Anomalous dimensions from the S-matrix

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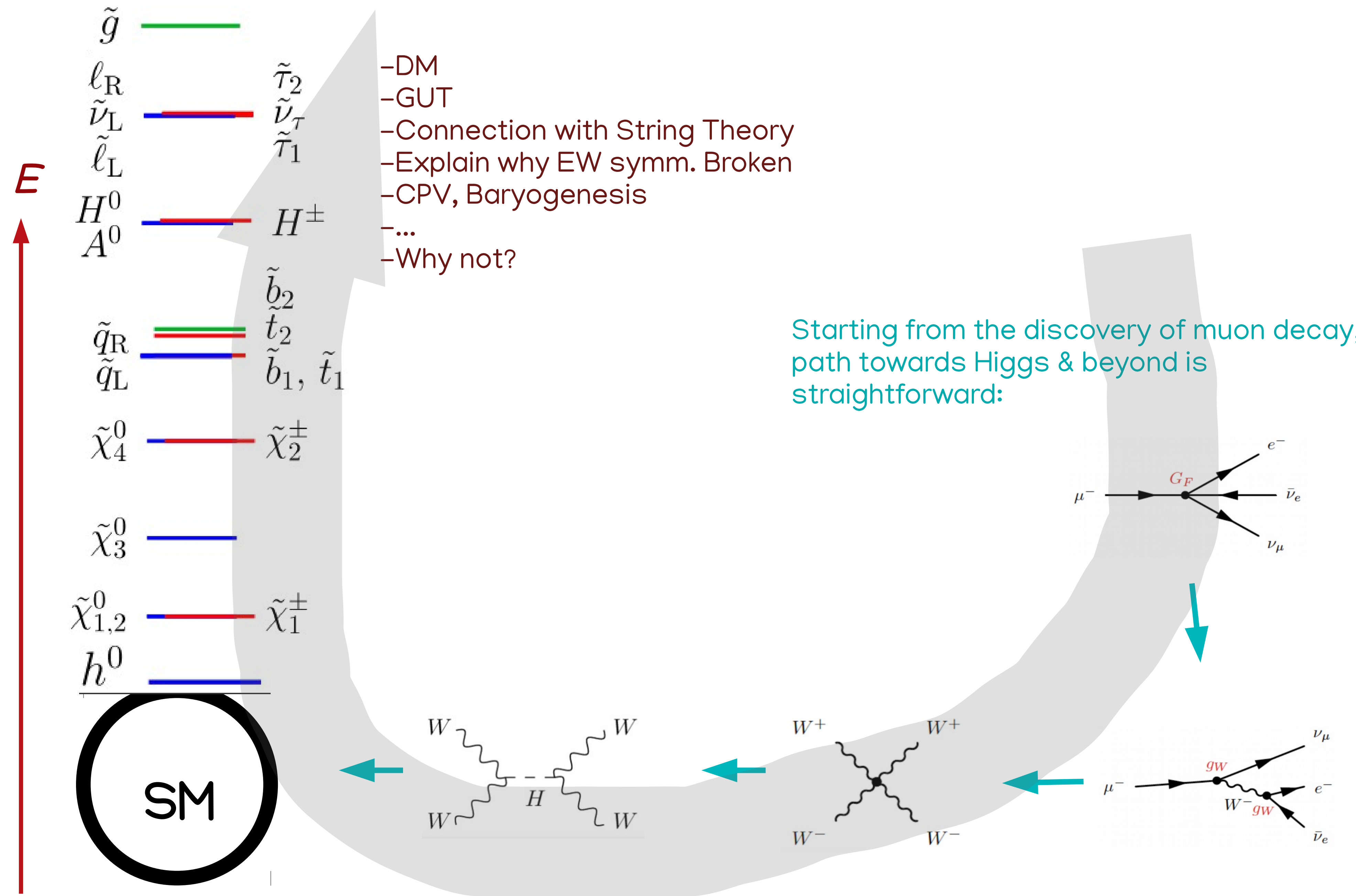
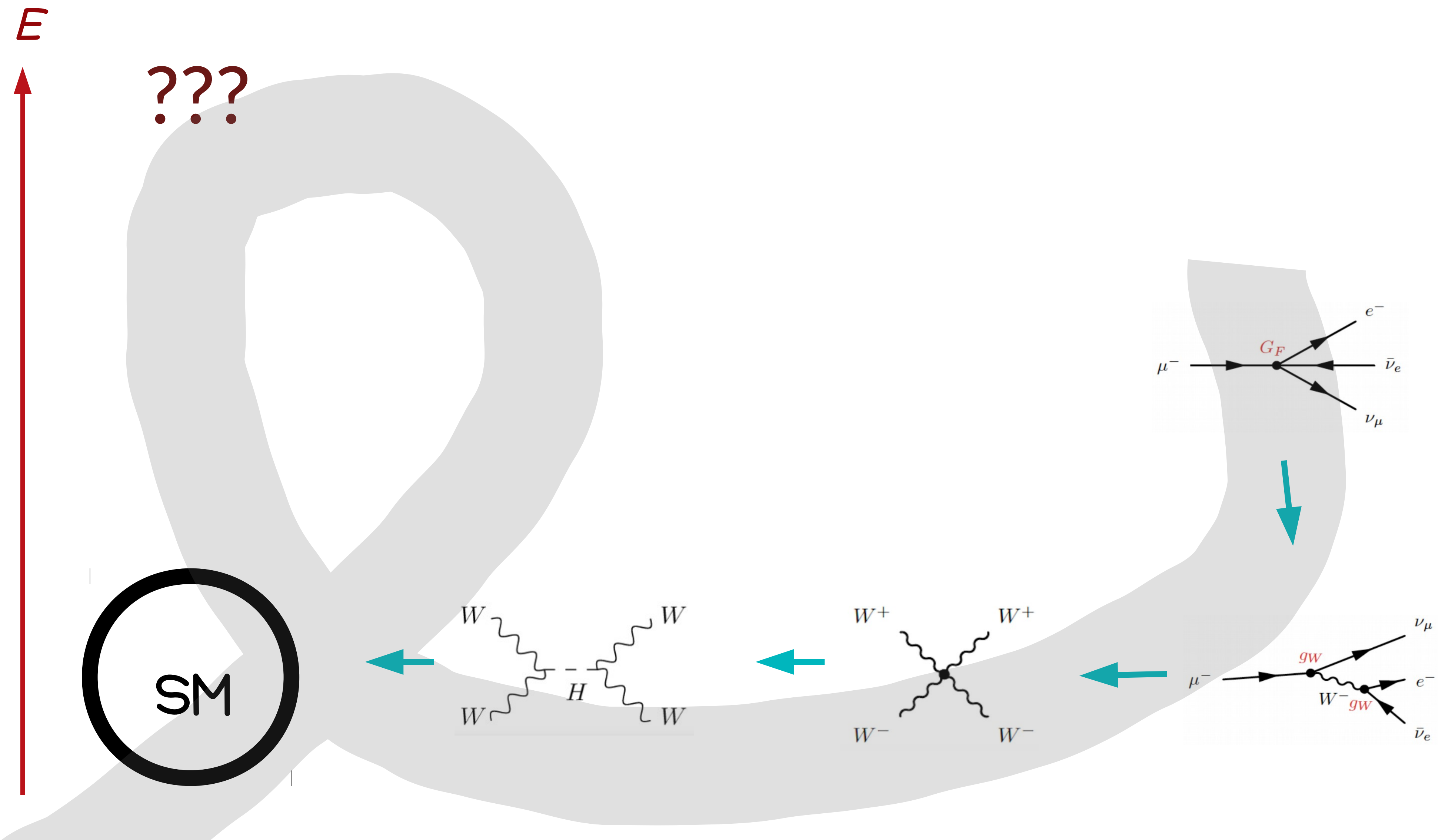
30th September 2020

Based on J. Elias-Miro, J. Ingoldby, MR; 2005.06983

XXth Century particle physics from a XXIst Century perspective:




E
???
SM
W
W
H
W
W
W+
W+
W−
W−
μ−
gW
W−
gW
νμ
e−
ν̄e
μ−
GF
e−
ν̄e
νμ

$$\mathcal{L}\,?$$

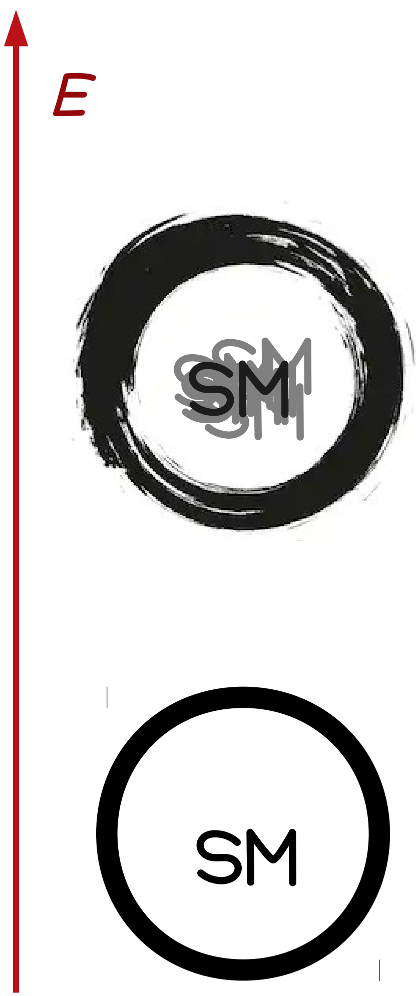
E

Particle Physics is back to the origin, is again the exploration of the unknown.

SM

$$\mathcal{L} = \mathcal{L}_{\rm SM}$$

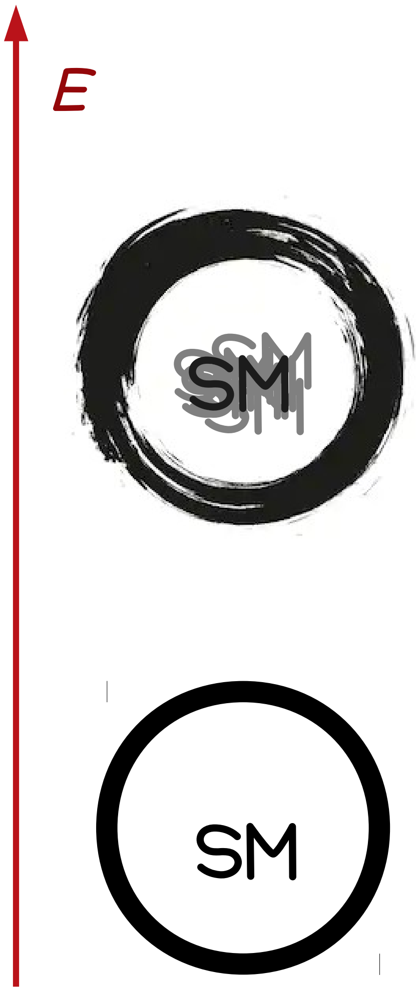
$$\mathcal{L}\,?$$



$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda}\mathcal{O}_i$$

EFT operators encode information about the heavy dynamics, and tells us in which way the SM is deformed.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}}$$



$\mathcal{L}$?

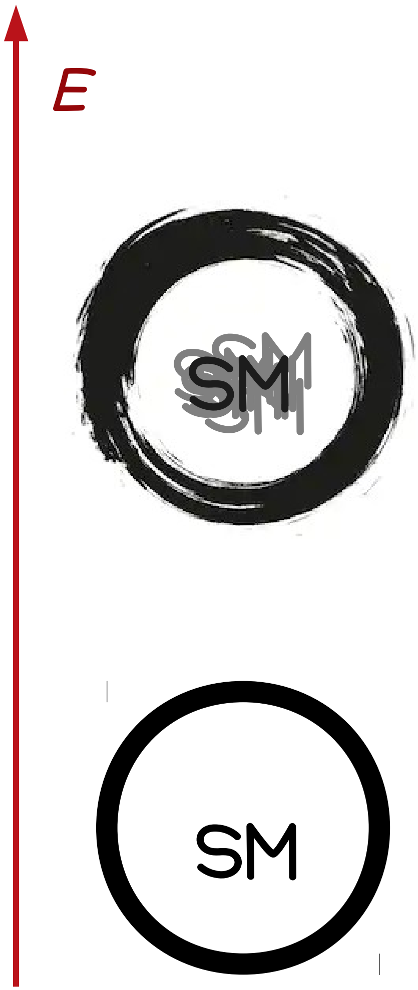
While waiting for the next collider,
we might get some hints from precision experiments.

– Flavour

– mu to e transitions (several orders of mag. improvement)

– EDMs

– ...



$\mathcal{L}$?

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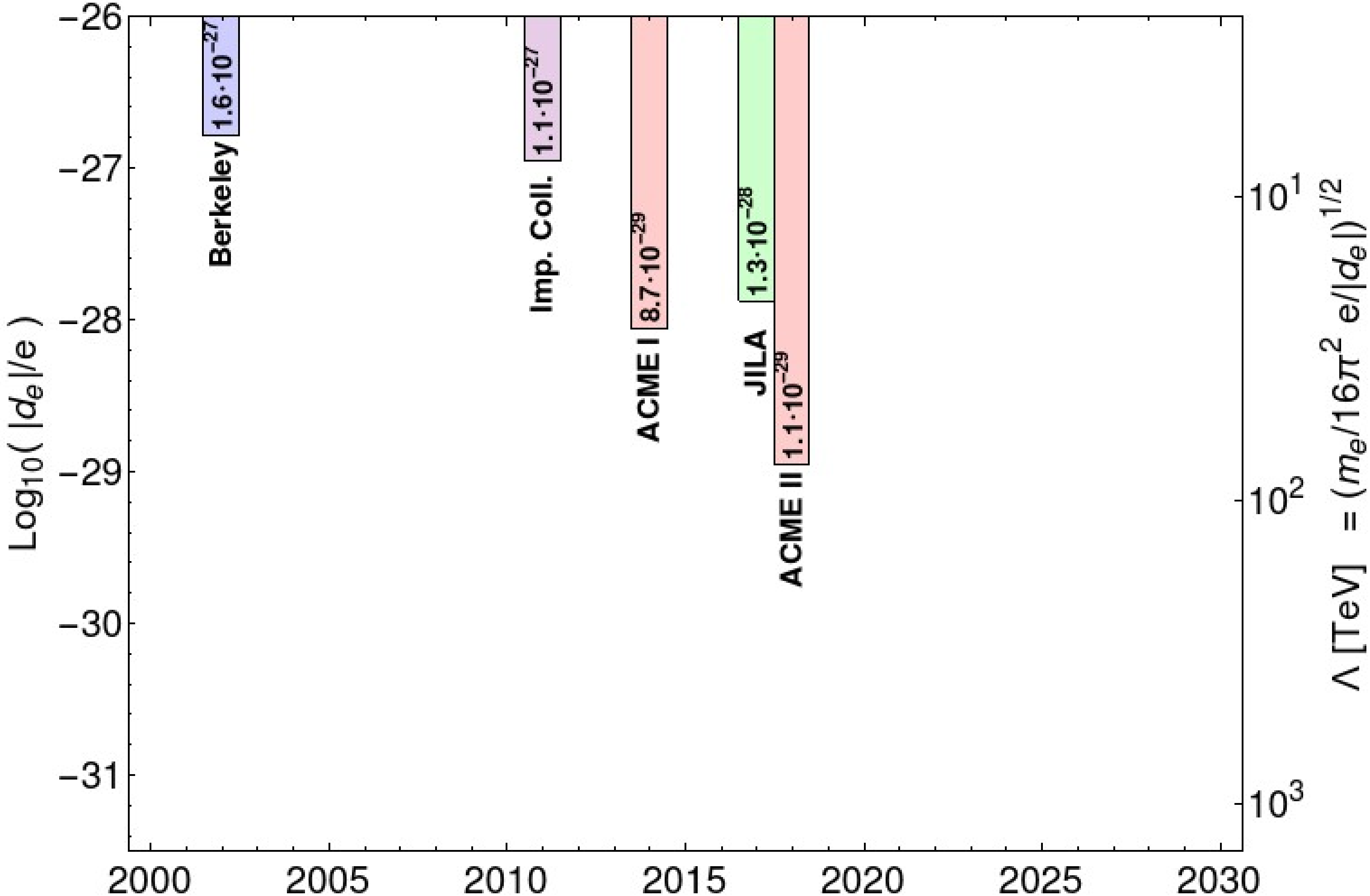
Current bounds already testing dynamics at several TeV
even if only affect the dipole at two loops [Pomarol, Panico, MR]

Precision experiments might explore dynamics at two loops…
but how to get this precision?

This talk is about a new way to compute anomalous dimensions, potentially reaching higher loops.

But before, I will exemplify the potential usefulness of this approach For the case of the electron EDM.

Evolution of electron EDM constraints



Current: ACME II $|d_e| < 1.1 \cdot 10^{-29}$ e cm

Translation of ACME constraints to particle physics:

$$\frac{d_e}{e} \sim \frac{1}{(16\pi^2)^2} \frac{m_e}{\Lambda^2} \qquad \to \Lambda > 3\,\mathrm{TeV}$$

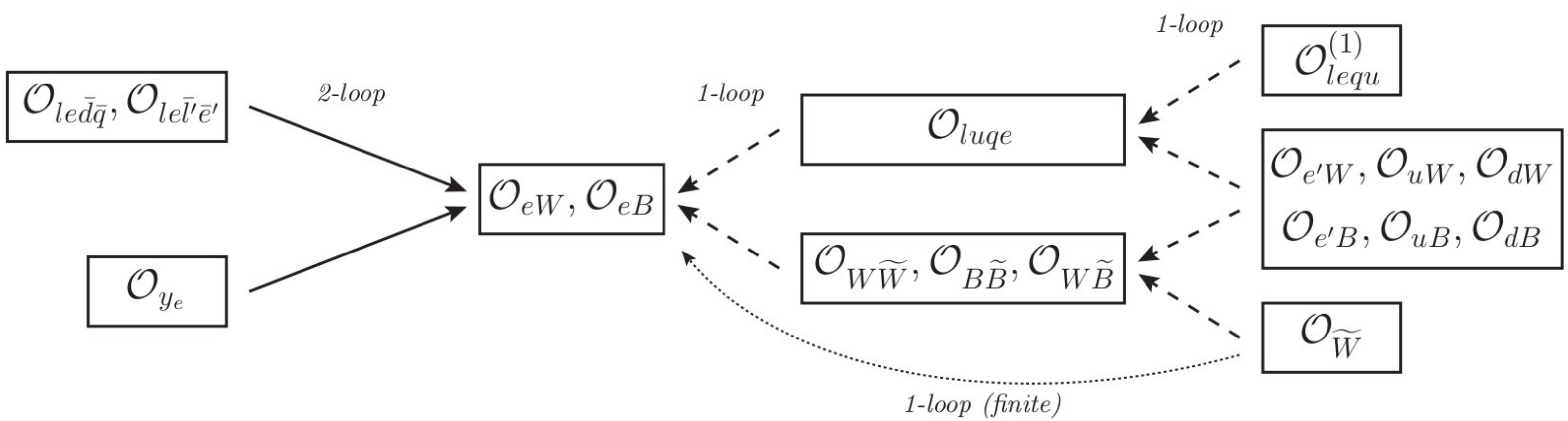
Relevant constraints even at two loops.

We want to characterize all effects that enter with

Two loops   Chirality flip   log enhanced

This is the key to help organize the contributions

G. Panico, A. Pomarol, MR [1810.09413]



ACME–II implications for BSM:

Fix $\Lambda = 10\ TeV$.

tree-level

C_{eW}	$5.5 \times 10^{-5}\, y_e g$
C_{eB}	$5.5 \times 10^{-5}\, y_e g'$

one-loop

C_{luqe}	$1.0 \times 10^{-3}\, y_e y_t$
$C_{W\widetilde{W}}$	$4.7 \times 10^{-3}\, g^2$
$C_{B\widetilde{B}}$	$5.2 \times 10^{-3}\, g'^2$
$C_{W\widetilde{B}}$	$2.4 \times 10^{-3}\, gg'$
$C_{\widetilde{W}}$	$6.4 \times 10^{-2}\, g^3$

two-loops

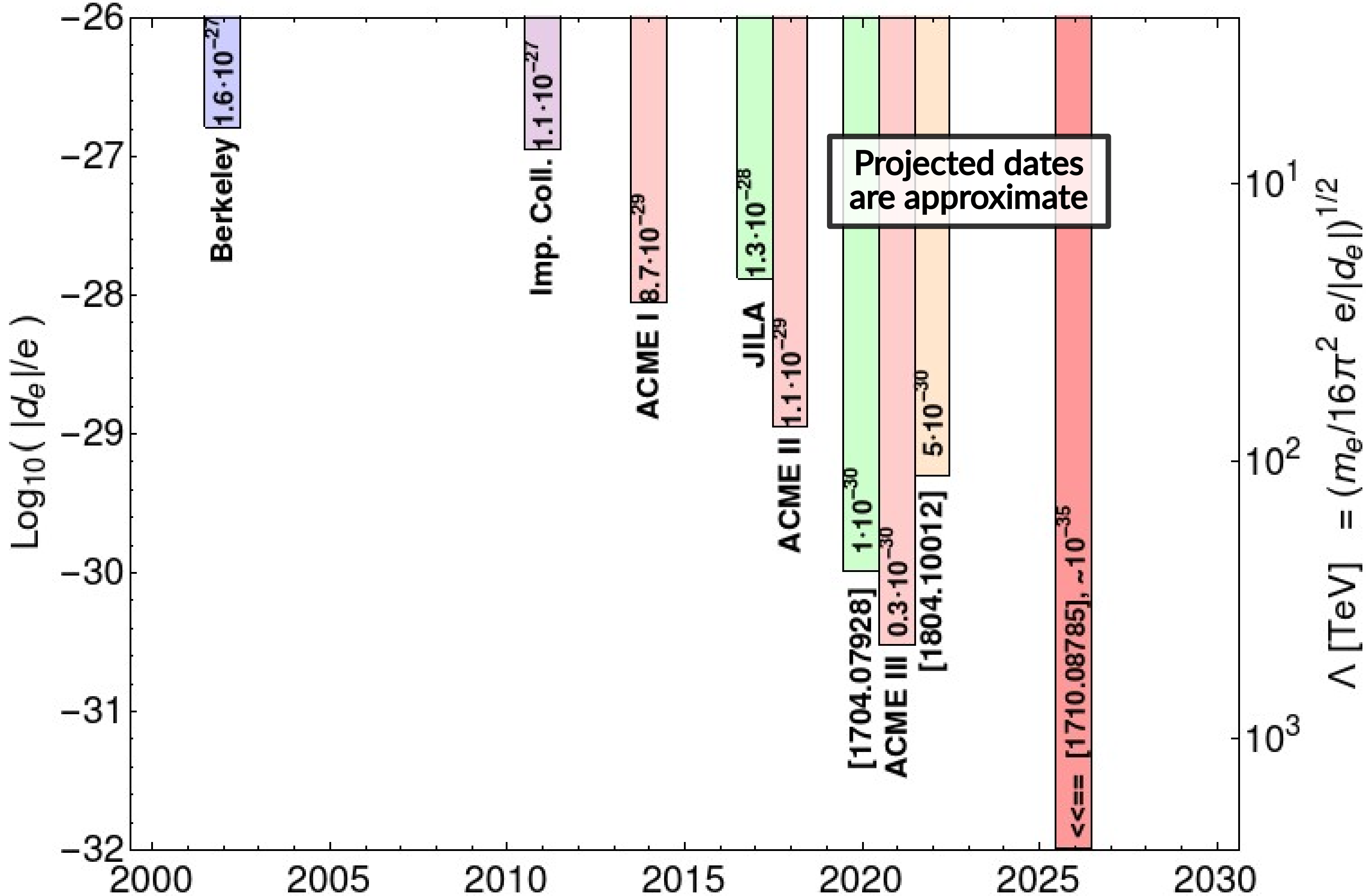
C_{lequ}	$3.8 \times 10^{-2}\, y_e y_t$
$C_{\tau W}$	$260\, y_\tau g$
$C_{\tau B}$	$380\, y_\tau g'$
C_{tW}	$6.9 \times 10^{-3}\, y_t g$
C_{tB}	$1.2 \times 10^{-2}\, y_t g'$
C_{bW}	$64\, y_b g$
C_{bB}	$47\, y_b g'$
$C_{led\bar{q}}$	$10\, y_e y_t (y_t/y_b)$
$C_{le\bar{e}'\bar{l}'}$	$0.63\, y_e y_t (y_t/y_\tau)$

two-loops finite

C_{y_e}	$14\, y_e \lambda_h$
C_{y_t}	$14\, y_t \lambda_h$
C_{y_b}	$2.9 \times 10^3\, y_b \lambda_h$
C_{y_τ}	$3.1 \times 10^4\, y_\tau \lambda_h$

G. Panico, A. Pomarol, MR [1810.09413]

Evolution of electron EDM constraints



- After some time of promises of improvements with nothing happening, it seems that there will be further progress in a short time scale.
- If there is a positive signal, we'll have confirmation very quickly.
- There are some proposals for a total breakthrough.

So two-loop RGE relevant for the electron EDM.

Might be other experiments are also sensitive to some two-loop RGEs

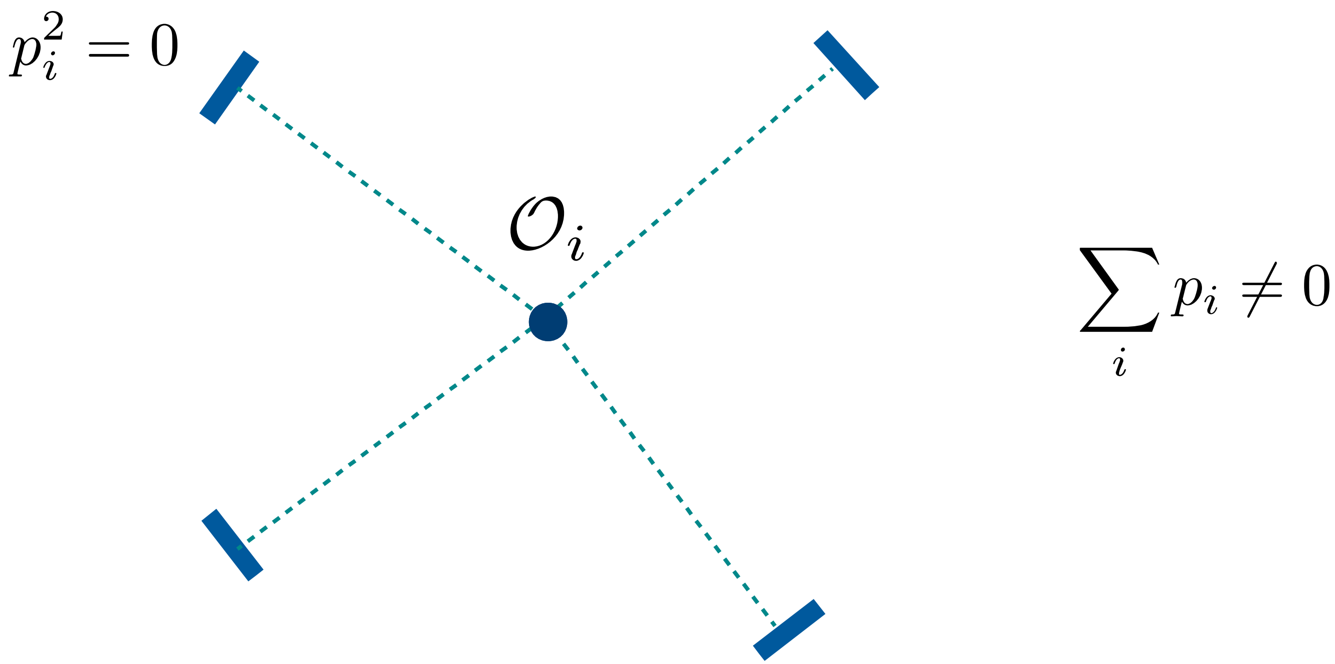
Only if there was a way to compute anomalous dimensions easily...

SMEFT anomalous dimensions from the S-matrix

Joan Elias Miró, James Ingoldby, MR [2005.06983]

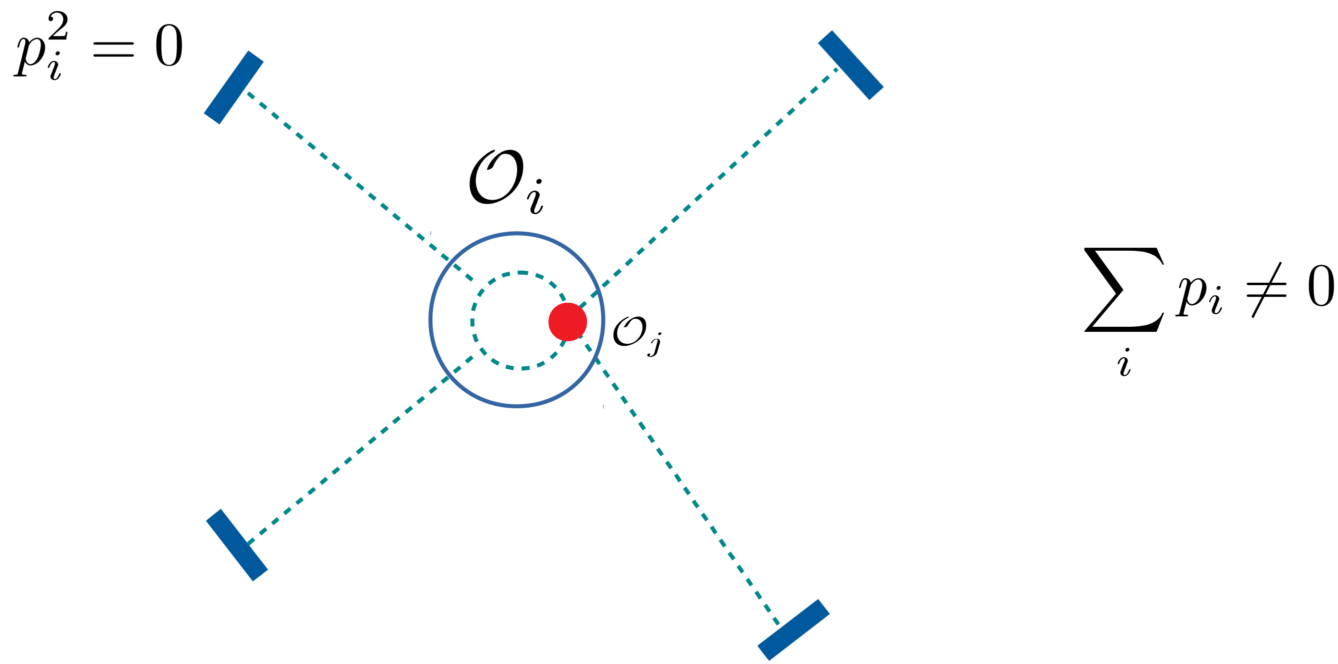
Form Factors of specific operators give the response of a given state once we insert that operator

$$F_{\mathcal{O}_i} = \langle p_1, p_2, p_3, p_4 | \mathcal{O}_i | 0 \rangle \stackrel{\text{e.g.}}{=} \delta_{ab}(s+t)$$



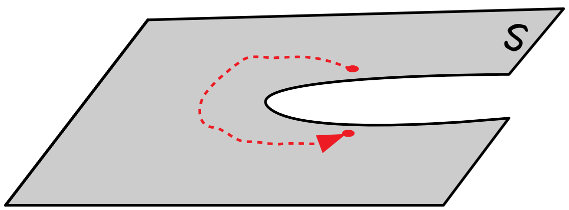
They receive, of course, loop corrections.

$$F_{\mathcal{O}_i} = \langle p_1, p_2, p_3, p_4 | \mathcal{O}_i | 0 \rangle \overset{\text{e.g.}}{=} \delta_{ab}(s+t)\left(1 + c_j \frac{g^2}{16\pi^2} \log \frac{s}{\Lambda}\right)$$



In perturbation theory (and perhaps beyond), a Form Factor is related across both sides of the cut by the **reality condition**

$$F(s_{ij}+i\epsilon)=F^*(s_{ij}-i\epsilon)$$



This is generated by a complex rotation of the momenta,

$$F=e^{-i\pi\sum_i p_i\frac{\partial}{\partial p_i}}F^*=e^{-i\pi D}F^*$$

On the other hand, **unitarity** implies

CPT

$$F={}_{out}\langle\alpha|\mathcal{O}|0\rangle=\sum_\beta {}_{out}\langle\alpha|\beta\rangle_{in\,in}\langle\beta|\mathcal{O}|0\rangle=SF^*$$

So,

CPT
Unitarity
Analyticity

$$e^{-i\pi D}F^*=SF^*$$

(This is a generalization of the Watson equation)

The dilatation operator is proportional to the phase of the S-matrix

see [Caron-Huot, Wilhelm '16] for alternative derivation

[Caron-Huot, Wilhelm '16]

$$e^{-i\pi D}F = SF$$

Dilatation operator related to anomalous dimensions by RG equation:

$$DF \sim \mu\frac{\partial}{\partial\mu}F \sim (\gamma_{UV} - \gamma_{IR} + \beta(g^2)\frac{\partial}{\partial g})F$$

Convolution of FF with S-matrix:

$$S = 1 + i\mathcal{M}$$

At LO, dependence on beta ignored,

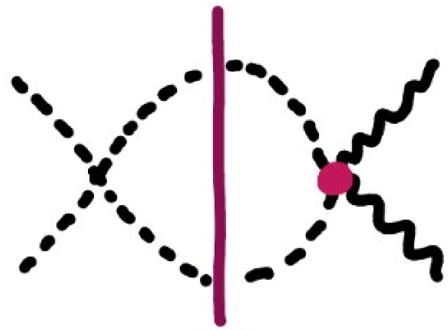
$$(\gamma_{UV} - \gamma_{IR})\langle\alpha|\mathcal{O}|0\rangle = -\frac{1}{\pi}\langle\alpha|\mathcal{M}\otimes\mathcal{O}|0\rangle$$

For this talk, we focus on elements with no IR divergences

$$\gamma_{i\leftarrow j} = -\frac{1}{\pi}\frac{\langle\alpha|\mathcal{M}\otimes\mathcal{O}_j|0\rangle}{\langle\alpha|\mathcal{O}_i|0\rangle}$$

Example 1: Self renormalization of $\mathcal{O}_{FF} = |H|^2 F_{\mu\nu}^2$

$$\langle \alpha | \mathcal{M} \otimes \mathcal{O}_{FF} | 0 \rangle =$$



$$\langle 1_i 2_j^* | \mathcal{M} | 3_k 4_l^* \rangle = 2\lambda(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})$$

$$\langle 1_{\phi_i} 2_{\phi_j^*} 3^- 4^- | H^\dagger H F_{\mu\nu} F_{\mu\nu} | 0 \rangle = 2\delta_{ij} \langle 34 \rangle^2$$

$$= \int [\mathrm{d}p_{1'}][\mathrm{d}p_{2'}] 4\lambda \langle 34 \rangle^2 (\delta_{ij}\delta_{i'j'} + \delta_{ii'}\delta_{jj'})\delta_{i'j'}$$

$$= (2\delta_{ij}\langle 34 \rangle^2)\, 4\lambda(n_s + 1) \int [\mathrm{d}p_{1'}][\mathrm{d}p_{2'}]$$

$$= (2\delta_{ij}\langle 34 \rangle^2)\, 4\lambda(n_s + 1) \frac{1}{16\pi}$$

next slide I'll tell you.
Not important now.

So, $\gamma_{FF \leftarrow FF} = \dfrac{\lambda}{16\pi^2} 4(n_s + 1)$

Spinor notation for massless momenta:

See [Elvang-Huang] for a review

$$p_\mu \sigma^\mu_{a\dot{b}} = p_{a\dot{b}} = -|p]_a \langle p|_{\dot{b}} \qquad |p\rangle^{\dot{a}} = \sqrt{2E}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\phi} \end{pmatrix}$$

The important point is that angle and square brackets carry opposite little group weight, or helicity.
Example:

$$\bar{f}\gamma_\mu f \phi \overleftrightarrow{D}_\mu \phi \to \bar{u}_\pm(p_1)(\not{p}_3 - \not{p}_4)v_\mp(p_2) \begin{cases} \xrightarrow{+-} [13]\langle 32\rangle + [14]\langle 42\rangle \\ \xrightarrow{-+} \langle 13\rangle [32] + \langle 14\rangle [42] \end{cases}$$

Phase space integral:

The previous integral was trivial, but in general it is not.
It will be useful to write it as

$$\int [\mathrm{d}p][\mathrm{d}q] = \frac{1}{16\pi}\int_0^{\pi/2} 2\cos\theta\sin\theta \mathrm{d}\theta \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}$$

The angles parametrize the rotation to base spinors,

$$\begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta e^{i\phi} \\ s_\theta e^{-i\phi} & c_\theta \end{pmatrix} \begin{pmatrix} |p\rangle \\ |q\rangle \end{pmatrix}$$

Example 2: 4–fermions

– There are two fundamentally different types of 4–fermions, with and without net helicity

$$(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma_\mu \ell) = \langle 1|\gamma_\mu|2]\langle 3|\gamma_\mu|4] = 2\cdot\langle 13\rangle[24]$$

$$(\bar{\ell}u)(\bar{q}e) = \langle 12\rangle\langle 34\rangle$$

– We'll compute anomalous dimensions of the second type due to a U(1):

$$\mathcal{A}(f^- f^+ f^- f^+) \;=\; g^2\langle 13\rangle[24]\left(\frac{1}{s}+\frac{1}{u}\right)$$

– Only two flavour structures will be independent due to Schouten identity:

$$0 = \langle \ell u\rangle\langle qe\rangle + \langle \ell q\rangle\langle eu\rangle + \langle \ell e\rangle\langle uq\rangle$$

$$f_1 f_2 \in \{\ell u, eq, \ell e, eu, \ell q, qu\}$$

$$\gamma = \sum_{f_1 f_2}\langle f_1 f_2|\mathcal{M}|xy\rangle\langle f_3 f_4 xy|\mathcal{O}_{\ell equ}|0\rangle$$

$$= \int \mathrm{d}\Omega g^2\left(Y_\ell Y_u \frac{\langle \ell u\rangle[xy]\langle xe\rangle\langle qy\rangle}{s_{\ell x}} + Y_e Y_q \frac{\langle eq\rangle[xy]\langle \ell x\rangle\langle yu\rangle}{s_{ex}} + \ldots\right)$$

$$= \int \mathrm{d}\Omega\, g^2\,(Y_\ell Y_u + Y_\ell Y_q + Y_e Y_u + Y_e Y_q)\langle \ell u\rangle\langle qe\rangle + \;\langle \ell e\rangle\langle qu\rangle\ldots$$

$$= \frac{g^2}{16\pi^2}(Y_\ell + Y_e)(Y_q + Y_u)\langle \ell u\rangle\langle qe\rangle$$

This language trivializes the non-renormalization theorems of [Alonso, Jenkins, Manohar] [Elias Miro, Espinosa, Pomarol] [Cheung, Shen]

See also [Bern, Parra-Martinez, Sawyer]

The spinor language naturally splits the dim-6 operators into 5 categories:

$$\langle\,\cdot\,\rangle^3,\quad \langle\,\cdot\,\rangle^2,\quad \langle\,\cdot\,\rangle,\quad 1\quad \text{and}\quad \langle\,\cdot\,\rangle[\,\cdot\,]$$

	Operator	MFF	
O_{3F}	$\frac{f^{ABC}}{2\cdot 3!} F^{\mu}_{A\,\nu} F^{\nu}_{B\,\rho} \tilde{F}^{\rho}_{C\,\mu}$	$F_3(1^-_A 2^-_B 3^-_C)$	$\langle 12\rangle\langle 23\rangle\langle 31\rangle$
O_{FF}	$\frac{1}{2} H^\dagger H\, F^A_{\mu\nu} \tilde{F}^{A\,\mu\nu}$	$F_{FF}(1_i 2^*_j 3^-_A 4^-_B)$	$\langle 34\rangle\langle 34\rangle$
O_{qF}	$\bar{Q}\sigma^{\mu\nu} T^A q H F^A_{\mu\nu}$	$F_{qF}(1^-_i 2^- 3_k 4^-_A)$	$\langle 14\rangle\langle 42\rangle$
O_{4F_1}	$(\bar{Q}_i u)\epsilon_{ij}(\bar{Q}_j d)$	$F_{4F_1}(1^-_i 2^- 3^-_j 4^-)$	$\langle 12\rangle\langle 34\rangle$
O_y	$\|H\|^2 \bar{Q} q H$	$F_y(1_i 2^*_j 3_k 4^-_l 5^-)$	$\langle 45\rangle$
O_6	$\|H\|^6$	$F_6(1_a 2_b 3_c 4^*_d 5^*_e 6^*_f)$	1
O_{4F_2}	$(\bar{Q} T^A \gamma^\mu Q)(\bar{Q} T^A \gamma_\mu Q)$	$F_{4F_2}(1^-_i 2^-_j 3^+_k 4^+_l)$	$\langle 12\rangle[34]$
O_{QH}	$(\bar{Q} T^A \gamma^\mu Q)\,(i H^\dagger T^A \overleftrightarrow{D}_\mu H)$	$F_{QH}(1_i 2^*_j 3^-_k 4^+_l)$	$\langle 31\rangle\,[14]$
$O_\perp$	$(H^\dagger D_\mu H)(D^\mu H)^\dagger H$	$F_\perp(1_i 2_j 3^*_k 4^*_l)$	$\langle 13\rangle[13]$
$O_\parallel$	$\|H\|^2 (D^\mu H)^\dagger (D_\mu H)$	$F_\parallel(1_i 2_j 3^*_k 4^*_l)$	$\langle 14\rangle[14]$

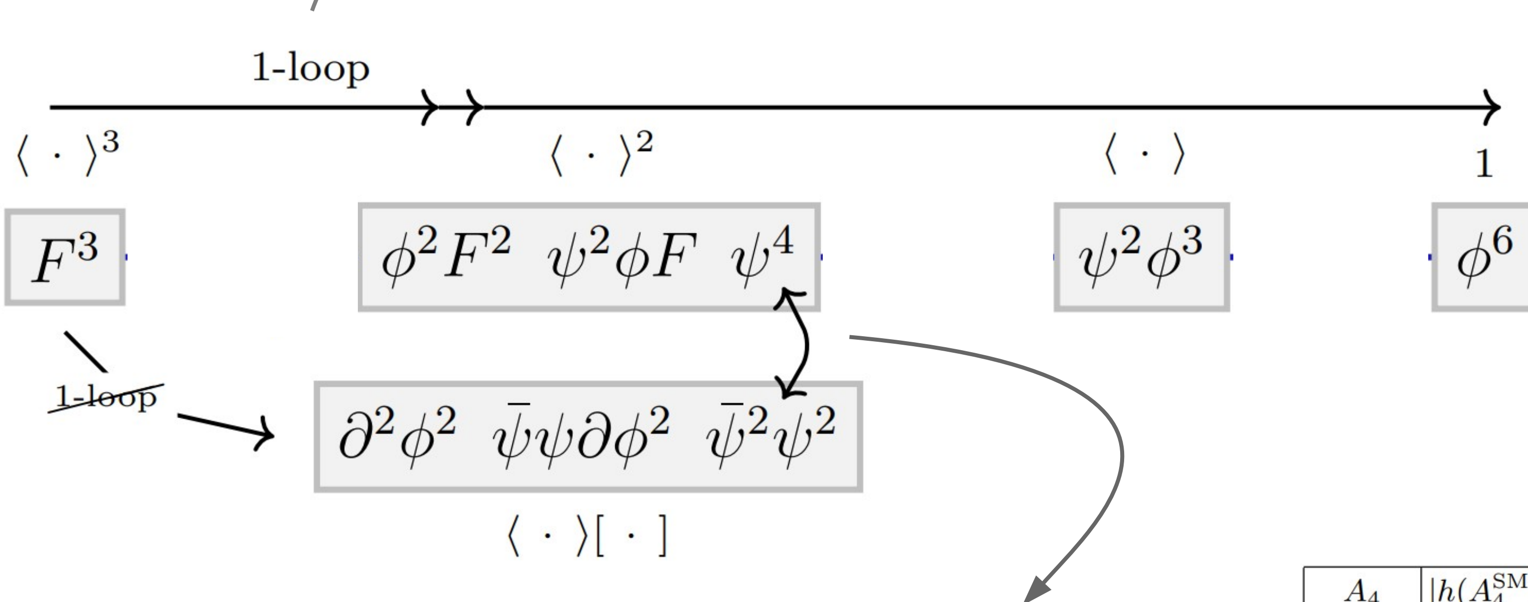
similar to [Henning, Melia]

Non-renormalization theorems

One loop structure:

A two-particle cut
removes two legs of the operator and
the S-matrix inertion adds, at least, two legs.

So operators can only renormalize operators with the same number of legs or more.



Between the two 4-particle class (and from F^3 to <>[]) there is another effect.

Transitions between them require 4-particle amplitudes to violate helicity.
but there is only one of them in the SM.

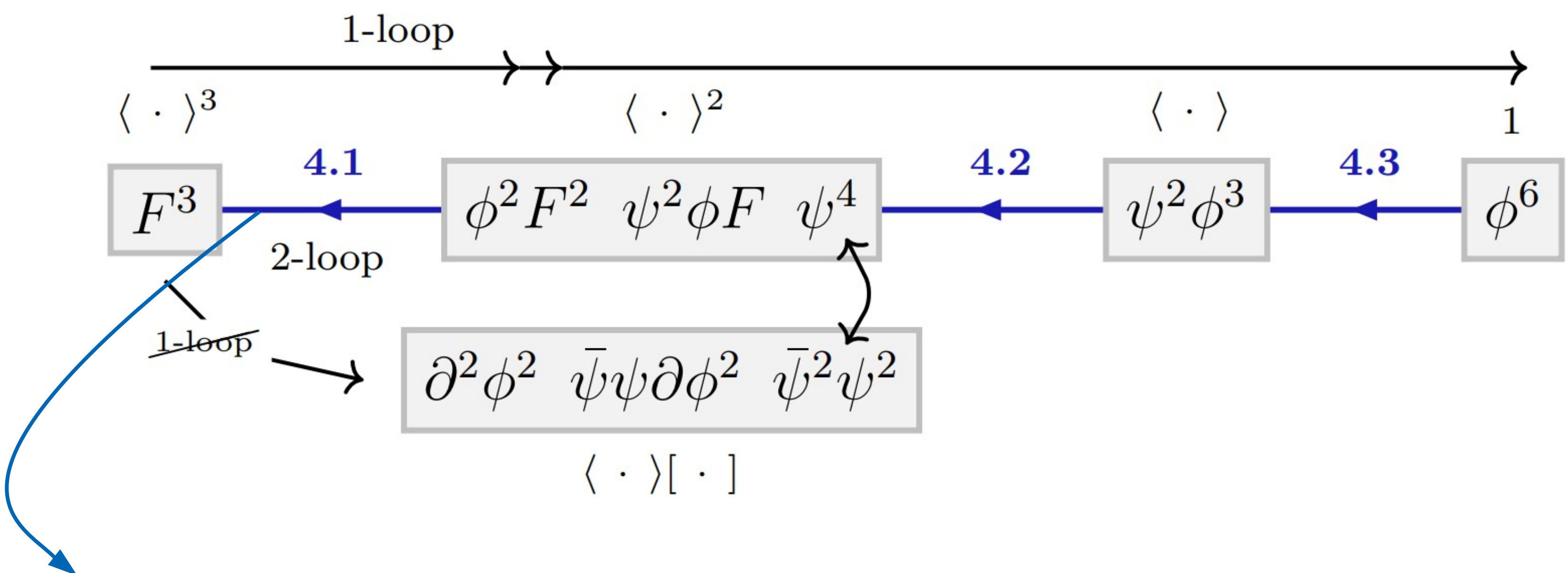
The only non-vanishing transition is between 4-fermion operators

A_4	$\lvert h(A_4^{SM})\rvert$
$VVVV$	0
$VV\phi\phi$	0
$VV\psi\psi$	0
$V\psi\psi\phi$	0
$\psi\psi\psi\psi$	2,0
$\psi\psi\phi\phi$	0
$\phi\phi\phi\phi$	0

[Azatov, Contino, Machado, Riva]

Non-renormalization theorems

Two loop structure:

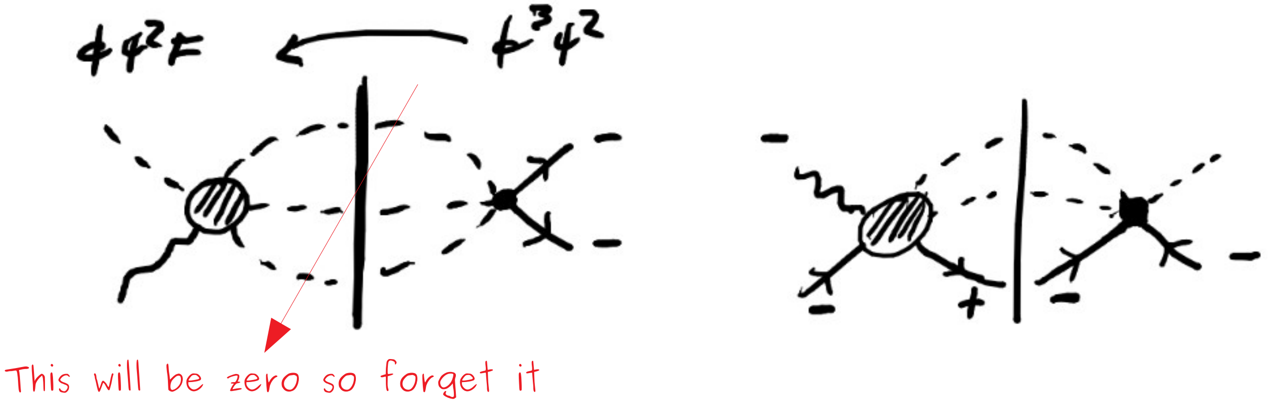


At two loops, RGEs can shorten operator legs by one, or avoid helicity selection rules.

The former type are particularly easy to compute with this method!

Example 3: A 2-loop example, Yukawa to dipole

Two types of diagrams, involving 3-particle cut and 5-point amplitudes:



The entire difficulty of this calculation is to write down the 5-point amplitude in a simple way so that the integral is easily doable. Let's focus on the pure gauge part:

$$\mathcal{A}(f^- f^+ \phi\phi\gamma^-) = g^3 \left(Q_f Q_\phi^2 \frac{[23][24]}{[12][35][45]} - Q_f^2 Q_\phi \frac{[23][24]}{[15][25][34]} \right)$$

Example 3: A 2-loop example, Yukawa to dipole

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0

Since integral symmetric under 3, 4 exchange, and this term is odd

$$\text{integral} = s_{14} \int d\mu \frac{[xy][xz]}{[1x][y4][z4]} \langle x2 \rangle = \langle 14 \rangle \langle 42 \rangle \cdot N$$

dipole FF

with

$$N = \int_0^{\pi/2} 2 s_{\theta_1} c_{\theta_1} \mathrm{d}\theta_1 \int_0^{\pi/2} 4 s_{\theta_2}^3 c_{\theta_2} \mathrm{d}\theta_2 \int_0^{\pi/2} 2 s_{\theta_3} c_{\theta_3} \mathrm{d}\theta_3 \, \frac{c_{\theta_1}^2}{s_{\theta_2}^2} = 1$$

Adding the flavour structure, one gets the result in the literature,

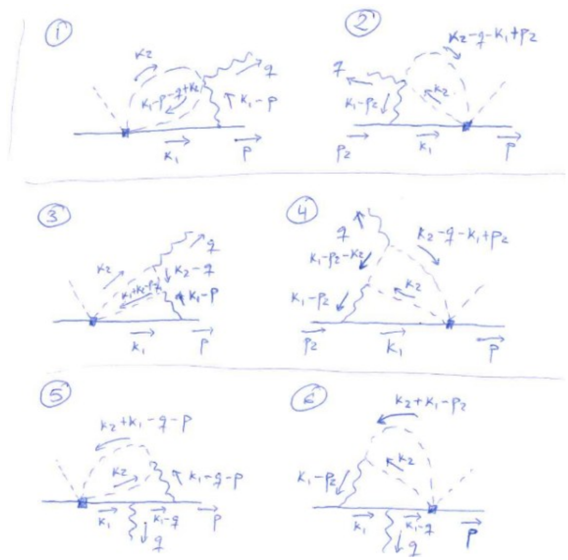
$$\frac{d}{d \ln \mu} \begin{pmatrix} C_{eB} \\ C_{eW} \end{pmatrix} = \frac{g^3}{(16\pi^2)^2} \frac{3}{4} \begin{pmatrix} t_{\theta_W} Y_H + 4 t_{\theta_W}^3 Y_H^2 (Y_L + Y_e) \\ \frac{1}{2} + \frac{2}{3} t_{\theta_W}^2 Y_H (Y_L + Y_e) \end{pmatrix} C_{y_e}$$

from [Pomarol, Panico, MR]

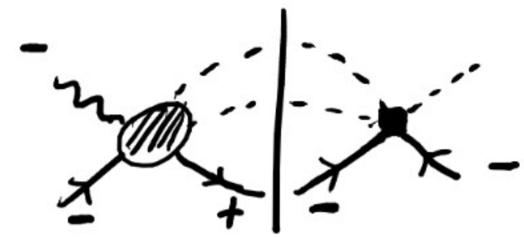
Example 3:

My brain is not a standard candle, but to get an idea...

The Verdict

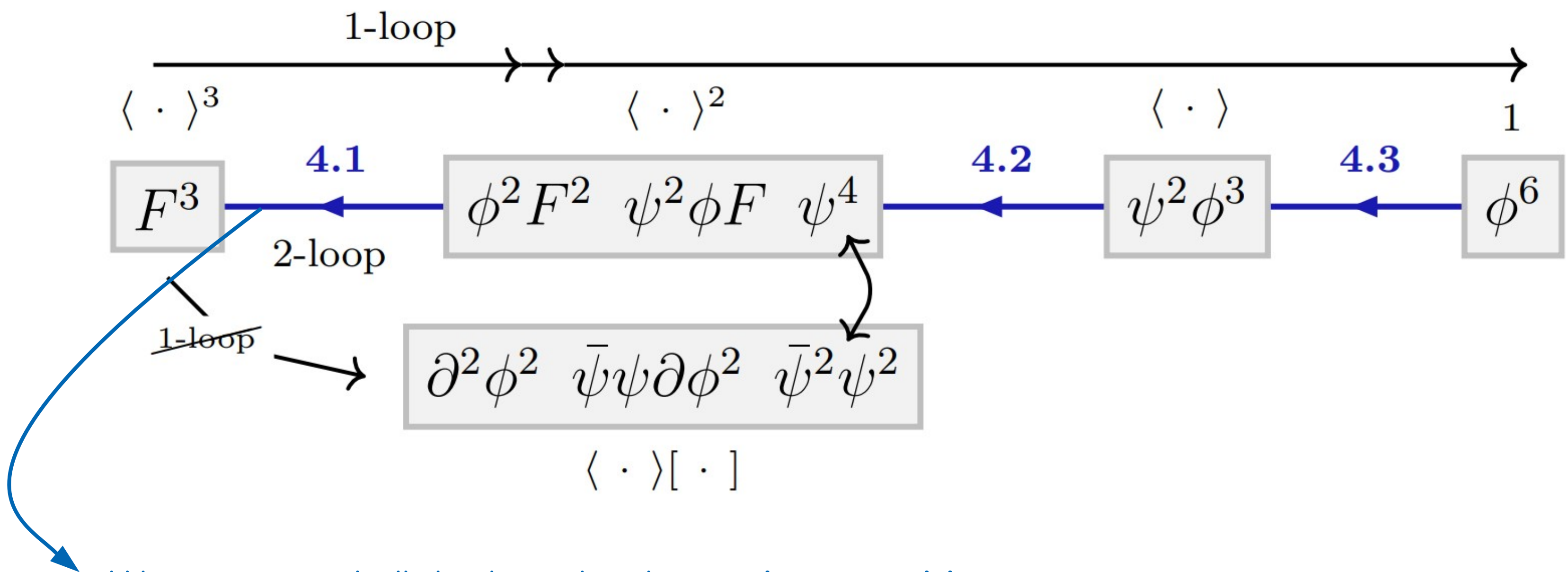


VS



One month+ of struggle,
of sign-chasing, of looking
for factors of two and comparing
with collaborators...

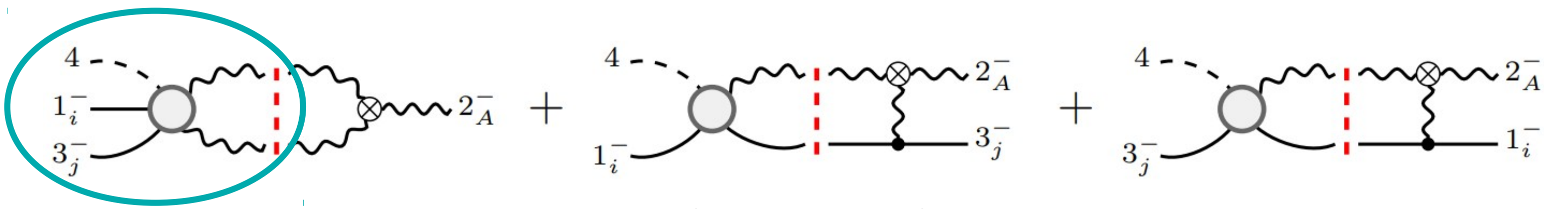
One day to get the amplitude
in a nice form.
30min of writing the rotation and
do the integrals in Mathematica.



We computed all the length-shortening transitions.

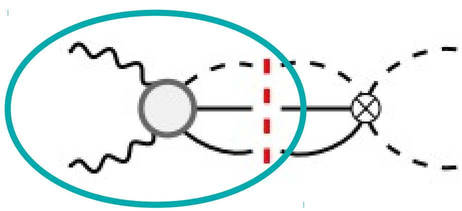
Since they are the leading contribution, they are a single log
and in this method they are especially simple:
a tree amplitude and a tree FF with a 3-particle cut.

Last but not least, an important aspect of the method is that it is GREEN

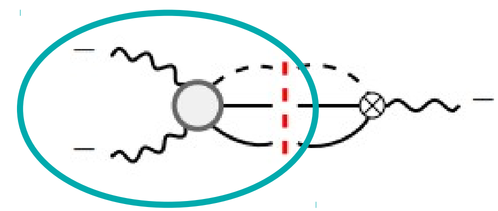


$$\psi^2\phi F \leftarrow F^3$$

$$\phi^2 F^2 \leftarrow \psi^2\phi^3$$



$$F^3 \leftarrow \psi^2\phi F$$

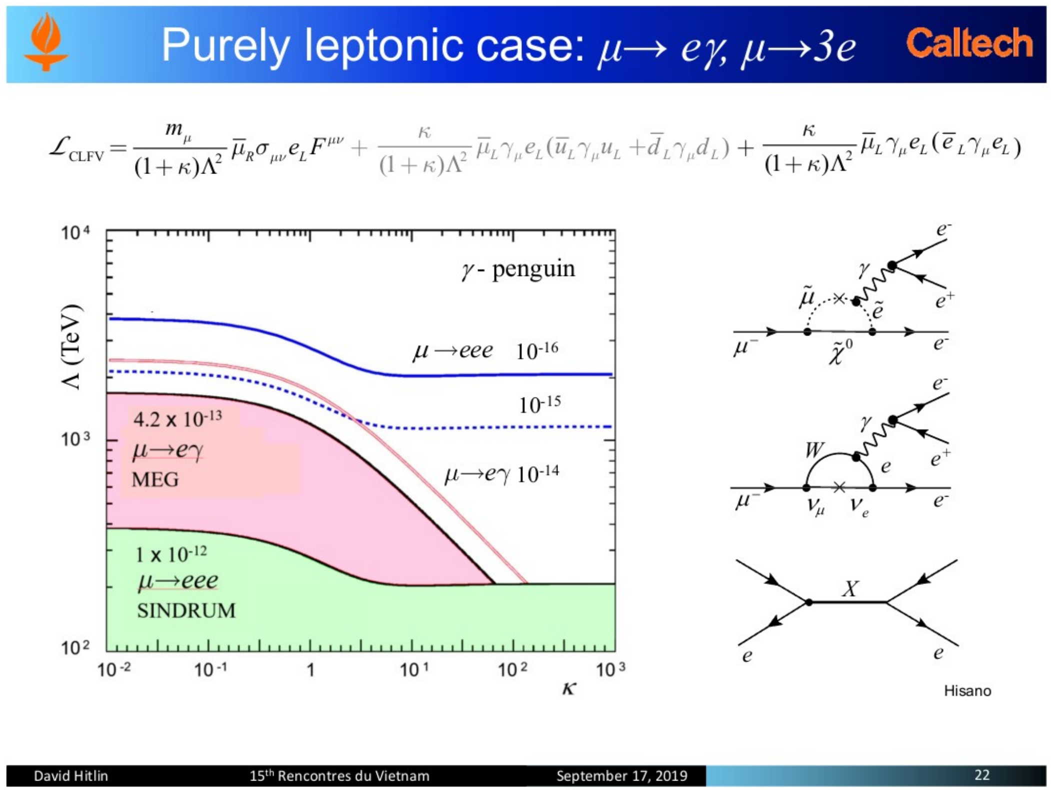


Same amplitude appears in several computations, in this case $\mathcal{A}(f^- f^- \phi\, g^+ g^+)$

Electron EDM searches are not the only low energy probes to get a boost in the next years,

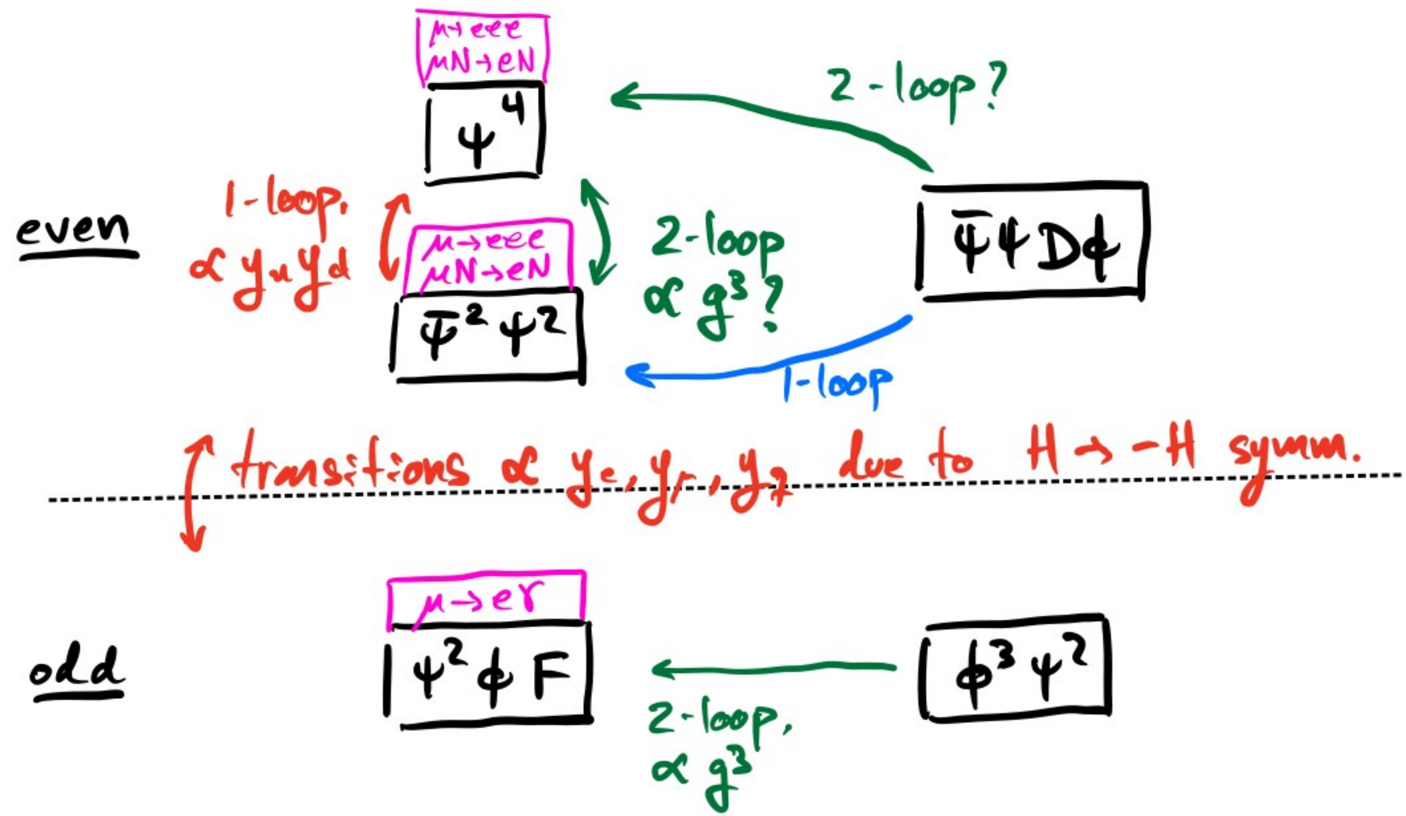
As you all know much better than me,
muon to electron conversion searches (+photon, +ee, conversion in hadrons) are expected to improve by orders of magnitude

Slide from **David Hitlin**,
Rencontres du Vietnam, Sept 17th 2019



$$\mathrm{Br}(\mu \to e\gamma) < 6 \cdot 10^{-14} \quad \Longrightarrow \quad \Lambda \geq 10\mathrm{TeV},\ \text{for } \mathcal{L} \supset \frac{\sqrt{y_e y_\mu}}{\Lambda^2}\left(\frac{g^4}{(16\pi^2)^2}\log\frac{m_w^2}{\Lambda^2}\right)\bar{\mu}\sigma_{\mu\nu}eHF_{\mu\nu}$$

The full map of two-loop effects for mu to e transition experiments seem to look like this:



Would be great to learn if there are new effects compared with previous literature, e.g. A. Crivellin et al [1702.03020]

Conclusions:

- Unitarity and alanyticity give us new perspectives on the computation of anomalous dimensions.

- The method is particularly useful for computing a set of two-loop transitions

- Two loop precision is required to correctly interpret the electron EDM constraints, and might be useful for other future experiments

Thank you!