac{dP(t)}{dt} = \left(\frac{\rho(t)-\beta}{\Lambda}\right)P + \sum_{i=1}^{6}\lambda_i C_i(t) + S_0$$

$$\frac{dC_i(t)}{dt} = \left(\frac{\beta_i}{\Lambda}\right)P(t) - \lambda_i C_i(t), \qquad i = 1 \ldots 6$$

Where:

$P(t)$	=	Prompt neutron power [W]
$\rho(t)$	=	Reactivity
β	=	Total delayed neutron fraction
Λ	=	Prompt neutron generation time [s]
λ_i	=	Decay constant of i-th precursor group [1/s]
$C_i(t)$	=	Power of i-th delayed neutron precursor group [W]
β_i	=	Fraction of i-th delayed neutron group
S_0	=	Initial neutron source [W/s]



DNP drift

- Leads to lower effective DN fraction,
- Looks like a negative reactivity insertion, and
- Introduces a "reactivity bias" barrier to criticality for a given flow

Relatively lower (higher) DN emission in core as core DNP inventory decreases (increases)

Fuel flow (e.g. as driven by fuel pump) has direct reactivity implications

Fluid Fuel Point Reactor Kinetics Model

$$\frac{dP(t)}{dt} = \left(\frac{\rho(t) - \bar{\beta}(t)}{\Lambda}\right) P(t) + \sum_{i=1}^{6} \lambda_i C_i^C + S_0$$

$$\frac{dC_i^C(t)}{dt} = \overset{\text{A}}{\left(\frac{\beta_i}{\Lambda}\right) P(t)} - \overset{\text{B}}{\left(\lambda_i + {}^{1}/_{\tau_C}\right) C_i^C(t)} + \overset{\text{C}}{\left(\frac{V_L}{\tau_L V_C}\right) C_i^L(t - \tau_L)}, i = 1 \ldots 6$$

$$\frac{dC_i^L(t)}{dt} = \overset{\text{D}}{\left(\frac{V_C}{\tau_C V_L}\right) C_i^C(t)} - \overset{\text{E}}{\left(\lambda_i + {}^{1}/_{\tau_L}\right) C_i^L(t)}, \qquad i = 1 \ldots 6$$

F $$\bar{\beta}(t) = \beta - \beta_l(t) = \beta - \left(\frac{\Lambda}{P(t)}\right) \sum_{i=1}^{6} \lambda_i C_i^L(t)$$

Where:

$P(t)$	=	Prompt neutron power [W]
$\rho(t)$	=	Reactivity
Λ	=	Prompt neutron generation time [s]
λ_i	=	Decay constant of i-th precursor group [1/s]
S_0	=	Initial neutron source [W/s]
$\bar{\beta}(t)$	=	Effective delayed neutron fraction
β	=	Static delayed neutron fraction
β_i	=	Static fraction of i-th delayed neutron group
$\beta_l(t)$	=	Lost delayed neutron fraction
$C_i^C(t)$	=	Core cohort i-th delayed neutron precursor group power [W]
$C_i^L(t)$	=	Loop cohort i-th delayed neutron precursor group power [W]
τ_C	=	Residence time of precursors in the core [s]
τ_L	=	Residence time of precursors in the loop [s]
V_C	=	Fluid volume in core [m^3]
V_L	=	Fluid volume in loop [m^3]

A – "Core" (in-vessel) DNP gain by fission

B – "Core" DNP loss by decay and flow

C – "Core" DNP gain by "Loop" (ex-vessel) DNP flow

D – "Loop" DNP gain by "Core" DNP flow

E – "Loop" DNP loss by decay and flow

F – Definition of "effective" DN fraction

Note time-lag term **C**

- Numerically explicit source of "C" from "L"
- Could inform by tracking a time history
- Could approximate as:

$$C_i^L(t - \tau_L) = C_i^L(t) - \tau_L \frac{dC_i^L(t)}{dt}$$

- And thereby obtain an equation w/o time-lag term:

$$\frac{dC_i^C(t)}{dt} = \overset{\text{A}}{\left(\frac{\beta_i}{\Lambda}\right) P(t)} - \overset{\text{B}}{\left(\lambda_i + {}^{2}/_{\tau_C}\right) C_i^C(t)} + \overset{\text{C}}{\left(\frac{V_L}{V_C}\right) \left(\lambda_i + {}^{2}/_{\tau_L}\right) C_i^L(t)}.$$

Fluid Fuel Point Reactor Kinetics Model

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To obtain a finalized form convenient for solution, modify the power/reactivity equation by substituting the definition of "effective" DN fraction:

$$\frac{dP(t)}{dt} = \left(\frac{\rho(t) - \beta + 2\beta_{l,0}}{\Lambda}\right) P(t) - \sum_{i=1}^{6} \lambda_i C_i^L(t) + \sum_{i=1}^{6} \lambda_i C_i^C(t) + S_0$$

Where:

$\beta_{l,0}$ = Initial lost delayed neutron fraction = $\beta_l(t = t_0) = \left(\frac{\Lambda}{P(t_0)}\right) \sum_{i=1}^{6} \lambda_i C_i^L(t_0)$

Thus the final set of thirteen equations:

$$\frac{dP(t)}{dt} = \left(\frac{\rho(t) - \beta + 2\beta_{l,0}}{\Lambda}\right) P(t) - \sum_{i=1}^{6} \lambda_i C_i^L(t) + \sum_{i=1}^{6} \lambda_i C_i^C(t) + S_0$$

$$\frac{dC_i^C(t)}{dt} = \left(\frac{\beta_i}{\Lambda}\right) P(t) - \left(\lambda_i + {}^2\!/_{\tau_C}\right) C_i^C(t) + \left(\frac{V_L}{V_C}\right) \left(\lambda_i + {}^2\!/_{\tau_L}\right) C_i^L(t), \qquad i = 1 \dots 6$$

$$\frac{dC_i^L(t)}{dt} = \left(\frac{V_C}{\tau_C V_L}\right) C_i^C(t) - \left(\lambda_i + {}^1\!/_{\tau_L}\right) C_i^L(t), \qquad i = 1 \dots 6$$

FFPRK Model – Steady State Initialization

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Assume criticality (all time derivatives zeroed) without a source and steady-state flow, then:

- Initial power is P_0 and no source is present ($S_0 = 0$)
- Feedback and external reactivity is zero
- Time-zero reactivity $\rho(t_0)$ equals "bias reactivity" $\Delta\rho_0$, i.e. reactivity required to compensate for DNP drift

$$\frac{dP(t_0)}{dt} = 0 = \left(\frac{\Delta\rho_0 - \bar{\beta}(t_0)}{\Lambda}\right)P_0 + \sum_{i=1}^{6}\lambda_i C_{i,0}^C = \left(\frac{\Delta\rho_0 - \beta + \beta_l(t_0)}{\Lambda}\right)P_0 + \sum_{i=1}^{6}\lambda_i C_{i,0}^C$$

$$\frac{dC_i^C(t_0)}{dt} = 0 = \left(\frac{\beta_i}{\Lambda}\right)P_0 - \left(\lambda_i + {}^2/_{\tau_C}\right)C_{i,0}^C + \left(\frac{V_L}{V_C}\right)\left(\lambda_i + {}^2/_{\tau_L}\right)C_{i,0}^L, \qquad i = 1 \dots 6$$

$$\frac{dC_i^L(t_0)}{dt} = 0 = \left(\frac{V_C}{\tau_C V_L}\right)C_{i,0}^C - \left(\lambda_i + {}^1/_{\tau_L}\right)C_{i,0}^L, \qquad i = 1 \dots 6$$

Solving:

$$C_{i,0}^C = \alpha_i P_0, \qquad i = 1 \dots 6$$

$$C_{i,0}^L = \gamma_i \alpha_i P_0 = \gamma_i C_{i,0}^C, \qquad i = 1 \dots 6$$

$$\gamma_i = \left(\frac{V_C}{\tau_C V_L}\right) \Big/ \left(\lambda_i + {}^1/_{\tau_L}\right), \qquad i = 1 \dots 6$$

$$\alpha_i = \left(\frac{\beta_i}{\Lambda}\right) \Big/ \left[\left(\lambda_i + {}^2/_{\tau_C}\right) - \gamma_i\left(\left(\frac{V_L}{V_C}\right)\left(\lambda_i + {}^2/_{\tau_L}\right)\right)\right], \qquad i = 1 \dots 6$$

$$\Delta\rho_0 = \beta - \left(\frac{\Lambda}{P_0}\right)\sum_{i=1}^{6}\lambda_i\left(C_{i,0}^C + C_{i,0}^L\right) = \beta - \Lambda\sum_{i=1}^{6}\lambda_i\alpha_i(1+\gamma_i)$$

$$\bar{\beta}(t_0) = \beta - \beta_l(t_0) = \beta - \Lambda\sum_{i=1}^{6}\lambda_i\gamma_i\alpha_i$$

- Bias reactivity a constant component of $\rho(t)$
- Initial effective DN fraction used in solution

FFPRK Model – Reactivity

Total reactivity $\rho(t)$ includes feedback, external, and bias: $\rho(t) = \rho_{fb}(t) + \rho_{ext}(t) + \Delta\rho_0$

The "reactivity budget": $\left(\frac{\Lambda}{P(t)}\right)\left(\frac{dP(t)}{dt}\right) = \rho_{fb}(t) + \rho_{ext}(t) + \Delta\rho_0 - \bar{\beta}(t) + \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t) + S_0\right)$

Substituting for bias reactivity, effective DN fraction, lost DN fraction, and collecting terms yields:

$$\begin{aligned}\left(\frac{\Lambda}{P(t)}\right)\left(\frac{dP(t)}{dt}\right) &= \rho_{fb}(t) + \rho_{ext}(t) \\ &+ \left[\beta + \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) + \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right)\right] \\ &- \left[\beta + \left(\frac{\Lambda}{P_0}\right)\sum_{i=1}^{6} \lambda_i \left(C_{i,0}^L\right) + \left(\frac{\Lambda}{P_0}\right)\sum_{i=1}^{6} \lambda_i \left(C_{i,0}^C\right)\right] + \left(\frac{\Lambda}{P(t)}\right) S_0\end{aligned}$$

Define "flow reactivity": $\Delta\rho(t) = \beta - \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) - \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right)$

Note the bias reactivity equals the initial flow reactivity

Thus obtain the reactivity budget in terms of flow effects: $\left(\frac{\Lambda}{P(t)}\right)\left(\frac{dP(t)}{dt}\right) = \rho_{fb}(t) + \rho_{ext}(t) + \Delta\rho_0 - \Delta\rho(t) + \left(\frac{\Lambda}{P(t)}\right) S_0$

- Dependence of criticality and power on flow via reactivity effects is more obvious from this budget
- For criticality during a flow transient, $\rho_{fb}(t) + \rho_{ext}(t)$ must balance deviation of flow reactivity from bias $\Delta\rho_0 - \Delta\rho(t)$

FFPRK Model – Reactivity

CVH-FFPKM-REACT-FLOW	$\Delta\rho(t) = \beta - \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) - \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right)$
CVH-FFPKM-REACT-FLOW-CORE	$\Delta\rho_C(t) = \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right) - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_{i,0}^C\right)$
CVH-FFPKM-REACT-FLOW-LOOP	$\Delta\rho_L(t) = \left(\frac{\Lambda}{P(t)}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_{i,0}^L\right)$
CVH-FFPKM-REACT-FEEDBACK	$\rho_{fb}(t)$
CVH-FFPKM-REACT-CONTROL	$\rho_{ext}(t)$
CVH-FFPKM-REACT-FLOWCONT (“perfect” flow control system model)	$\rho_{ext}(t) = \Delta\rho_0(t) - \Delta\rho_0 + 2\left(\beta_l(t) - \beta_{l,0}\right)$
CVH-FFPKM-REACT-TOTAL	$\rho(t) = \rho_{fb}(t) + \rho_{ext}(t) + \Delta\rho_C(t) - \Delta\rho_L(t)$
CVH-FFPKM-REACT-BIAS	$\Delta\rho_0 = \beta - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_{i,0}^L\right) - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_{i,0}^C\right)$

FFPRK Model – “Perfect” Control System

Derive a prescription for external (e.g. control system) reactivity required to maintain criticality:

- Arbitrary flow transient i.e. flow reactivity $\Delta\rho(t)$ allowed to change arbitrarily as $C_i^C(t)$ and $C_i^L(t)$ evolve due to flow
- No source
- No feedback reactivity (e.g. hot-zero power condition) such that $\rho_{fb}(t) = 0$ and $P(t) = P_0$

The power/reactivity equation then reduces to:

$$0 = \rho_{ext}(t) + \Delta\rho_0 - \beta + 2\beta_{l,0} - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) + \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right)$$

Algebraically manipulating:

$$\begin{aligned}\rho_{ext}(t) - 2\left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) &= \left[\beta - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right)\right] \\ &- \left[\beta - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_{i,0}^L\right) - \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_{i,0}^C\right)\right] - 2\beta_{l,0}\end{aligned}$$

Finally:

$$\rho_{ext}(t) = \Delta\rho_0(t) - \Delta\rho_0 + 2\left(\beta_l(t) - \beta_{l,0}\right)$$

Where:

$$\Delta\rho_0(t) = \beta + \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^L(t)\right) + \left(\frac{\Lambda}{P_0}\right)\left(\sum_{i=1}^{6} \lambda_i C_i^C(t)\right)$$

FFPRK Model – Auxiliary Flow Quantities

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Gross characteristics of core and loop flow inform DNP cohort source/sink terms

- Transit times approximate the time for flow to traverse both active core and balance of primary loop
- Fluid volumes calculated from control volumes that comprise the core and loop
- "Core" quantities consider all CV's identified as belonging to the core
- "Loop" quantities consider all CV's identified as belonging to the balance of the loop
- Resort to control volume averaged notions of flow path phasic (pool) flows

$$V_C = \sum_{j=1}^{N_C} V_j$$

$$V_L = \sum_{j=1}^{N_L} V_j$$

$$\tau_C = \frac{1}{V_C} \sum_{j=1}^{N_C} (vAV)_j$$

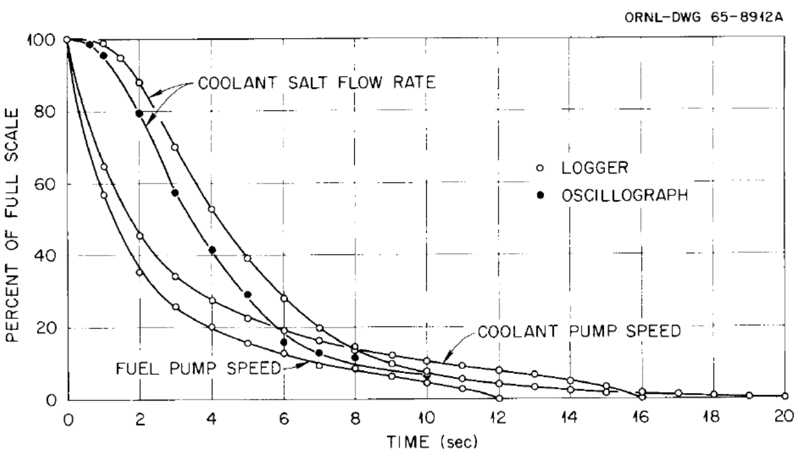
$$\tau_L = \frac{1}{V_L} \sum_{j=1}^{N_L} (vAV)_j$$

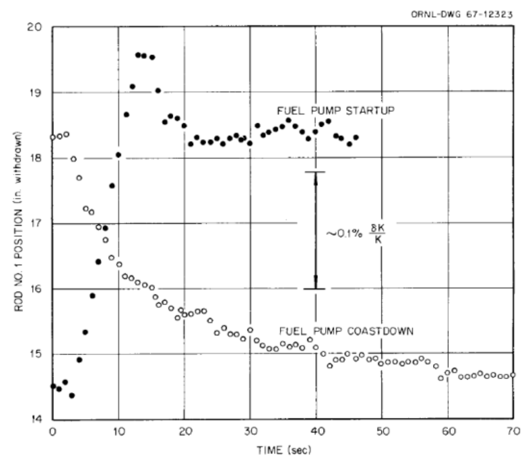
Where:

N_C	=	Number of control volumes comprising the core
N_L	=	Number of control volumes comprising the loop
V	=	Pool phase volume in CV
v	=	Volume-averaged pool phase velocity in CV
A	=	CV area in direction of flow in CV

FFPRK Model – Validation

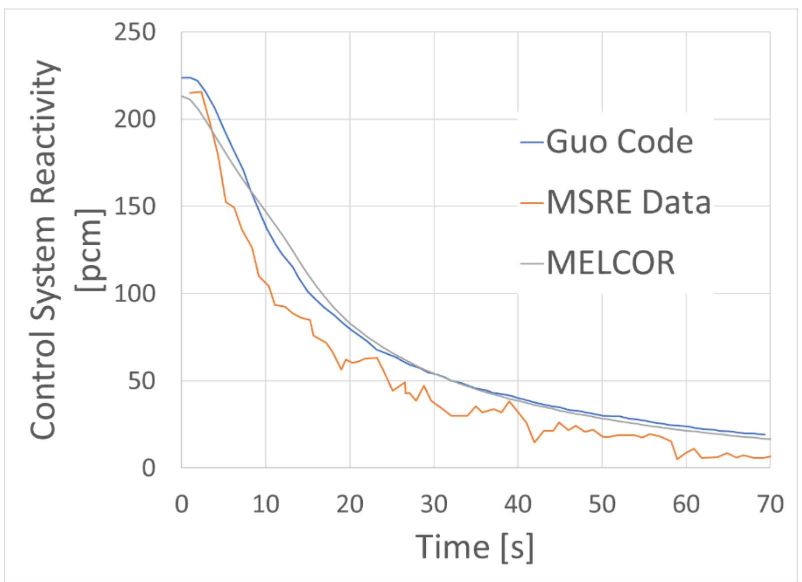
Zero-power coast-down (ORNL MSRE)

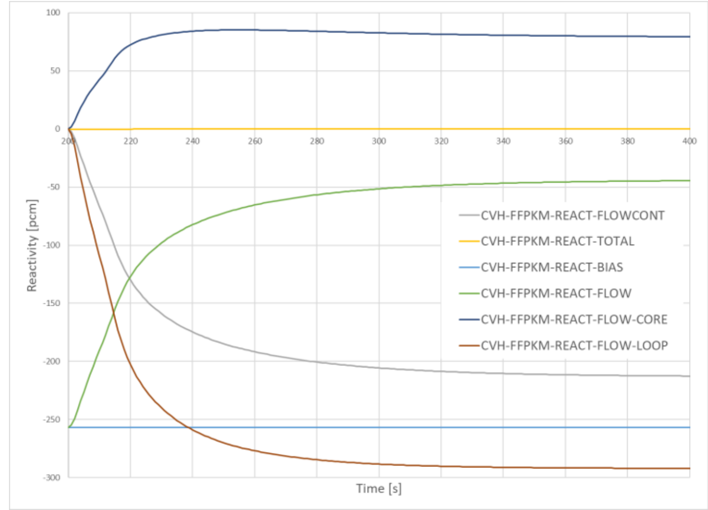


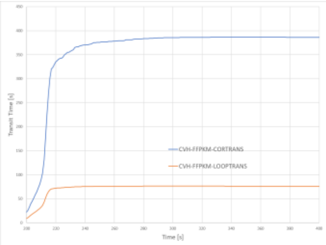


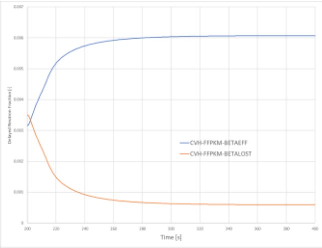
MSRE circulation worth:
0.212 +/- 0.004 δK/K

MELCOR calc. worth:
0.21325 δK/K





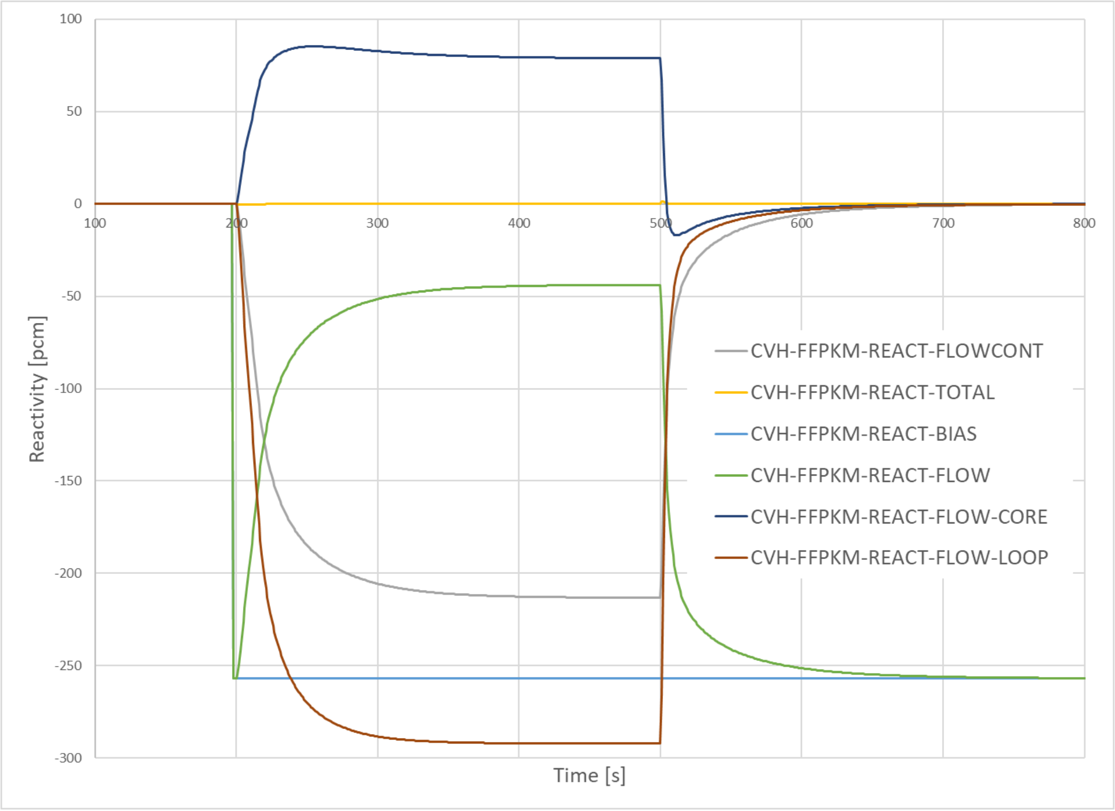


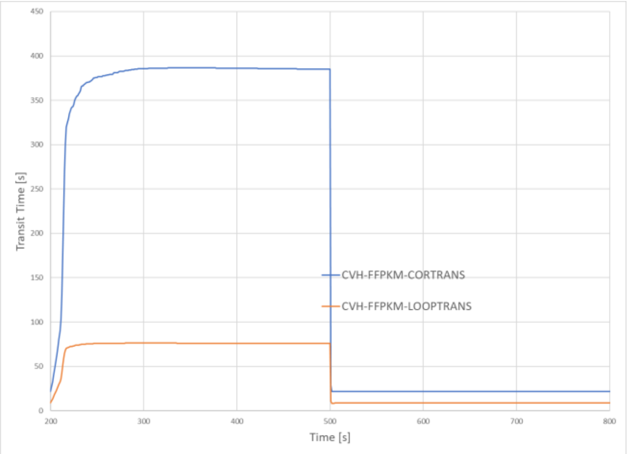


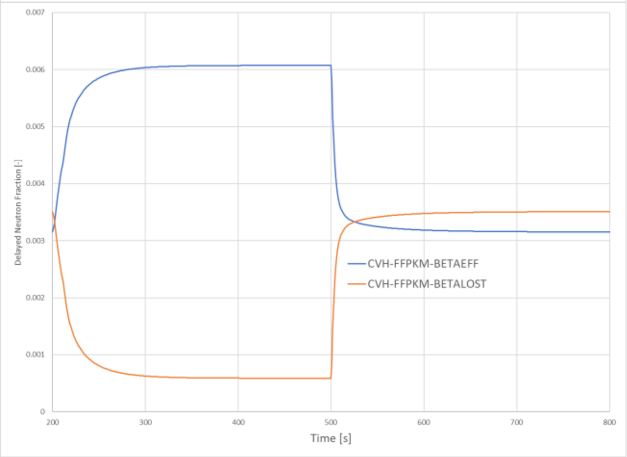
FFPRK Model – Flow Ramp-Up

Zero-power ramp-up (ORNL MSRE)

- Not truly a validation at this point, but a verification of expected FFPKM behavior
- Run the coast-down validation case followed by a quick pump ramp-up (return to flow)
- Returning to steady flow and reversing coast-down control system reactivity leads to initial configuration







Summary

Reviewed mathematical model

- Time-lagged source term approximation
- Bias reactivity and initial delayed neutron fraction
- Steady-state initialization
- Reactivity budget

Validation

- Good comparison with experimentally-measured “circulating flow worth”
- Good benchmark comparison to another code prediction from literature

Verification

- Flow ramp-up after coast-down
- Control reactivity balances flow reactivity to preserve criticality
- System returns to initial configuration