Gauge-Fixing in EFT Matching Calculations

Anders Eller Thomsen

Based on [2404.11640]

PSI physics seminar
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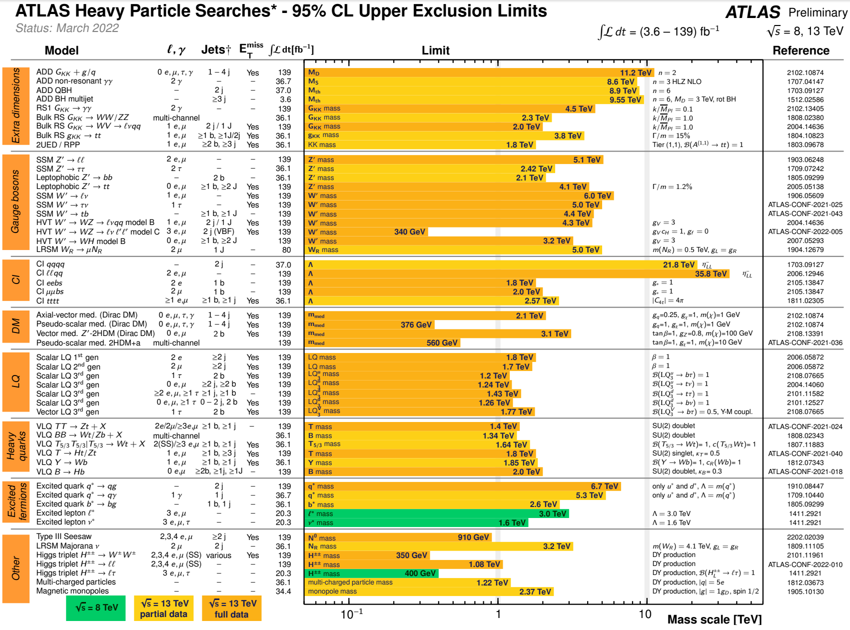
Direct searches for new physics



ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2022

ATLAS Preliminary

$\int\mathcal{L}\,dt$ = (3.6 – 139) fb^{-1}

$\sqrt{s}$ = 8, 13 TeV

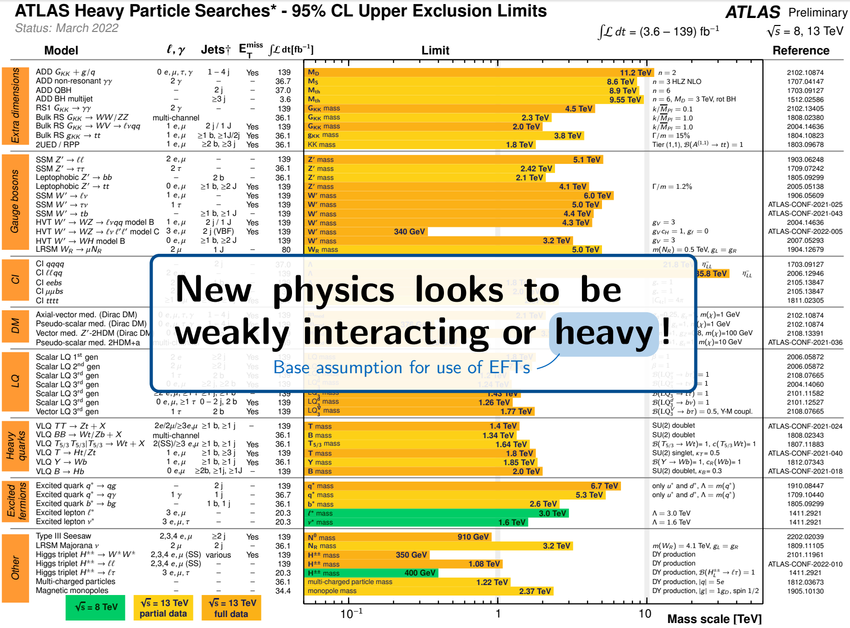
	Model	ℓ,γ	Jets†	E_T^{miss}	$\int\mathcal{L}\,dt[fb^{-1}]$	Limit		Reference
Extra dimensions	ADD $G_{KK}+g/q$	0 e,μ,τ,γ	1 – 4 j	Yes	139	M_D 11.2 TeV	$n=2$	2102.10874
	ADD non-resonant $\gamma\gamma$	2 γ	–	–	36.7	M_S 8.6 TeV	$n=3$ HLZ NLO	1707.04147
	ADD QBH	–	2 j	–	37.0	M_{th} 8.9 TeV	$n=6$	1703.09127
	ADD BH multijet	–	≥3 j	–	3.6	M_{th} 9.55 TeV	$n=6$, $M_D=3$ TeV, rot BH	1512.02586
	RS1 $G_{KK}\to\gamma\gamma$	2 γ	–	–	139	G_{KK} mass 4.5 TeV	$k/\overline{M}_{Pl}=0.1$	2102.13405
	Bulk RS $G_{KK}\to WW/ZZ$	multi-channel			36.1	G_{KK} mass 2.3 TeV	$k/\overline{M}_{Pl}=1.0$	1808.02380
	Bulk RS $G_{KK}\to WV\to\ell\nu qq$	1 e,μ	2 j / 1 J	Yes	139	G_{KK} mass 2.0 TeV	$k/\overline{M}_{Pl}=1.0$	2004.14636
	Bulk RS $g_{KK}\to tt$	1 e,μ	≥1 b, ≥1J/2j	Yes	36.1	g_{KK} mass 3.8 TeV	$\Gamma/m=15\%$	1804.10823
	2UED / RPP	1 e,μ	≥2 b, ≥3 j	Yes	36.1	KK mass 1.8 TeV	Tier (1,1), $\mathcal{B}(A^{(1,1)}\to tt)=1$	1803.09678
Gauge bosons	SSM $Z'\to\ell\ell$	2 e,μ	–	–	139	Z' mass 5.1 TeV		1903.06248
	SSM $Z'\to\tau\tau$	2 τ	–	–	36.1	Z' mass 2.42 TeV		1709.07242
	Leptophobic $Z'\to bb$	–	2 b	–	36.1	Z' mass 2.1 TeV		1805.09299
	Leptophobic $Z'\to tt$	0 e,μ	≥1 b, ≥2 J	Yes	139	Z' mass 4.1 TeV	$\Gamma/m=1.2\%$	2005.05138
	SSM $W'\to\ell\nu$	1 e,μ	–	Yes	139	W' mass 6.0 TeV		1906.05609
	SSM $W'\to\tau\nu$	1 τ	–	Yes	139	W' mass 5.0 TeV		ATLAS-CONF-2021-025
	SSM $W'\to tb$	–	≥1 b, ≥1 J	–	139	W' mass 4.4 TeV		ATLAS-CONF-2021-043
	HVT $W'\to WZ\to\ell\nu qq$ model B	1 e,μ	2 j / 1 J	Yes	139	W' mass 4.3 TeV	$g_V=3$	2004.14636
	HVT $W'\to WZ\to\ell\nu\,\ell'\ell'$ model C	3 e,μ	2 j (VBF)	Yes	139	W' mass 340 GeV	$g_V c_H=1$, $g_f=0$	ATLAS-CONF-2022-005
	HVT $W'\to WH$ model B	0 e,μ	≥1 b, ≥2 J		139	W' mass 3.2 TeV	$g_V=3$	2007.05293
	LRSM $W_R\to\mu N_R$	2 μ	1 J	–	80	W_R mass 5.0 TeV	$m(N_R)=0.5$ TeV, $g_L=g_R$	1904.12679
CI	CI $qqqq$	–	2 j	–	37.0	Λ 21.8 TeV η_{LL}^-		1703.09127
	CI $\ell\ell qq$	2 e,μ	–	–	139	Λ 35.8 TeV	η_{LL}^-	2006.12946
	CI $eebs$	2 e	1 b	–	139	Λ 1.8 TeV	$g_*=1$	2105.13847
	CI $\mu\mu bs$	2 μ	1 b	–	139	Λ 2.0 TeV	$g_*=1$	2105.13847
	CI $tttt$	≥1 e,μ	≥1 b, ≥1 j	Yes	36.1	Λ 2.57 TeV	$\lvert C_{4t}\rvert=4\pi$	1811.02305
DM	Axial-vector med. (Dirac DM)	0 e,μ,τ,γ	1 – 4 j	Yes	139	m_{med} 2.1 TeV	g_q=0.25, g_χ=1, $m(\chi)$=1 GeV	2102.10874
	Pseudo-scalar med. (Dirac DM)	0 e,μ,τ,γ	1 – 4 j	Yes	139	m_{med} 376 GeV	g_q=1, g_χ=1, $m(\chi)$=1 GeV	2102.10874
	Vector med. Z'-2HDM (Dirac DM)	0 e,μ	2 b	Yes	139	m_{med} 3.1 TeV	tanβ=1, g_Z=0.8, $m(\chi)$=100 GeV	2108.13391
	Pseudo-scalar med. 2HDM+a	multi-channel			139	m_{med} 560 GeV	tanβ=1, g_χ=1, $m(\chi)$=10 GeV	ATLAS-CONF-2021-036
LQ	Scalar LQ 1st gen	2 e	≥2 j	Yes	139	LQ mass 1.8 TeV	$\beta=1$	2006.05872
	Scalar LQ 2nd gen	2 μ	≥2 j	Yes	139	LQ mass 1.7 TeV	$\beta=1$	2006.05872
	Scalar LQ 3rd gen	1 τ	2 b	Yes	139	LQ$_3^u$ mass 1.2 TeV	$\mathcal{B}(\mathrm{LQ}_3^u\to b\tau)=1$	2108.07665
	Scalar LQ 3rd gen	0 e,μ	≥2 j, ≥2 b	Yes	139	LQ$_3^u$ mass 1.24 TeV	$\mathcal{B}(\mathrm{LQ}_3^u\to t\nu)=1$	2004.14060
	Scalar LQ 3rd gen	≥2 e,μ, ≥1 τ	≥1 j, ≥1 b	–	139	LQ$_3^d$ mass 1.43 TeV	$\mathcal{B}(\mathrm{LQ}_3^d\to t\tau)=1$	2101.11582
	Scalar LQ 3rd gen	0 e,μ, ≥1 τ	0 – 2 j, 2 b	Yes	139	LQ$_3^d$ mass 1.26 TeV	$\mathcal{B}(\mathrm{LQ}_3^d\to b\nu)=1$	2101.12527
	Vector LQ 3rd gen	1 τ	2 b	Yes	139	LQ$_3^V$ mass 1.77 TeV	$\mathcal{B}(\mathrm{LQ}_3^V\to b\tau)=0.5$, Y-M coupl.	2108.07665
Heavy quarks	VLQ $TT\to Zt+X$	2e/2μ/≥3e,μ	≥1 b, ≥1 j	–	139	T mass 1.4 TeV	SU(2) doublet	ATLAS-CONF-2021-024
	VLQ $BB\to Wt/Zb+X$	multi-channel			36.1	B mass 1.34 TeV	SU(2) doublet	1808.02343
	VLQ $T_{5/3}T_{5/3}\lvert T_{5/3}\to Wt+X$	2(SS)/≥3 e,μ	≥1 b, ≥1 j	Yes	36.1	$T_{5/3}$ mass 1.64 TeV	$\mathcal{B}(T_{5/3}\to Wt)$= 1, $c(T_{5/3}Wt)$= 1	1807.11883
	VLQ $T\to Ht/Zt$	1 e,μ	≥1 b, ≥3 j	Yes	139	T mass 1.8 TeV	SU(2) singlet, κ_T= 0.5	ATLAS-CONF-2021-040
	VLQ $Y\to Wb$	1 e,μ	≥1 b, ≥1 j	Yes	36.1	Y mass 1.85 TeV	$\mathcal{B}(Y\to Wb)$= 1, $c_R(Wb)$= 1	1812.07343
	VLQ $B\to Hb$	0 e,μ	≥2b, ≥1j, ≥1J	–	139	B mass 2.0 TeV	SU(2) doublet, κ_B= 0.3	ATLAS-CONF-2021-018
Excited fermions	Excited quark $q^*\to qg$	–	2 j	–	139	q* mass 6.7 TeV	only u^* and d^*, $\Lambda=m(q^*)$	1910.08447
	Excited quark $q^*\to q\gamma$	1 γ	1 j	–	36.7	q* mass 5.3 TeV	only u^* and d^*, $\Lambda=m(q^*)$	1709.10440
	Excited quark $b^*\to bg$	–	1 b, 1 j	–	36.1	b* mass 2.6 TeV		1805.09299
	Excited lepton ℓ^*	3 e,μ	–	–	20.3	ℓ^* mass 3.0 TeV	$\Lambda=3.0$ TeV	1411.2921
	Excited lepton ν^*	3 e,μ,τ	–	–	20.3	ν^* mass 1.6 TeV	$\Lambda=1.6$ TeV	1411.2921
Other	Type III Seesaw	2,3,4 e,μ	≥2 j	Yes	139	N^0 mass 910 GeV		2202.02039
	LRSM Majorana ν	2 μ	2 j	–	36.1	N_R mass 3.2 TeV	$m(W_R)=4.1$ TeV, $g_L=g_R$	1809.11105
	Higgs triplet $H^{\pm\pm}\to W^\pm W^\pm$	2,3,4 e,μ (SS)	various	Yes	139	$H^{\pm\pm}$ mass 350 GeV	DY production	2101.11961
	Higgs triplet $H^{\pm\pm}\to\ell\ell$	2,3,4 e,μ (SS)	–	–	139	$H^{\pm\pm}$ mass 1.08 TeV	DY production	ATLAS-CONF-2022-010
	Higgs triplet $H^{\pm\pm}\to\ell\tau$	3 e,μ,τ	–	–	20.3	$H^{\pm\pm}$ mass 400 GeV	DY production, $\mathcal{B}(H_L^{\pm\pm}\to\ell\tau)=1$	1411.2921
	Multi-charged particles	–	–	–	36.1	multi-charged particle mass 1.22 TeV	DY production, $\lvert q\rvert=5e$	1812.03673
	Magnetic monopoles	–	–	–	34.4	monopole mass 2.37 TeV	DY production, $\lvert g\rvert=1g_D$, spin 1/2	1905.10130

🟩 $\sqrt{s}$ = 8 TeV

🟨 $\sqrt{s}$ = 13 TeV partial data

🟧 $\sqrt{s}$ = 13 TeV full data

Direct searches for new physics

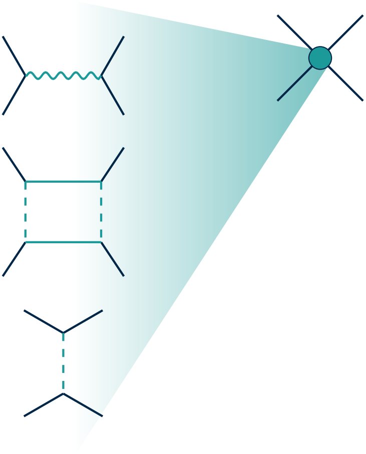


New physics looks to be weakly interacting or heavy!

Base assumption for use of EFTs

Effective field theory

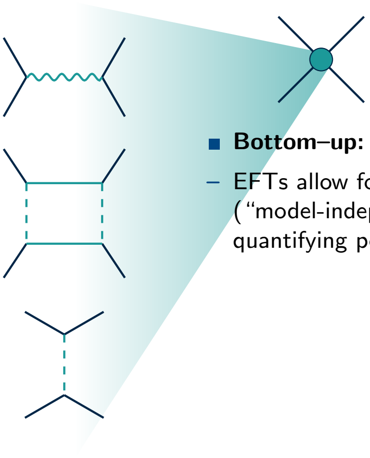
High-energy physics manifests as contact interactions in EFTs



$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{D=5} \sum_{k} \overset{\text{UV Physics}}{\frac{C_{D,k}}{\Lambda^{D-4}}} \mathcal{O}_{D,k}$$

Effective field theory

High-energy physics manifests as contact interactions in EFTs



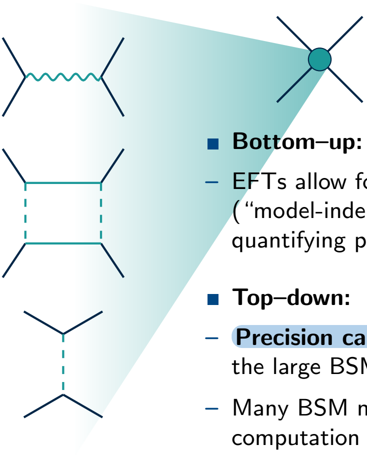
UV Physics

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{D=5} \sum_{k} \frac{C_{D,k}}{\Lambda^{D-4}} \mathcal{O}_{D,k}$$

- **Bottom–up:**
  - EFTs allow for a **model-comprehensive** ("model-independent") analysis of deviations from the SM, quantifying possible deviations as an expansion in E/Λ

Effective field theory

High-energy physics manifests as contact interactions in EFTs

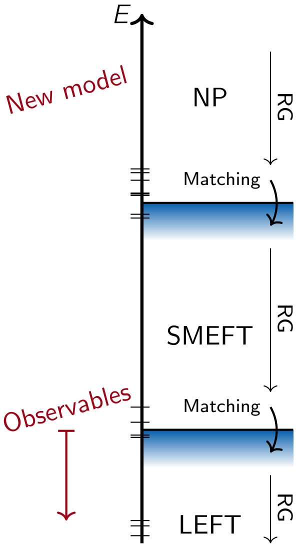


UV Physics

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{D=5}\sum_{k} \frac{C_{D,k}}{\Lambda^{D-4}}\mathcal{O}_{D,k}$$

- **Bottom–up:**
  - EFTs allow for a **model-comprehensive** (“model-independent”) analysis of deviations from the SM, quantifying possible deviations as an expansion in E/Λ
- **Top–down:**
  - **Precision calculations** necessitates the use of EFTs to separate the large BSM energy scales
  - Many BSM models results in the same EFT, ensuring that computation are **reusable**: you only need to compute once in the EFT

Top–down EFT workflow



E

New model

NP

RG

Matching

SMEFT

RG

Matching

Observables

LEFT

RG

Jenkins, Manohar, Trott [1308.2627]
Jenkins, Manohar, Trott [1310.4838]
Alonso *et al.* [1312.2014]

Jenkins, Manohar, Stoffer [1709.04486]
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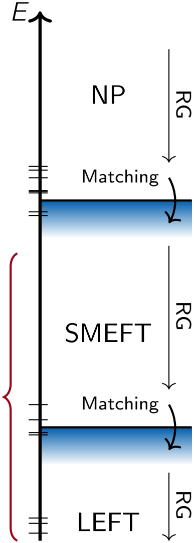

Included in codes
wilson and **DsixTools**

Aebischer, Kumar, Straub [1804.05033]
Cellis *et al.* [1704.04504]
Fuentes-Martín *et al.* [2010.16341]

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The repetitive nature of EFT computations call for **automated tools!**

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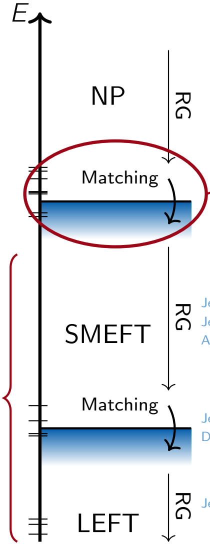

MATCHETE

Carmona *et al.* [2112.10787];
Fuentes-Martín, König, Pagès, **AET**, Wilsch [2212.04510]

Jenkins, Manohar, Trott [1308.2627]
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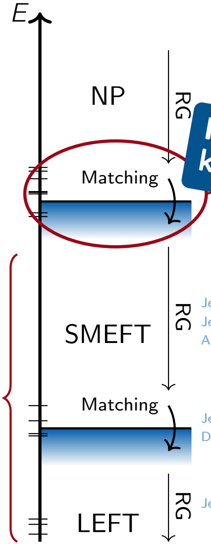

How to handle broken gauge theories?

Fuentes-Martín,
Wilsch [2212.04510]

Jenkins, Manohar, Trott [1308.2627]
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The repetitive nature of EFT computations call for **automated tools!**

On-shell EFT matching

Given

$$\mathcal{L}_{\mathrm{UV}}[\Phi, \phi] = \mathcal{L}_{\mathrm{kin}}[\Phi, \phi] + \sum_{a} g_a \, Q_a[\Phi, \phi] \qquad M_\Phi \sim \Lambda \gg m_\phi$$

Heavy fields

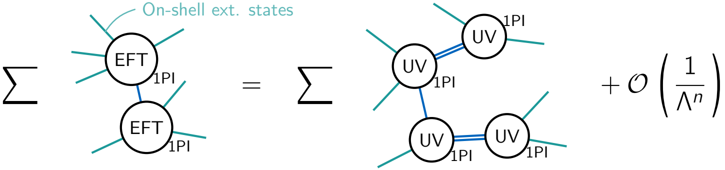
determine

$$\mathcal{L}_{\mathrm{EFT}}[\phi] = \mathcal{L}_{\mathrm{kin}}[\phi] + \sum_{k} C_k(g) \, \mathcal{O}_k[\phi]$$

On-shell EFT matching

Weak (physical) matching condition

$$\langle f | S^{\mathrm{EFT}}_{\mathrm{mat.}} | i \rangle = \langle f | S^{\mathrm{UV}}_{\mathrm{mat.}} | i \rangle_{+\mathcal{O}(\Lambda^{-n},\, (16\pi^2)^{-\ell})}, \qquad \forall i, f \in \{\text{low energy}\}$$



Given $\quad \mathcal{L}_{\mathrm{UV}}[\Phi, \phi] = \mathcal{L}_{\mathrm{kin}}[\Phi, \phi] + \sum_a g_a\, Q_a[\Phi, \phi] \qquad M_\Phi \sim \Lambda \gg m_\phi$

Heavy fields

determine $\quad \mathcal{L}_{\mathrm{EFT}}[\phi] = \mathcal{L}_{\mathrm{kin}}[\phi] + \sum_k C_k(g)\, \mathcal{O}_k[\phi]$

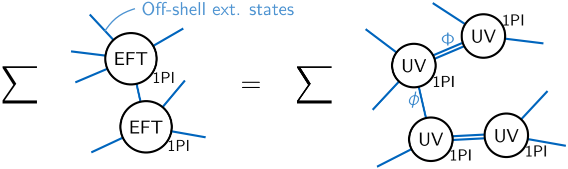
- Physical condition (works whenever decoupling is possible)
- Multiple solutions for $C_k(g)$: it is surprisingly difficult to determine an EFT basis
- **Challenging to compute on-shell matrix element!**

Off-shell matching

Strong matching condition

$$\mathcal{W}_{\text{EFT}}[J_\phi] = \mathcal{W}_{\text{UV}}[J_\Phi = 0,\ J_\phi]$$

*The vacuum functional $\mathcal{W}$ generates all connected Green's functions



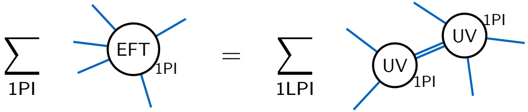
- Unphysical matching condition (it is stronger than needed)
- Strong matching condition $\implies$ weak condition
- Non-trivial that a solution exists: **Green's functions depend on gauge choice**

Off-shell matching

Strong matching condition (equivalent)

$$\Gamma_{\mathsf{EFT}}[\hat{\phi}] = \Gamma_{\mathsf{UV}}\big[\widehat{\Phi}[\hat{\phi}],\, \hat{\phi}\big], \qquad 0 = \frac{\delta\Gamma_{\mathsf{UV}}}{\delta\Phi}\big[\widehat{\Phi}[\hat{\phi}],\, \hat{\phi}\big]$$

*The quantum effective action Γ generates all 1PI Green's functions



- Reduced number of diagrams
- We may directly solve for $S_{\mathsf{EFT}} \subset \Gamma_{\mathsf{EFT}}$

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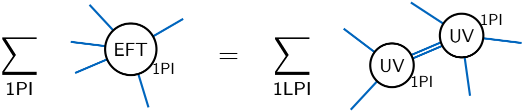

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At tree-level we may **integrate out the heavy fields** with their EOM solution

$$S^{(0)}_{\mathrm{EFT}}[\phi] = S_{\mathrm{UV}}\big[\widehat{\Phi}[\phi],\, \phi\big], \qquad \frac{\delta S_{\mathrm{UV}}}{\delta \Phi}\big[\widehat{\Phi}[\phi],\, \phi\big] = 0$$

Off-shell matching

Strong matching condition (equivalent)

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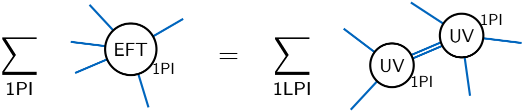

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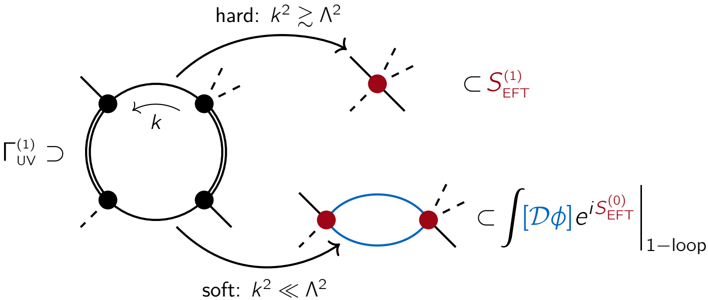
What about loop-level matching?

Separation of scales

Decomposition of UV loops:



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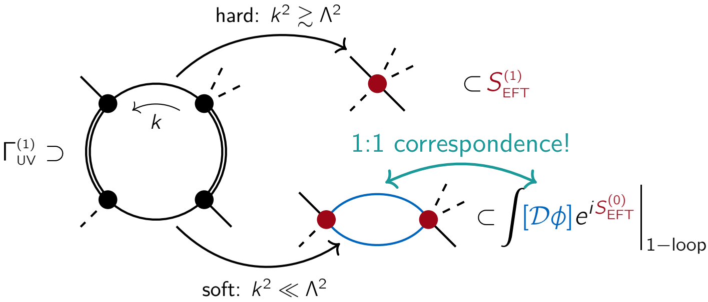

Hard-region matching formula

$$S_{\text{EFT}}[\phi] = \Gamma_{\text{UV}}[\widehat{\Phi}, \phi]\Big|_{\text{hard}}, \qquad \frac{\delta \Gamma_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\widehat{\Phi}, \phi] = 0$$

"hard" denotes the part without *any* soft loop momenta (it includes all tree-level contributions)

Fuentes-Martin, Palavrić, **AET** [2311.13630]

*Generalization of Fuentes-Martin *et al.* [1607.02142]; Zhang [1610.00710]

Matching gauge theories

Consider a gauge theory with **gauge group *G***

$$S_{\text{UV}}[\eta_g] = S_{\text{UV}}[\eta], \qquad \forall g \in G$$

With no Higgs-mechanism, we are looking for a low-energy EFT action with the same symmetry

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What about the hard-region matching formula?

$$\underset{G\text{-inv.}}{S_{\text{EFT}}[\phi]} \overset{?}{=} \overset{\text{BRST invariant?}}{\Gamma_{\text{UV}}[\hat{\eta}]\Big|_{\text{hard}}}, \qquad \frac{\delta \Gamma_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\hat{\eta}] = 0$$

Γ_{UV} loses G invariance for the smaller BRST invariance with ordinary gauge-fixing

Background field gauge

Background field method: $\neq$ background field gauge

$$\Gamma[\overline{A}] = -i \log \int \mathcal{D}A \, \mathcal{D}\overset{\text{(anti-)ghosts}}{\boldsymbol{\omega}} \, \exp\left[i \left(S[A + \overline{A}] + S^{G}_{\text{fix}}[A + \overline{A}, \, \boldsymbol{\omega}, \, \Theta] + \int_x J^{\mu}_{A} A^{A}_{\mu} \right) \right]$$

Generalization of the R_ξ gauges:

$$S^{G}_{\text{fix}}[A, \, \boldsymbol{\omega}, \, \Theta] = - \int_x \left(\frac{1}{2\xi} \left[F^{A}_{\mu B}[\Theta] (A - \Theta)^{B}_{\mu} \right]^2 + \overline{\omega}_A F^{A}_{\mu B}[\Theta] D^{\mu}[A] \omega^B \right)$$

$$F^{A}_{\mu B}[\Theta] = \delta^{A}{}_{B} \partial_\mu + f^{A}{}_{CB} \Theta^{C}_{\mu}$$

Background field gauge

Background field method: $\neq$ background field gauge

$$\Gamma[\overline{A}] = -i \log \int \mathcal{D}A\, \overset{\text{(anti-)ghosts}}{\mathcal{D}\boldsymbol{\omega}}\, \exp\left[i \left(S[A+\overline{A}] + S^{G}_{\text{fix}}[A+\overline{A},\, \boldsymbol{\omega},\, \Theta] + \int_x J^{\mu}_{A} A^{A}_{\mu} \right) \right]$$

Generalization of the R_ξ gauges:

$$S^{G}_{\text{fix}}[A,\, \boldsymbol{\omega},\, \Theta] = -\int_x \left(\frac{1}{2\xi} \left[F^{A}_{\mu B}[\Theta](A-\Theta)^{B}_{\mu} \right]^2 + \overline{\omega}_A F^{A}_{\mu B}[\Theta] D^{\mu}[A] \omega^{B} \right)$$

$$F^{A}_{\mu B}[\Theta] = \delta^{A}{}_{B} \partial_\mu + f^{A}{}_{CB} \Theta^{C}_{\mu}$$

Background field gauge: it is more than a mere gauge

$$S^{G}_{\text{fix}}[A+\overline{A},\, \boldsymbol{\omega},\, \overline{A}] = -\int_x \left(\frac{1}{2\xi} \left[D_\mu[\overline{A}] A^{A}_{\mu} \right]^2 + \overline{\omega}_A D_\mu[\overline{A}] D^{\mu}[A+\overline{A}] \omega^{B} \right)$$

bkg. G invariance: $\quad \overline{\delta}_\alpha \overline{A}^{A}_{\mu} = D_\mu[\overline{A}]\alpha^{A}, \quad \overline{\delta}_\alpha A^{A}_{\mu} = -f^{A}{}_{BC} \alpha^{B} A^{C}_{\mu}$

Gauge-invariant effective action

Gauge-invariant effective action of the **background field (BF) gauge**

$$\overline{\Gamma}[\overline{\eta}] = -i \log \int \mathcal{D}\eta\, \mathcal{D}\underbrace{\boldsymbol{\omega}}_{\text{(anti-)ghosts}} \exp\Bigg[i \Bigg(S[\eta + \overline{\eta}] + \overset{\text{Gauge-fixing } \eta \text{ (quantum) using } \overline{\eta} \text{ (bkg.)}}{S^{G}_{\text{fix}}[\eta + \overline{\eta},\, \boldsymbol{\omega},\, \overline{\eta}]} + \int_x J_I \eta^I \Bigg) \Bigg]$$

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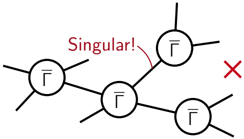
$\overline{\Gamma}[\overline{\eta}]$ Not the Legendre trans. of a vacuum functional!



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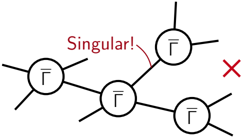
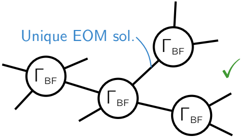


$$\Gamma_{\text{BF}}[\overline{\eta}] = \overline{\Gamma}[\overline{\eta}] + S^G_{\text{bkg.}}[\overline{\eta}]$$



Abbott *et al.* '83; Hart '83; Rebhan, Wirthumer '84

Vacuum functional is constructed with **gauge-fixed bkg. fields**:

$$\mathcal{W}_{\text{BF}}[\overline{J}] = \Gamma_{\text{BF}}[\overline{\eta}] + \int_x \overline{J}_I\overline{\eta}^I, \qquad \overline{J}_I = -\frac{\delta\Gamma_{\text{BF}}[\overline{\eta}]}{\delta\overline{\eta}^I},$$

$$\Gamma_{\text{BF}} \xrightarrow{\text{Legendre trans.}} \mathcal{W}_{\text{BF}} \xrightarrow{\text{on-shell}} S\text{-matrix}$$

Matching in an ordinary BF gauge

A version of the strong matching condition is

$$\Gamma^{\mathrm{EFT}}_{\mathrm{BF}}[\overline{\phi}] \;=\; \Gamma^{\mathrm{UV}}_{\mathrm{BF}}[\overline{\eta}], \qquad\qquad \frac{\delta\Gamma^{\mathrm{UV}}_{\mathrm{BF}}}{\delta\Phi}[\overline{\eta}] = 0$$

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Choosing identical $S^G_{\text{bkg.}}[\overline{\phi}]$ for UV and EFT yields

$$\overline{\Gamma}_{\text{EFT}}[\overline{\phi}] \;=\; \overline{\Gamma}_{\text{UV}}[\overline{\eta}], \qquad\qquad \frac{\delta\overset{S^G_{\text{bkg.}}\text{ is }\Phi\text{ independent}}{\overline{\Gamma}_{\text{UV}}}}{\delta\Phi}[\overline{\eta}] = 0$$

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$S^{G}_{\text{bkg.}}$ is Φ independent

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Quantum gauge-fixing can also be chosen identically for UV and EFT:
we can demonstrate a 1:1 correspondence of soft-region loops

Hard-region matching in unbroken gauge theories

$$S_{\text{EFT}}[\overline{\phi}] \; = \; \underset{\text{GIEA!}}{\overline{\Gamma}_{\text{UV}}}[\overline{\eta}]\Big|_{\text{hard}}, \qquad \frac{\delta \overline{\Gamma}_{\text{UV}}|_{\text{hard}}}{\delta \Phi}[\overline{\eta}] = 0$$

AET [2404.11640]

See also Henning, Lu, Murayama [1412.1837]; Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

SSB in BSM

- Many BSM scenarios involve **spontaneously broken gauge symmetries**
- Popular patterns include
  - $\mathrm{SU}(4) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SU}(4) \times \mathrm{SU}(3) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1) \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SU}(5), \mathrm{SO}(10) \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SU}(2)_{12} \times \mathrm{SU}(2)_3 \longrightarrow \mathrm{SU}(2)_\mathrm{L}$
  - $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_\mathrm{em}$
- Generically, the gauge group G is broken to a smaller group H in the IR
- Matching must accommodate the reduction in symmetry

Spontaneous symmetry breaking

A scalar VEV breaks $G \to H$:

$$\langle \varphi'^a \rangle = v^a \neq 0, \qquad \varphi'^a \subset \eta^I$$

The G-covariant derivative decomposes as

Gauge bosons of H

$$D_\mu = d_\mu - iV_\mu^i x_i, \qquad d_\mu = \partial_\mu - iB_\mu^\alpha t_\alpha$$

Massive gauge bosons

All types may contain multiple irreducible representations of H

and the scalars as

Would-be GBs

$$\varphi'^a = v^a + \varphi^a, \qquad \varphi^a = \varphi_h^a + \chi^i M_i^{-1} f_i{}^a$$

“Higgs” fields

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“Higgs” fields

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The scalar kinetic term gives masses to the V^i_μ gauge bosons:

$$\tfrac{1}{2} D_\mu \big(\varphi'^a\big)^2 = \tfrac{1}{2} d_\mu \varphi^a_h h_{ab} d^\mu \varphi^b_h + \tfrac{1}{2} d_\mu \chi_i d^\mu \chi^i + \tfrac{1}{2} M_i^2 V_i^\mu V^i_\mu + d^\mu \chi_i M_i V^i_\mu$$
$$+ \tfrac{1}{2} i V^i_\mu (\varphi_a x^a_{ib} \overleftrightarrow{d}^\mu \varphi^b) + \tfrac{1}{2} V^i_\mu V^{j\mu} \varphi_a (x_i x_j \varphi)^a - i V^i_\mu V^{j\mu} f_{ia} x^a_{jb} \varphi^b$$

And much more algebra...

Background field gauge

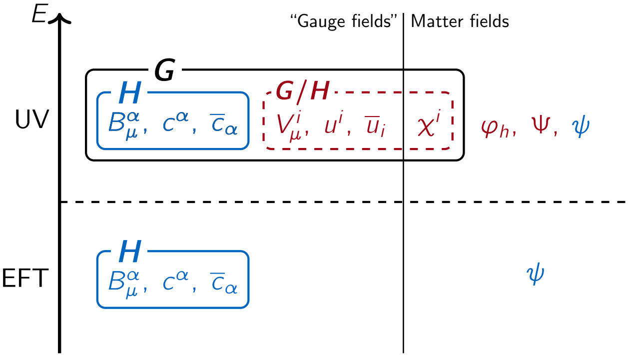
The gauge-fixing terms in ordinary BF gauge for $G \to H$ gauge theories

$$
\begin{aligned}
\mathcal{L}^{G}_{\text{vec.}} =& -\frac{1}{2\xi}\hat{a}^{-1}_{\alpha\beta}\overline{d}^{\mu}B^{\alpha}_{\mu}\overline{d}^{\nu}B^{\beta}_{\nu} - \frac{1}{2\xi}\overline{d}^{\mu}V^{i}_{\mu}\overline{d}_{\nu}V^{\nu}_{i} + M_{i}\chi_{i}\overline{d}^{\mu}V^{i}_{\mu} - \frac{\xi}{2}M_{i}^{2}\chi_{i}\chi^{i} - \frac{1}{\xi}\overline{d}^{\mu}B^{\alpha}_{\mu}\hat{a}^{-1}_{\alpha\beta}f^{\beta}{}_{jk}\overline{V}^{j}_{\nu}V^{k\nu} \\
& -\frac{1}{2\xi}\hat{a}^{-1}_{\alpha\beta}f^{\alpha}{}_{ij}\overline{V}^{i}_{\mu}V^{j\mu}f^{\beta}{}_{k\ell}\overline{V}^{k}_{\nu}V^{\ell\nu} + (\overline{d}^{\mu}B^{\alpha}_{\mu} + f^{\alpha}{}_{ij}\overline{V}^{i}_{\mu}V^{j\mu})(\chi_{k}f^{k}{}_{\alpha\ell}\overline{\chi}^{\ell} - i\varphi_{h\,a}t^{a}_{\alpha b}\overline{\varphi}^{b}_{h}) \\
& -\frac{\xi}{2}\hat{a}^{\alpha\beta}(\chi_{i}f^{i}{}_{\alpha j}\overline{\chi}^{j} - i\varphi_{h\,a}t^{a}_{\alpha b}\overline{\varphi}^{b}_{h})(\chi_{k}f^{k}{}_{\beta\ell}\overline{\chi}^{\ell} - i\varphi_{h\,c}t^{a}_{\beta d}\overline{\varphi}^{d}_{h}) - \frac{1}{\xi}\overline{d}^{\mu}V^{i}_{\mu}\overline{V}^{j}_{\nu}(f_{ij\alpha}B^{\alpha\nu} + f_{ijk}V^{k\nu}) \\
& -\frac{1}{2\xi}\overline{V}^{j}_{\mu}\overline{V}^{\ell}_{\nu}(f_{ij\alpha}B^{\alpha\mu} + f_{ijk}V^{k\mu})(f^{i}{}_{\ell\beta}B^{\beta\nu} + f^{i}{}_{\ell m}V^{m\nu}) + \frac{\xi}{2}(\varphi_{a}x^{a}_{ib}\overline{\varphi}^{b})\kappa^{ij}(\varphi_{c}x^{c}_{jd}\overline{\varphi}^{d}) \\
& +(M_{i}\chi_{i} - i\varphi_{a}x^{a}_{ib}\overline{\varphi}^{b})\overline{V}^{j}_{\mu}(f^{i}{}_{j\alpha}B^{\alpha\mu} + f^{i}{}_{jk}V^{k\mu}) - i\overline{d}^{\mu}V^{i}_{\mu}\varphi_{a}x^{a}_{ib}\overline{\varphi}^{b} + i\xi M_{i}\chi^{i}\varphi_{a}x^{a}_{ib}\overline{\varphi}^{b},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{G}_{\text{gh.}} =& -\overline{c}_{\alpha}\overline{d}^{2}c^{\alpha} - \overline{u}_{i}\overline{d}^{2}u^{i} + \overline{d}^{\mu}\overline{c}_{\alpha}f^{\alpha}{}_{\beta\gamma}B^{\beta}_{\mu}c^{\gamma} + \overline{d}^{\mu}\overline{u}_{i}f^{i}{}_{\alpha j}B^{\alpha}_{\mu}u^{j} + \overline{d}^{\mu}\overline{c}_{\alpha}f^{\alpha}{}_{ij}(\overline{V}^{i}_{\mu} + V^{i}_{\mu})u^{j} \\
& +\overline{d}^{\mu}\overline{u}_{i}(\overline{V}^{j}_{\mu} + V^{j}_{\mu})(f^{i}{}_{j\alpha}c^{\alpha} + f^{i}{}_{jk}u^{k}) - \overline{c}_{\alpha}f^{\alpha}{}_{ij}\overline{V}^{i}_{\mu}\overline{d}^{\mu}u^{j} - \overline{u}_{i}\overline{V}^{j}_{\mu}(f^{i}{}_{j\alpha}\overline{d}^{\mu}c^{\alpha} + f^{i}{}_{jk}\overline{d}^{\mu}u^{k}) \\
& -\overline{c}_{\alpha}f^{\alpha}{}_{ij}f^{j}{}_{\alpha k}\overline{V}^{i}_{\mu}B^{\alpha\mu}u^{k} - \overline{u}_{i}\overline{V}^{j}_{\mu}B^{\beta\mu}(f^{i}{}_{j\alpha}f^{\alpha}{}_{\beta\gamma}c^{\gamma} + f^{i}{}_{jk}f^{k}{}_{\beta\ell}u^{\ell}) \\
& -\overline{V}^{k\mu}(\overline{V}^{\ell}_{\mu} + V^{\ell}_{\mu})[\overline{c}_{\alpha}f^{\alpha}{}_{km}f^{m}{}_{\ell\beta}c^{\beta} + \overline{c}_{\alpha}f^{\alpha}{}_{km}f^{m}{}_{\ell j}u^{j} + \overline{u}_{i}f^{i}{}_{km}f^{m}{}_{\ell\beta}c^{\beta} + \overline{u}_{i}(f^{i}{}_{k\alpha}f^{\alpha}{}_{\ell j} + f^{i}{}_{km}f^{m}{}_{\ell j})u^{j}] \\
& +\xi\overline{c}_{\alpha}\hat{a}^{\alpha\beta}[\overline{\chi}_{i}f^{i}{}_{\beta j}f^{j}{}_{\gamma k}(\overline{\chi} + \chi)^{k} - \overline{\varphi}_{h\,a}(t_{\alpha}t_{\beta})^{a}{}_{b}(\overline{\varphi}_{h} + \varphi_{h})^{b}]c^{\gamma} \\
& +\xi\overline{c}_{\alpha}\hat{a}^{\alpha\beta}[M_{j}\chi_{j}f^{j}{}_{\alpha i} - \overline{\varphi}_{a}(t_{\alpha}x_{i})^{a}{}_{b}(\overline{\varphi} + \varphi)^{b}]u^{i} \\
& -\xi\overline{u}_{i}[f^{i}{}_{\alpha j}M_{j}(\overline{\chi} + \chi)^{j} + \overline{\varphi}_{a}(x^{i}t_{\alpha})^{a}{}_{b}(\overline{\varphi} + \varphi)^{b}]c^{\alpha} \\
& -\xi\overline{u}_{i}[\delta^{i}{}_{j}M_{j}^{2} - if^{i}{}_{a}x^{a}_{jb}(\overline{\varphi} + \varphi)^{b} - if_{ja}\kappa^{ik}x^{a}_{kb}\overline{\varphi}^{b} + \overline{\varphi}_{a}(x^{i}x_{j})^{a}{}_{b}(\overline{\varphi} + \varphi)^{b}]u^{j}
\end{aligned}
$$

quantum : $B^{\alpha}_{\mu},\ V^{i}_{\mu},\ \varphi^{a}_{h},\ \chi^{i}$ background : $\overline{B}^{\alpha}_{\mu},\ \overline{V}^{i}_{\mu},\ \overline{\varphi}^{a}_{h},\ \overline{\chi}^{i}$

Decoupling



Matching a spontaneously broken (Higgsed) gauge theory ($G \to H$)

$$S_{\text{UV}}[\eta_g] = S_{\text{UV}}[\eta], \qquad \forall g \in G$$

$$S_{\text{EFT}}[\phi_h] = S_{\text{EFT}}[\phi], \qquad \forall h \in H \subseteq G$$

Matching spontaneously broken gauge theories

What happens to matching with the BF gauge?

$$\underbrace{S_{\text{EFT}}}_{H\text{-inv.}}[\phi] \overset{?}{=} \overset{G\text{-inv.}}{\overline{\Gamma}}_{\text{UV}}[\overline{\eta}]\Big|_{\text{hard}} \qquad\qquad \frac{\delta \overline{\Gamma}_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\overline{\eta}] = 0$$

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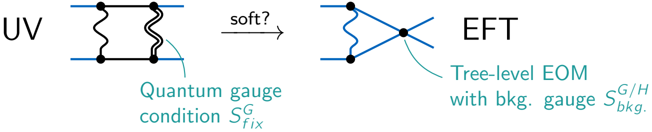
What about the **1:1 correspondence** between soft UV and EFT loops?



Quantum gauge condition S^{G}_{fix}

Tree-level EOM with bkg. gauge $S^{G/H}_{bkg.}$

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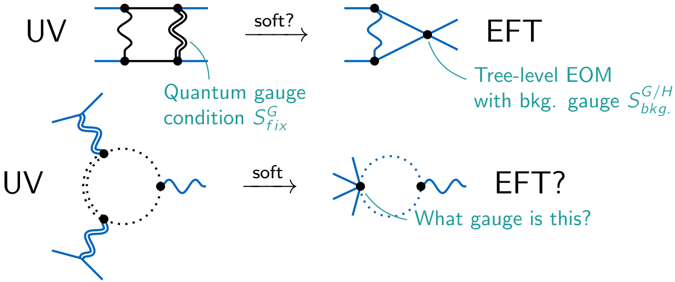

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Tree-level EOM with bkg. gauge $S^{G/H}_{bkg.}$

What gauge is this?

A 1:1 cancellation would depend intricately on S^{H}_{fix} (EFT), S^{G}_{fix} (UV), and $S^{G/H}_{\text{bkg.}}$

Matching spontaneously broken gauge theories

What happens to matching with the BF gauge?

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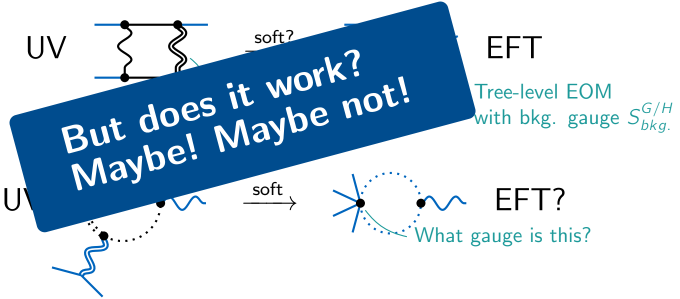

Tree-level EOM with bkg. gauge $S^{G/H}_{bkg.}$

What gauge is this?

But does it work?
Maybe! Maybe not!

A 1:1 cancellation would depend intricately on S^{H}_{fix} (EFT), S^{G}_{fix} (UV), and $S^{G/H}_{\text{bkg.}}$

Partial gauge fixing

Gauge-fixing a la Faddeev–Popov

$$Z = \int \mathcal{D}\eta \, e^{iS[\eta]} = \int \mathcal{D}\eta \, \delta\big(\mathcal{G}^A[\eta]\big) \, \mathrm{Det}\big(\mathcal{G}^A{}_{,I}[\eta] D^I{}_B[\eta]\big) e^{iS[\eta]}$$

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Factorization of the gauge-fixing condition:

Weinberg ‘80

$$G \to H, \qquad A^A_\mu = (\overset{H\text{ gauge fields}}{B^\alpha_\mu},\, V^i_\mu), \qquad \mathcal{G}^A[\eta] = \big(\mathcal{G}^\alpha[\eta],\, \underset{H\text{-covariant}}{\mathcal{G}^i}[\eta]\big)$$

$$Z = \int \mathcal{D}\eta\, \delta\big(\mathcal{G}^\alpha[\eta]\big)\delta\big(\mathcal{G}^i[\eta]\big)\, \mathrm{Det}\begin{pmatrix} \mathcal{G}^\alpha{}_{,I}[\eta] D^I{}_\beta[\eta] & \mathcal{G}^\alpha{}_{,I}[\eta] D^I{}_j[\eta] \\ \cancelto{0}{-f^i{}_{\beta k}\mathcal{G}^k} & \mathcal{G}^i{}_{,I}[\eta] D^I{}_j[\eta] \end{pmatrix} e^{iS[\eta]}$$

Partial gauge fixing

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Clever manipulations and the introduction of an auxiliary field yields

$$Z=\int\mathcal{D}\eta\mathcal{D}\mathbf{u}\,\underbrace{\delta\big(\mathcal{G}^\alpha[\eta]\big)\,\mathrm{Det}\big(\mathcal{G}^\alpha{}_{,I}[\eta]D^I{}_\beta[\eta]\big)}_{H\text{ gauge-fixing}}\exp\Big[i\,\Big(\underbrace{S[\eta]+S^{G/H}_{\text{fix}}[\eta,\,\mathbf{u}]}_{H\text{-inv.}}\Big)\Big]$$

where

Ferrari [1308.6802]

$$S^{G/H}_{\text{fix}}[\eta,\,\mathbf{u}]=-\int_x\bigg(\frac{1}{2\zeta}\mathcal{G}_i[\eta]\mathcal{G}^i[\eta]+\overline{u}_i\big(\mathcal{G}^i{}_{,I}[\eta]D^I{}_j[\eta]+f^i{}_{jk}\mathcal{G}^k[\eta]\big)u^j-\frac{\zeta}{2}\hat{a}^{\alpha\beta}f^i{}_{j\alpha}f^k{}_{\ell\beta}\overline{u}_i u^j\overline{u}_k u^\ell\bigg)$$

Partially fixed BF gauge

Proposal: Combine the partial fixing of G/H with a BF gauge for H

$$\overset{\text{bkg. } H \text{ inv.}}{\overline{\Gamma}}[\overline{\eta}] = -i\log\int \mathcal{D}\eta\,\mathcal{D}\mathbf{c}\,\mathcal{D}\mathbf{u}\,\exp\left[i\left(S[\eta+\overline{\eta}]+S^{G/H}_{\text{fix}}[\eta+\overline{\eta},\,\mathbf{u}]+S^{H}_{\text{fix}}[\eta+\overline{\eta},\,\mathbf{c},\,\overline{\eta}]+\int_x J_I\eta^I\right)\right]$$

with the BF effective action

$$\Gamma_{\text{BF}}[\overline{\eta}] = \overline{\Gamma}[\overline{\eta}] + S^{H}_{\text{bg.}}[\overline{\eta}]$$

$\overline{\Gamma}_{\text{UV}}$ of the partially fixed BF gauge possesses the symmetries of S_{EFT}!

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$\overline{\Gamma}_{\text{UV}}$ of the partially fixed BF gauge possesses the symmetries of S_{EFT}!

The gauge-fixing terms of the PFBF gauge are more "manageable:"

$$\mathcal{L}^{G/H}_{\text{vec.}} = -\frac{1}{2\zeta}(d_\mu V^\mu_i)(d^\nu V^i_\nu) + M_i \chi_i d^\mu V^i_\mu - \frac{\zeta}{2} M_i^2 \chi_i \chi^i,$$

$$\mathcal{L}^{G/H}_{\text{gh.}} = -\overline{u}_i (d^2 + \zeta M_i^2) u^i + \overline{u}_i (f^i{}_{jk} V^k_\mu d^\mu + f^i{}_{k\alpha} f^\alpha{}_{\ell j} V^{k\mu} V^\ell_\mu) u^j + \zeta \overline{u}_i (i f^i{}_a x^a_{jb} \varphi^b + f^i{}_{jk} M_k \chi^k) u^j + \frac{\zeta}{2} \hat{a}^{\alpha\beta} f^i{}_{j\alpha} f^k{}_{\ell\beta} \overline{u}_i u^j \overline{u}_k u^\ell,$$

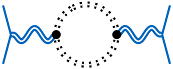
$$\mathcal{L}^{H}_{\text{vec.}} = -\frac{1}{2\xi} \hat{a}^{-1}_{\alpha\beta} \overline{d}^\mu B^\alpha_\mu \overline{d}^\nu B^\beta_\nu, \qquad \mathcal{L}^{H}_{\text{gh.}} = -\overline{c}_\alpha \overline{d}^\mu (\overline{d}_\mu c^\alpha + f^\alpha{}_{\beta\gamma} B^\beta_\mu c^\gamma)$$

Matching with the PFBF gauge

Massive ghosts of the PFBF gauge don't need to be mimicked by any EFT loops

$$\overline{\Gamma}_{\text{UV}} \supset \mathrm{Det}\big(\mathcal{G}^{i}_{,I} D^{I}{}_{j}\big) = \mathrm{Det}\big(\mathcal{G}^{i}_{,I} D^{I}{}_{j}\big)\Big|_{\text{hard}}$$

UV



$$\xrightarrow{\text{soft}} \quad 0 \qquad \text{EFT}$$

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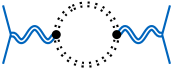

UV $\xrightarrow{\text{soft}}$ 0 EFT

Integrating out heavy vectors at tree-level gives identical vertices to the soft-region of UV loops

Also the GF condition for the quantum fields.

$$S^{(0)}_{\text{EFT}} = S_{\text{UV}} + S^{G/H}_{\text{fix}}, \qquad \frac{\delta(S_{\text{UV}} + S^{G/H}_{\text{fix}})}{\delta\Phi} = 0$$

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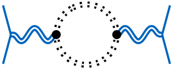
UV  $\xrightarrow{\text{soft}}$ 0 EFT

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Matching broken gauge theories with the PFBF gauge

$$S_{\text{EFT}}[\overline{\phi}] \;=\; \overline{\Gamma}_{\text{UV}}[\overline{\eta}]\Big|_{\text{hard}}, \qquad \frac{\delta\overline{\Gamma}_{\text{UV}}|_{\text{hard}}}{\delta\Phi}[\overline{\eta}] = 0$$

AET [2404.11640]

The soft-region cancellation of the one-loop functional traces can be explicitly demonstrated

Summary

- Practical matching relies on the hard-region matching formula
- The formula can be generalized to unbroken gauge theories with the BF gauge
- The partially fixed BF gauge allows for extension to broken gauge symmetries

Thank you!