

The background features a large, light blue watermark of the CERN logo, which consists of a stylized 'C' and 'N' with the word 'CERN' in the center.

Parton Showers beyond leading logarithmic accuracy

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In collaboration with

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LTP Theory Seminar @ PSI - December 2 2020

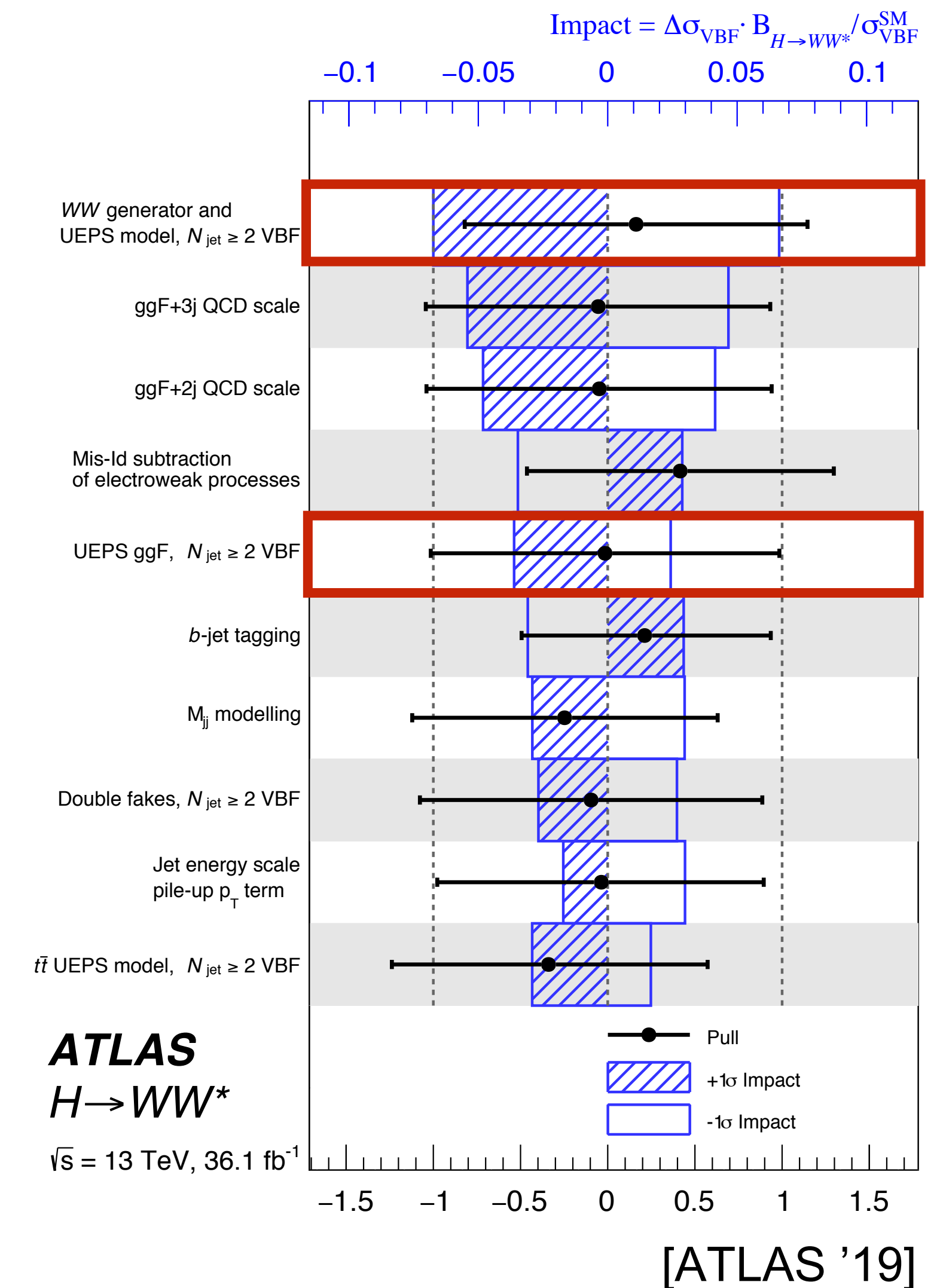
Inspecting the structure of collider events

- ▶ Great advances in the exploration of perturbative calculations (fixed/all orders) in QFTs in the last decade, instrumental to exploit LHC data (new physics searches, Higgs sector, SM, ...)
- ▶ However, more often than not the bridge between theory and experiments speaks the language of Monte Carlo (MC) parton showers ... with considerable uncertainties

e.g. Higgs prodⁿ in VBF:
(N)NNLO QCD and NLO EW available,
though Th. error dominates analyses

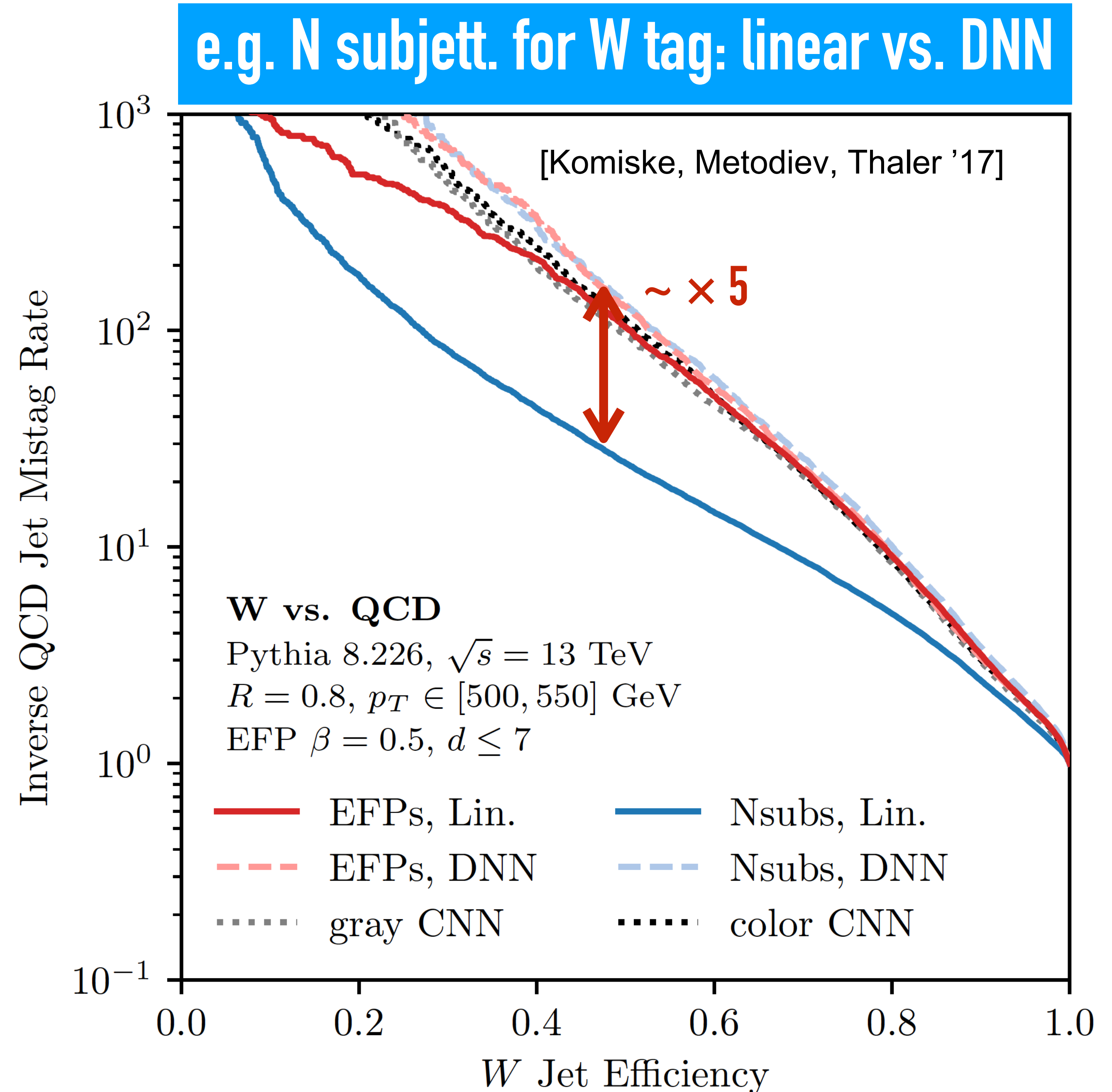
**Large difference in the simulation
of kinematic distributions with
different MC tools**

[Jaeger, Karlberg, Plaetzer, Scheller, Zaro '20]

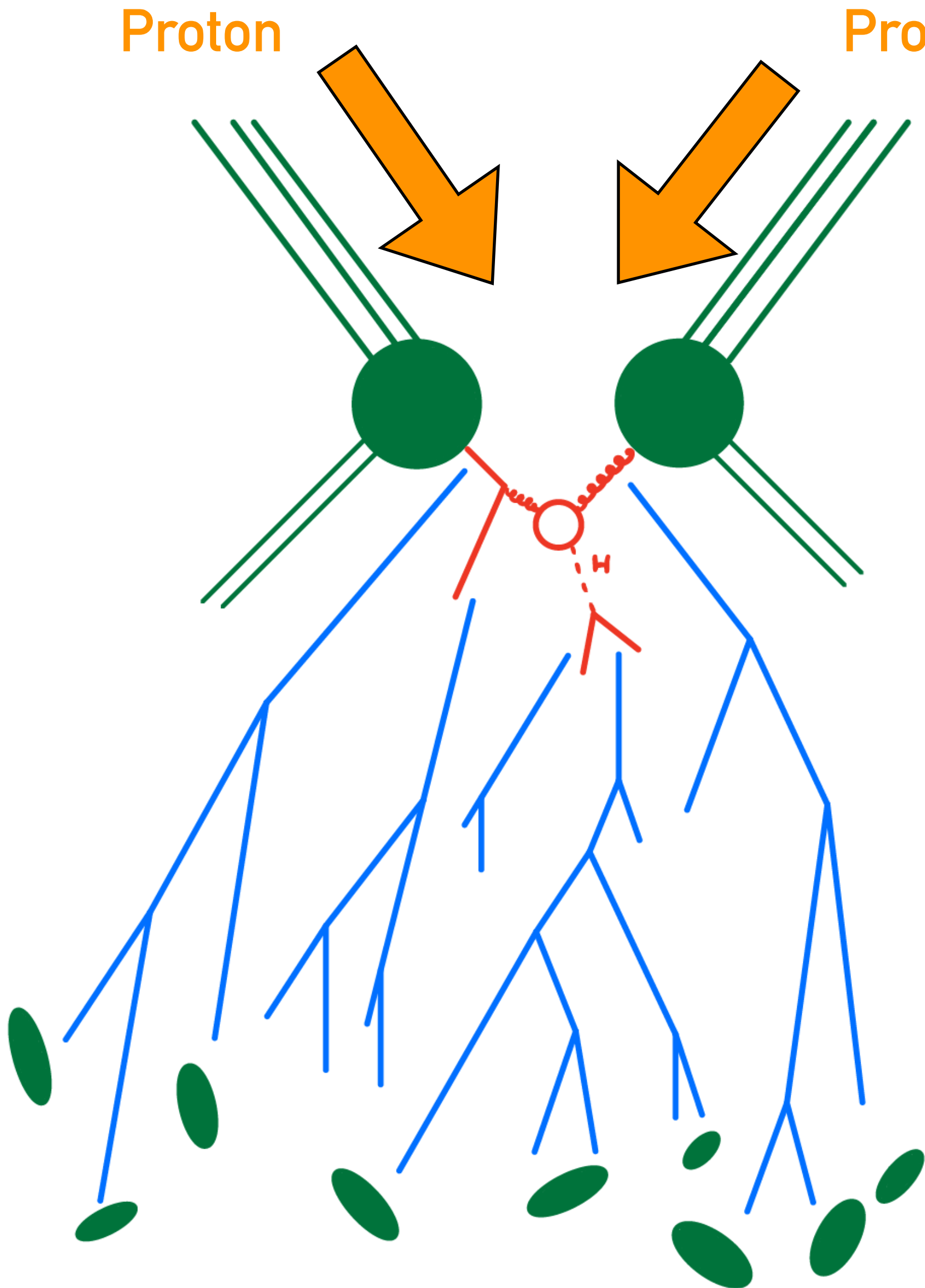


Inspecting the structure of collider events

- ▶ Outstanding experimental performance at the LHC opens new avenues to test the SM and perform indirect searches (constraints) of New Physics models
- ▶ In specific cases the sensitivity is augmented by Machine Learning technology:
e.g. substructure of jets, tagging of heavy particles, q/g discrimination, ...
- ▶ Dependence on the training data (MC) may be substantial.
Control over fine details needed !



Monte Carlo Parton Showers

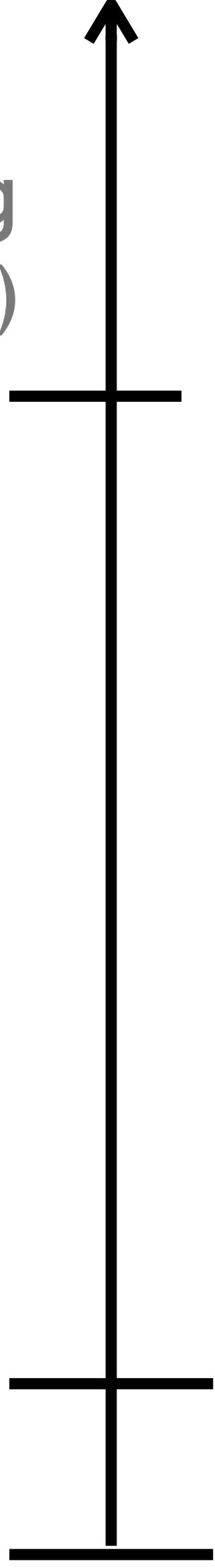


Hard scattering
($\sim 10^2 - 10^3$ GeV)

Hadron
formation
(~ 1 GeV)

Observation

Energy



Fixed order perturbation
theory: a lot of recent progress
[e.g. N(N)LO+PS matching &
multi-jet merging]

Parton Shower: multi-scale evolution
& large hierarchy of scales:
formal accuracy ??

Modelling of non-
perturbative dynamics

Perturbation theory in multi scale regimes

- ▶ When a hierarchy of scales is present

perturbative accuracy \equiv logarithmic accuracy (resummations)

- ▶ Two different (though perturbatively equivalent) definitions commonly used:

Perturbative order of EFT RGEs
(anomalous dimensions &
initial conditions)

Squared amplitudes in the relevant
kinematic limits (ordering)

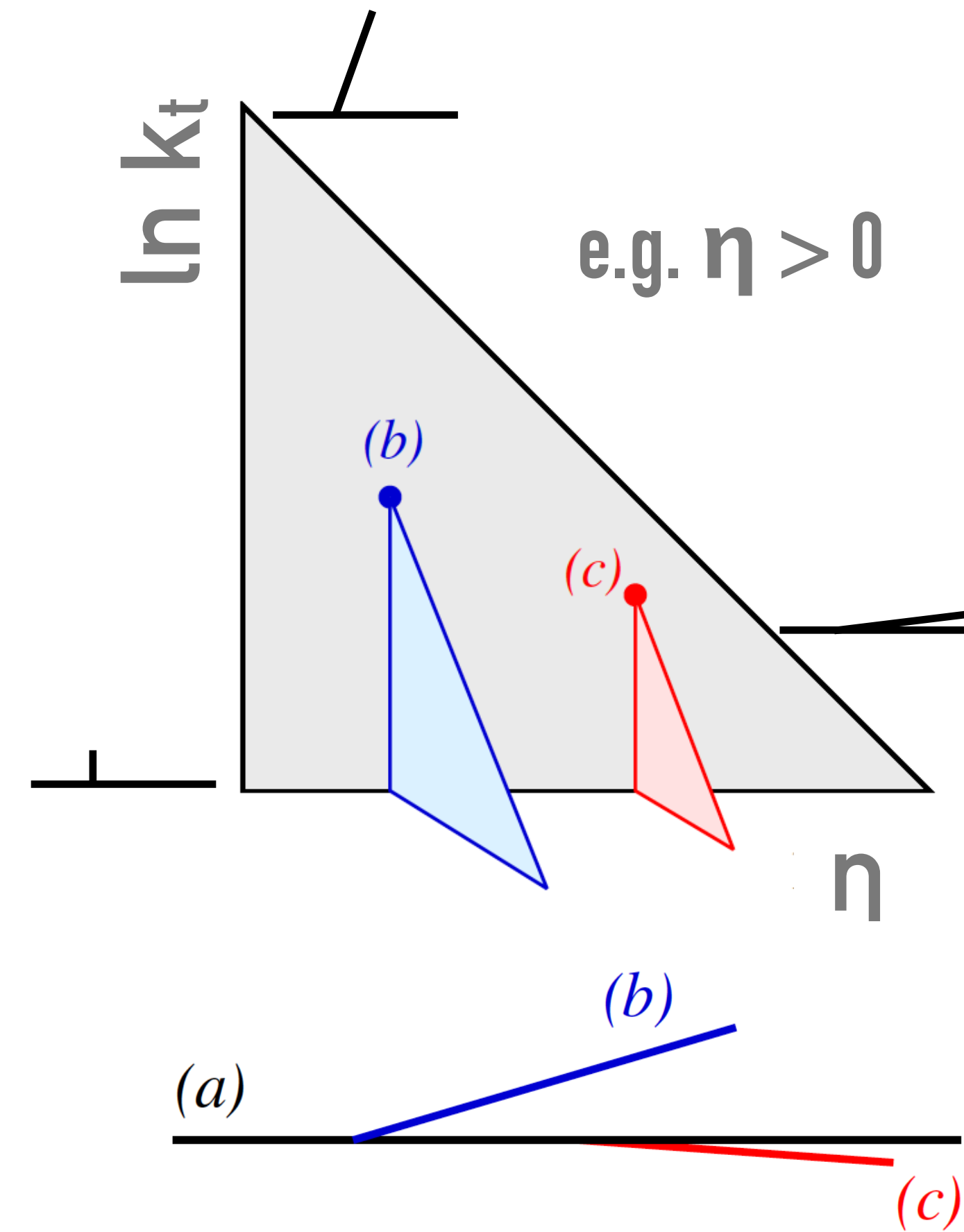
- ▶ e.g. cumulative distribution for an observable $v < e^{-L}$

LL (=0 sometimes) NLL

$$\Sigma(\alpha_s, \alpha_s L) = \exp \left[\alpha_s^{-1} g_1(\alpha_s L) + g_2(\alpha_s L) + \mathcal{O}(\alpha_s^n L^{n-1}) \right]$$

A geometric criterion: the Lund plane

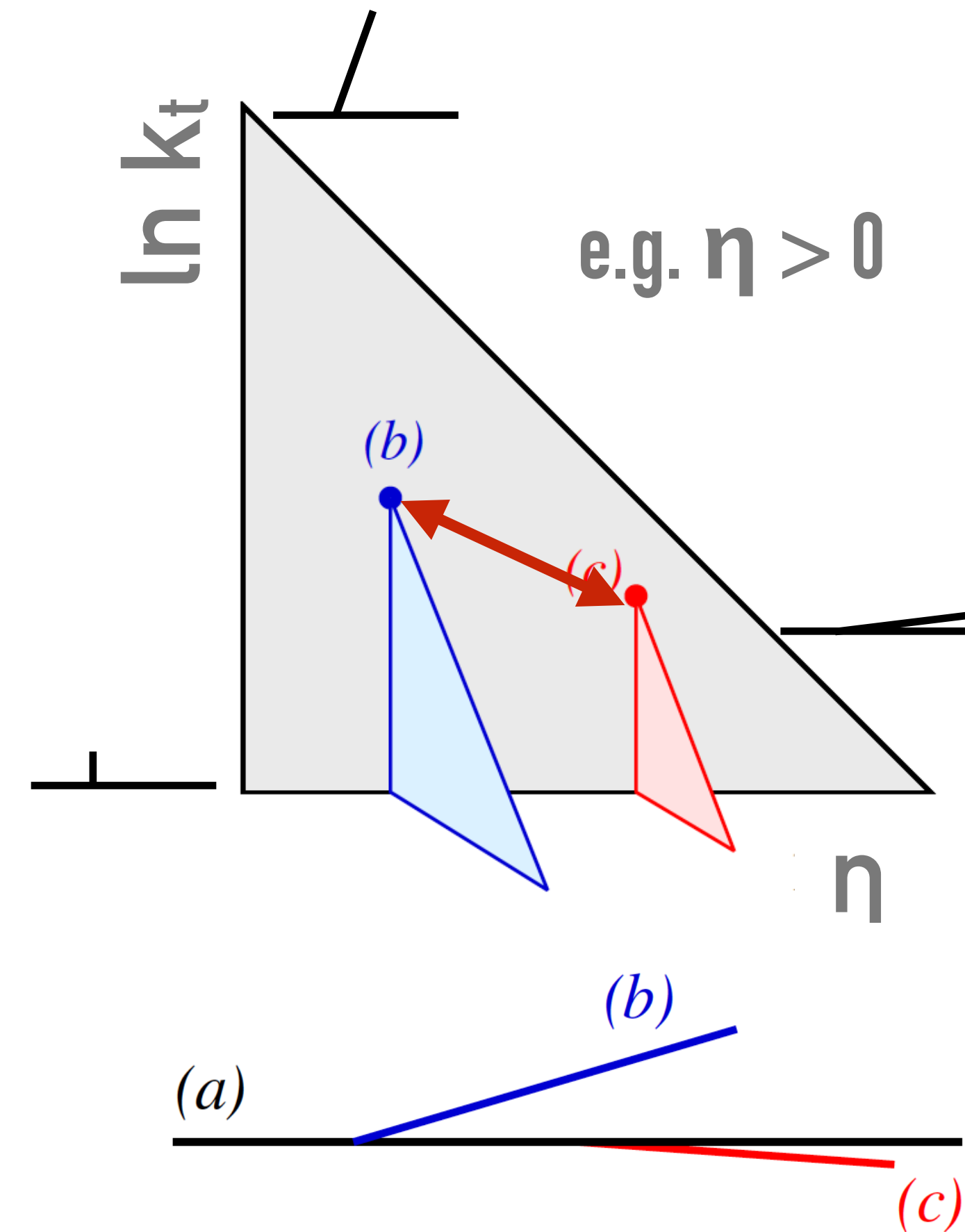
- ▶ 3 (phase space) variables per real radiation, two of which lead to logarithms, e.g. $\{k_T, \eta\}$; $\{E, \theta\}$
- ▶ LL: reproduce correct squared amplitude in limits where **both logarithmic variables** are strongly ordered across emissions



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Strong ordering in one log variable \equiv large Lund plane distance



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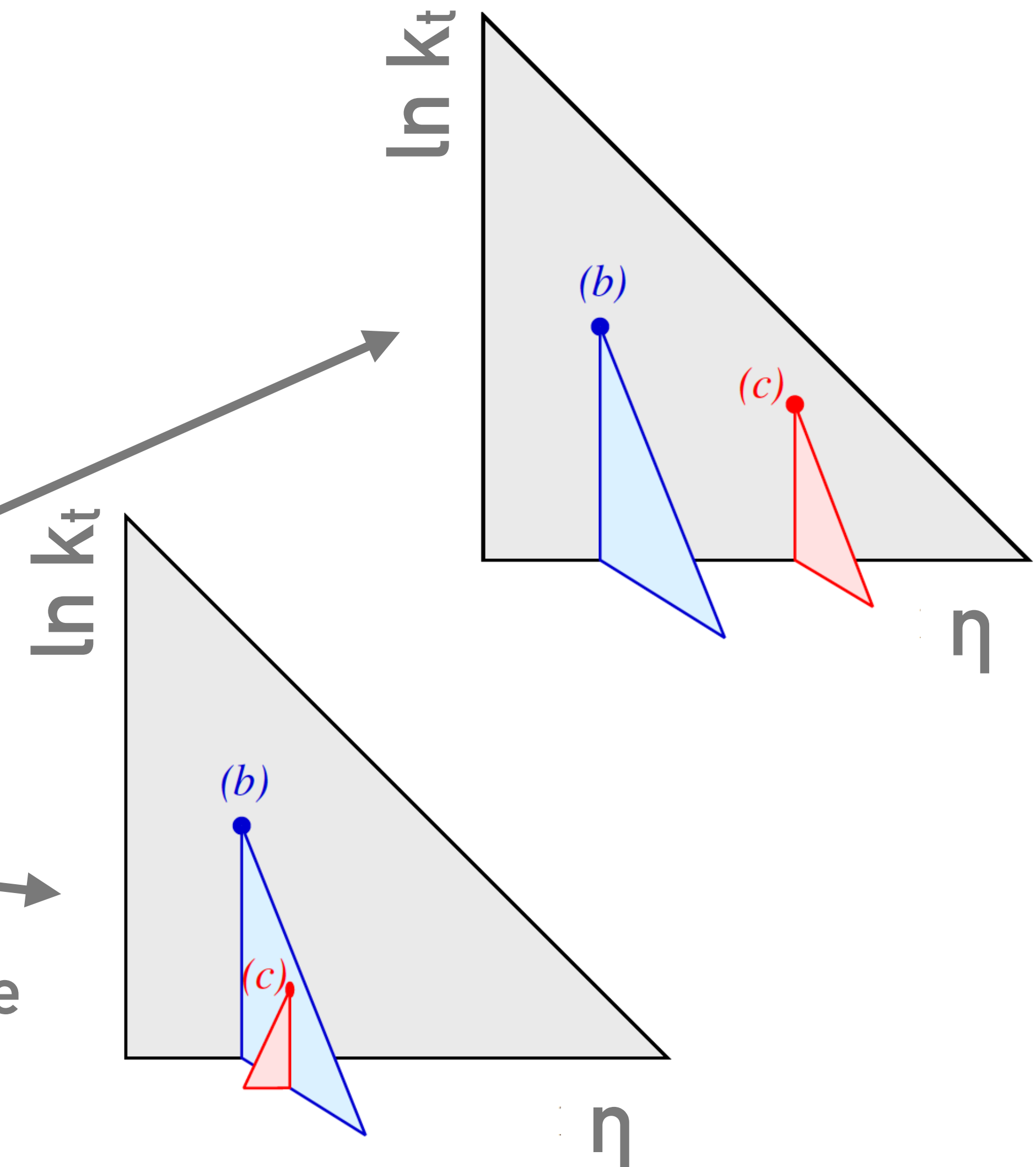
Strong ordering in one log variable \equiv large Lund plane distance

- ▶ NLL: reproduce correct squared amplitude in limit where **at least one logarithmic variable** is strongly ordered across emissions. E.g.

- ▶ Similar k_t and ordered in angle (or pseudo rapidity η)

- ▶ Similar angle and ordered in k_t (or energy)

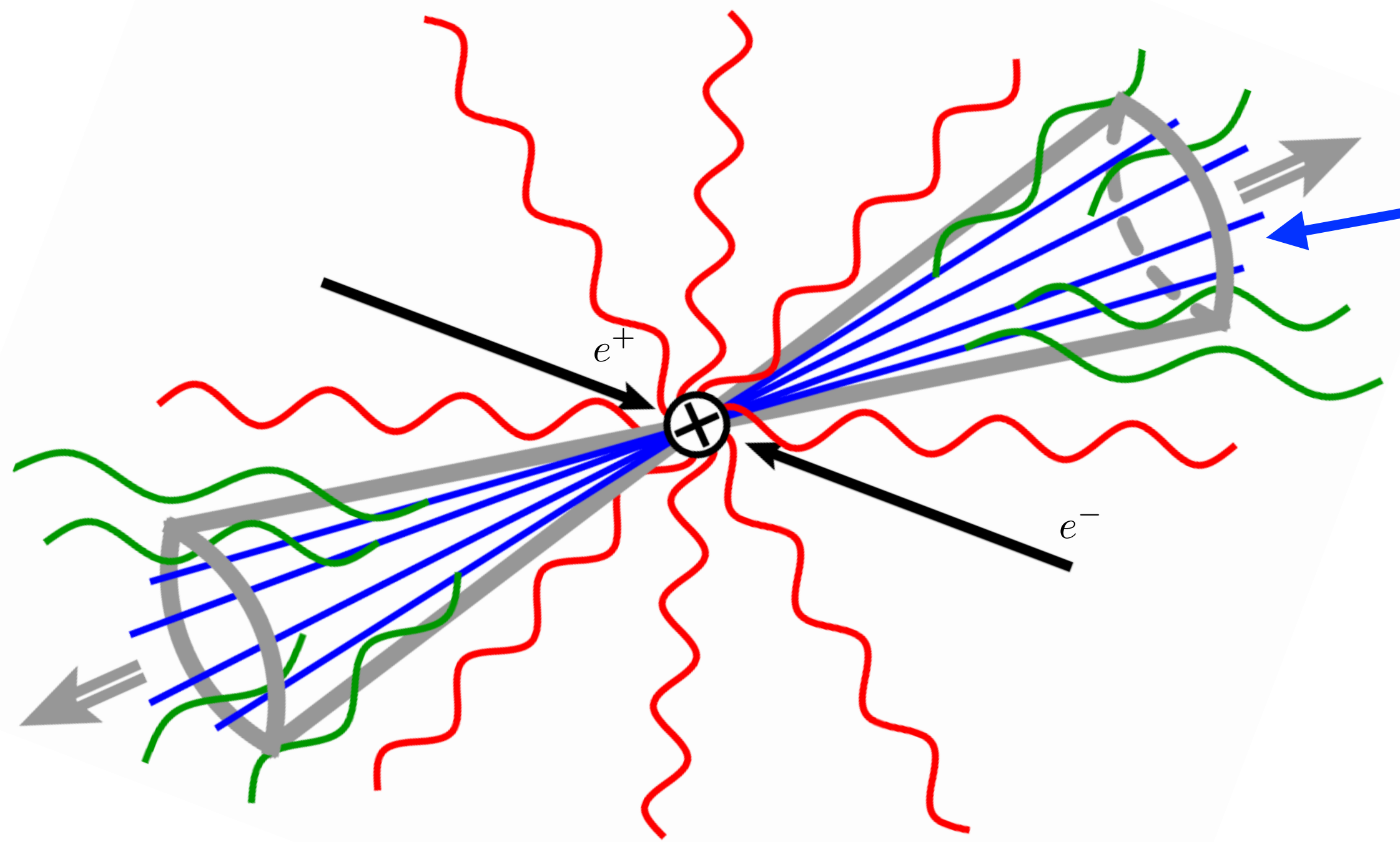
- ▶ When two emissions are close (in both variables), a mistake is allowed (NNLL)



e.g. NLL: building blocks

- ▶ rIRC safe, global observables described by emissions strongly ordered in angle, but with commensurate transverse momenta (recoil effects are relevant)
 - ▶ realised, e.g., in angular ordered parton showers

e.g. $e^+e^- \rightarrow q \bar{q} + X$ at NLL



$$dP_n \simeq \frac{C_F^n}{n!} \prod_{i=1}^n \left(\frac{\alpha_s^{\text{CMW}}(p_{\perp,i}^2)}{\pi} \frac{dp_{\perp,i}}{p_{\perp,i}} dz_i P_{q \rightarrow qg}(z_i) \frac{d\phi_i}{2\pi} \right)$$

collinear limit described by independent emissions
strongly ordered in angle

e.g. NLL: building blocks

▶ non-global logarithms described radiation at similar angles, but strongly ordered energy / transverse momentum

▶ angular ordering fails, dipole showers needed

[Banfi, Corcella, Dasgupta '06]

e.g. $e^+e^- \rightarrow q \bar{q} + X$ at NLL

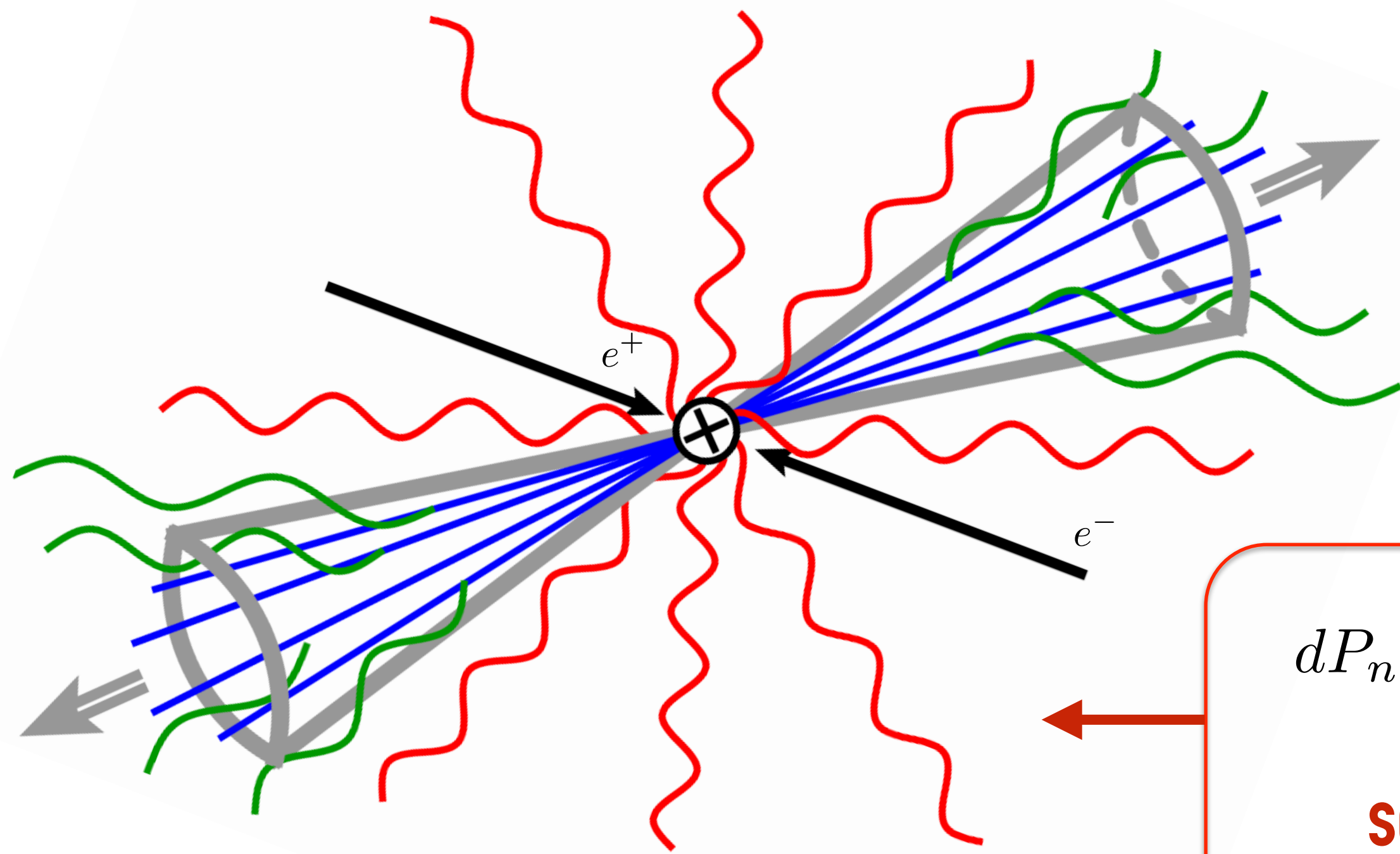


Image by T. Becher et al.

$$dP_n \simeq \frac{1}{n!} \prod_{i=1}^n \frac{\alpha_s}{\pi} \frac{d\omega_i}{\omega_i} \frac{d^2\Omega}{4\pi} N_c \sum_{\pi_n} \frac{p_1 \cdot p_2}{(p_1 \cdot k_{i_1})(k_{i_1} \cdot k_{i_2}) \dots (k_{i_n} \cdot p_2)}$$

soft wide angle limit described by a collection of soft colour dipoles strongly ordered in energy (planar limit)

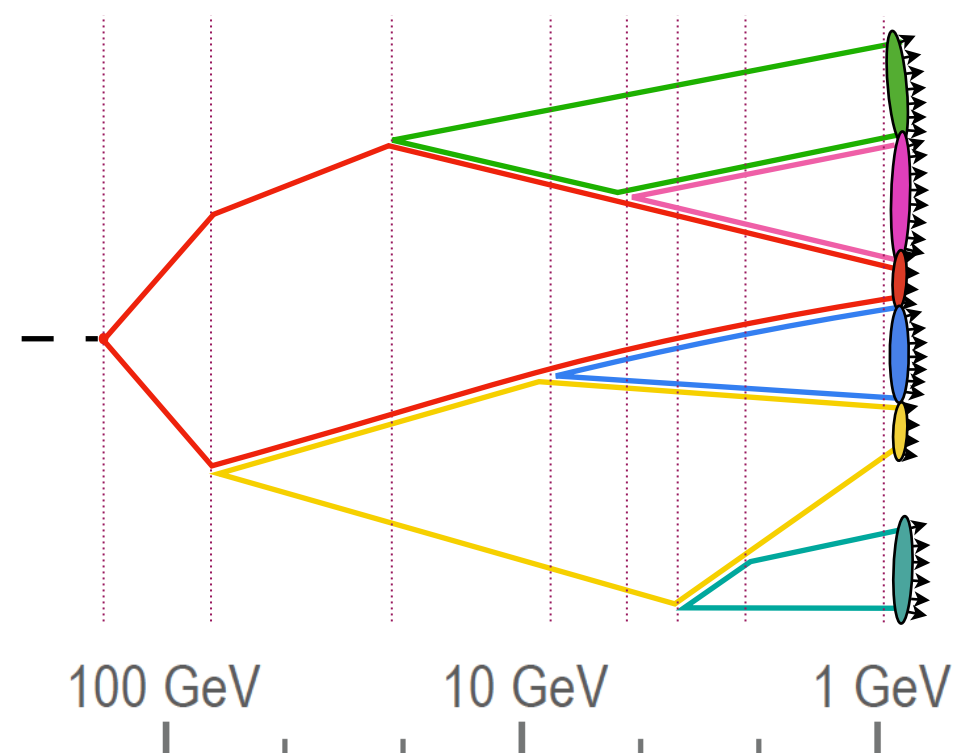
Dipole showers in a nutshell

- ▶ Squared amplitudes built recursively via a Markovian chain of emissions (planar limit). Virtual corrections strictly implemented through unitarity

Evolution from a state with n particles S_n
to one with $n+1$ particles S_{n+1}

$$\frac{d\mathcal{P}_{n \rightarrow n+1}}{d \ln v} = \sum_{\text{dipoles } \{\tilde{i}, \tilde{j}\}} \int d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t) + K \alpha_s^2(k_t)}{\pi} \times [g(\bar{\eta}) a_k P_{\tilde{i} \rightarrow ik}(a_k) + g(-\bar{\eta}) b_k P_{\tilde{j} \rightarrow jk}(b_k)]$$

H



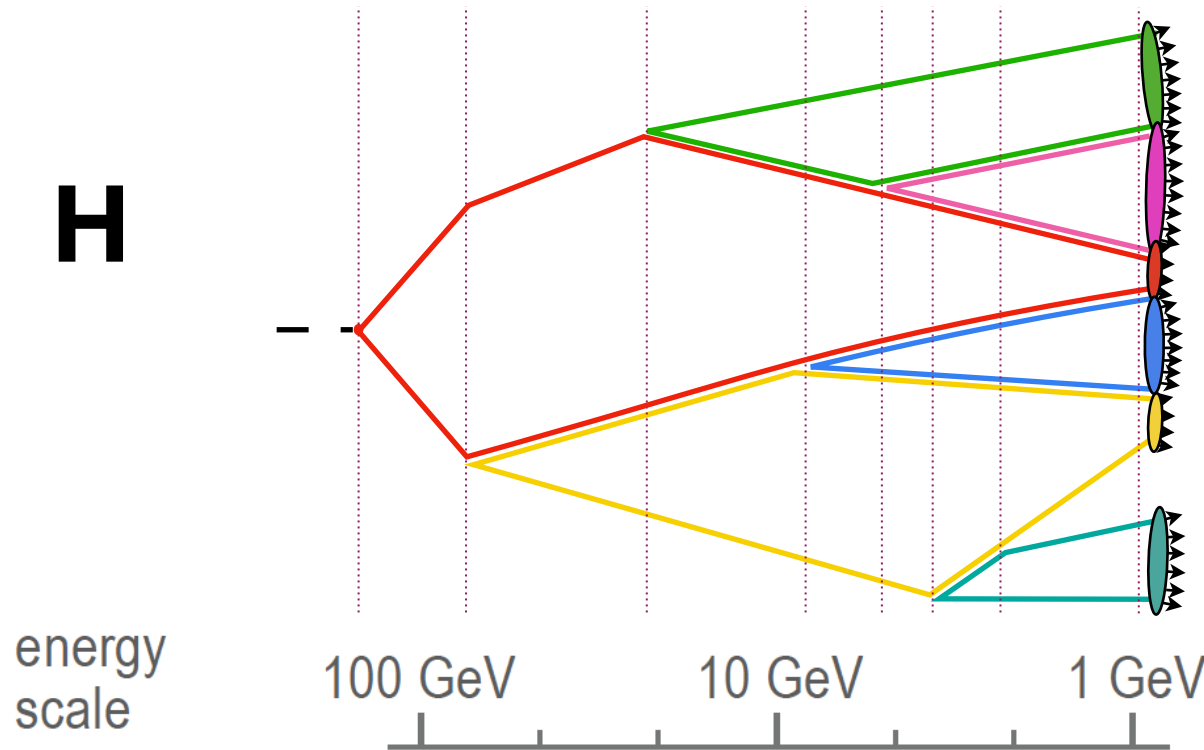
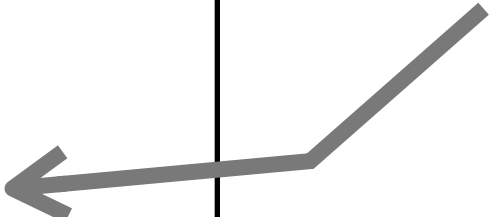
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Evolution variable,
e.g. k_T in the dipole
c.o.m. frame



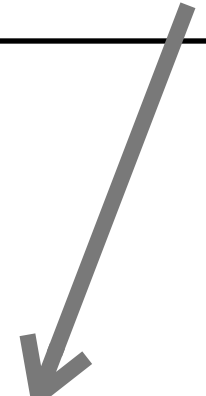
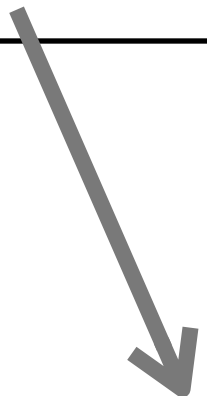
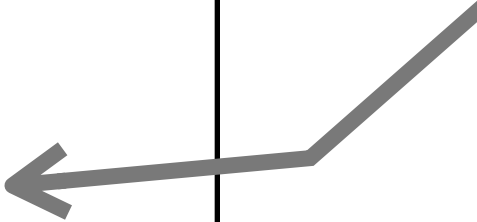
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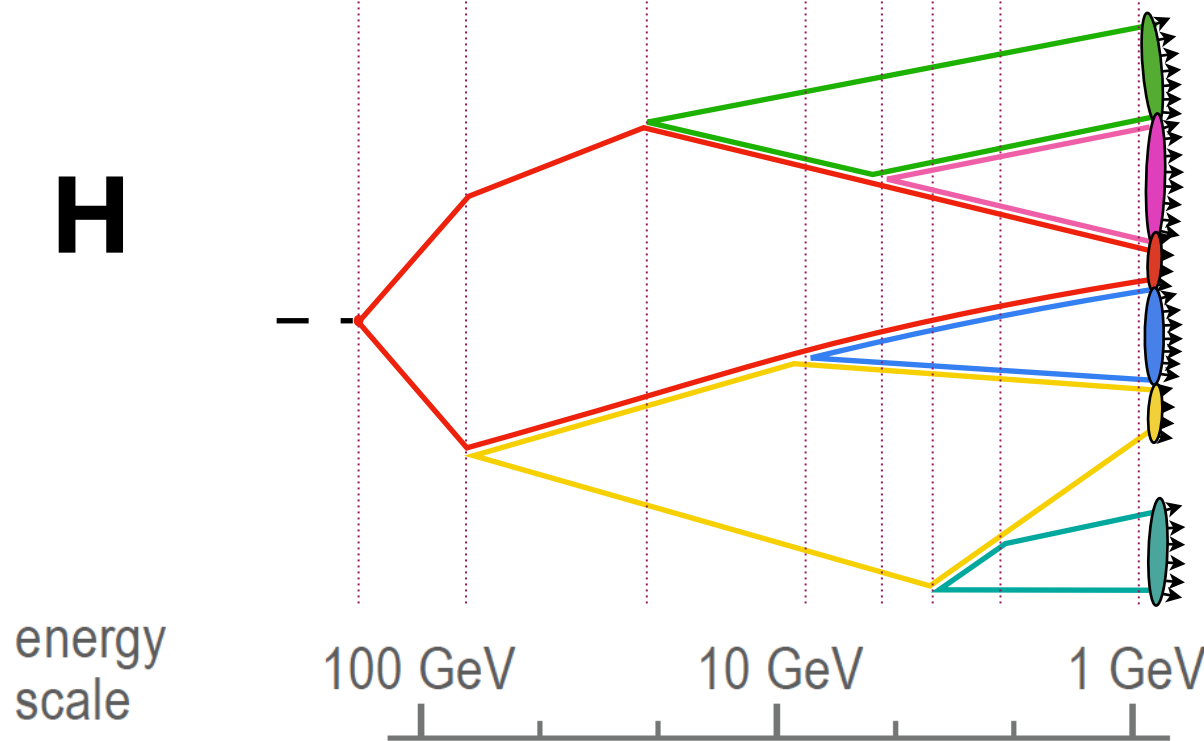
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Some notion of rapidity of the emission within the dipole, deciding how the dipole is partitioned.

Recoil assigned according to a **map** $S_n \rightarrow S_{n+1}$



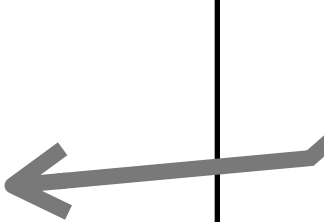
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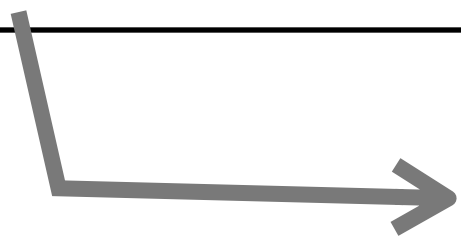
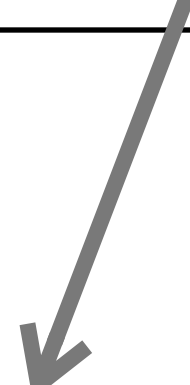
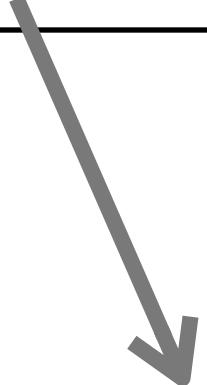
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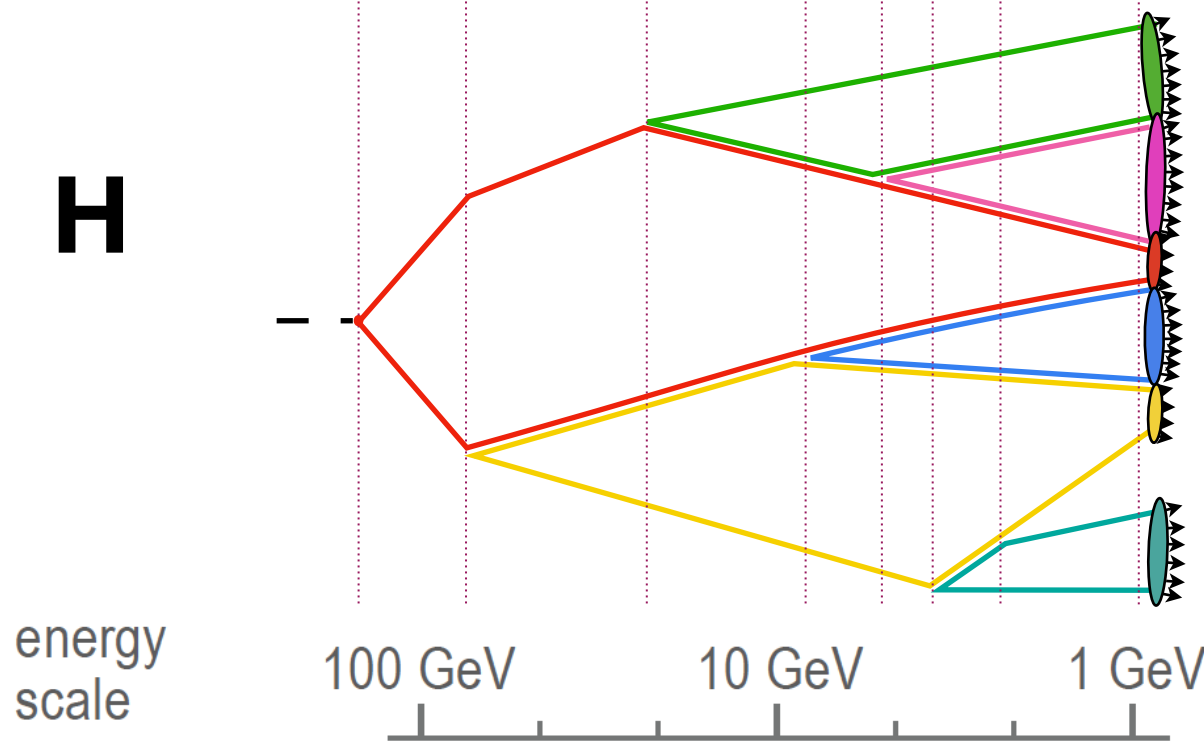


$\times [g(\bar{\eta}) a_k P_{\tilde{i} \rightarrow ik}(a_k) + g(-\bar{\eta}) b_k P_{\tilde{j} \rightarrow jk}(b_k)]$



LO Splitting functions

H



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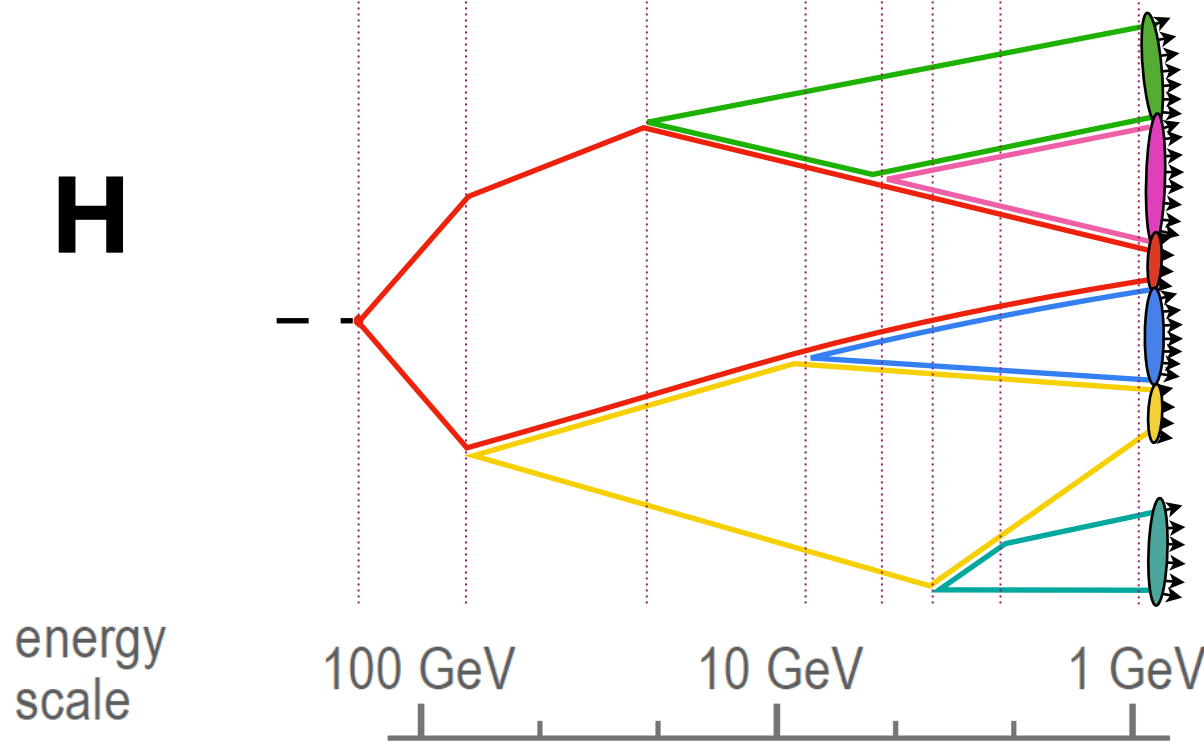
CMW scheme,
consistent inclusion
of $O(\alpha_s^2)$ soft current
up to NLL

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**A case study: k_t ordering & local recoil
(e.g. Pythia8, Dire_{v1}, CS dipole shower are of this type)**

A case study: k_t ordering & local recoil

- ▶ As an example, let us consider the $O(\alpha_s^2)$ soft emission off a fermion line. In the (NLL) limit of strong angular ordering, similar transverse momenta, one expects the emission probability (**squared amplitude x phase space**) to be

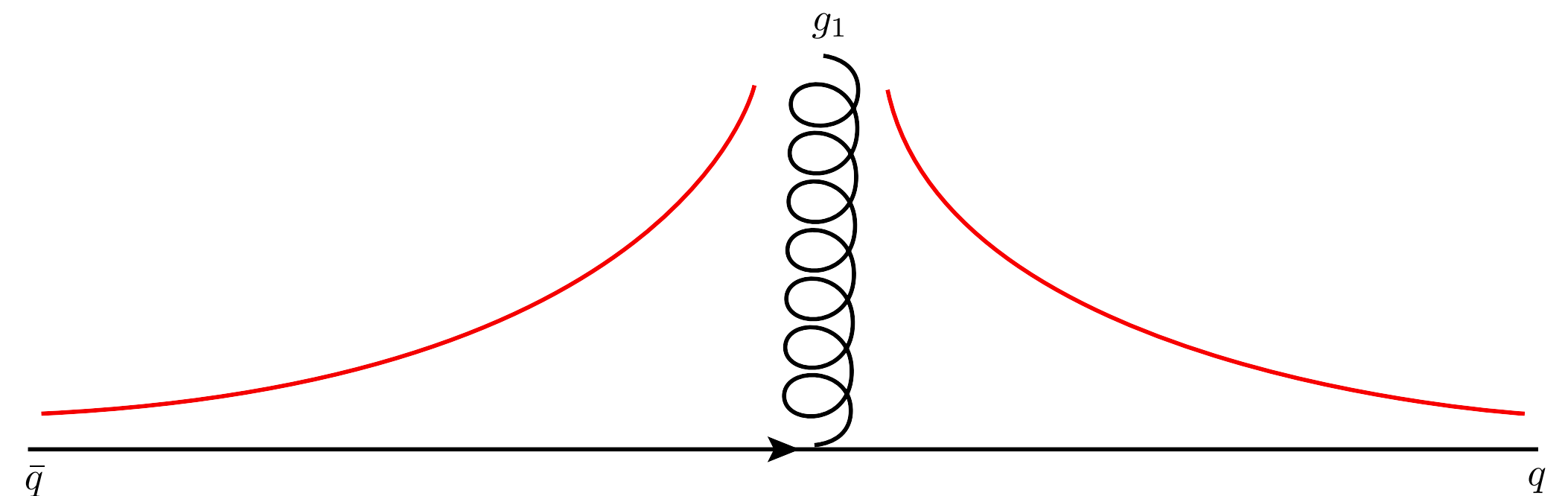
$$dP_2 = \frac{C_F^2}{2!} \prod_{i=1,2} \left(\frac{2\alpha_s(p_{\perp,i}^2)}{\pi} \frac{dp_{\perp,i}}{p_{\perp,i}} d\eta_i \frac{d\phi_i}{2\pi} \right)$$

i.e. recoil taken from the quark

- ▶ Instead, dipole local showers assign the recoil to either of the emitting dipole ends, according to the rapidity of the emission in the **dipole centre-of-mass frame**

- e.g.

- Start with an emission g_1



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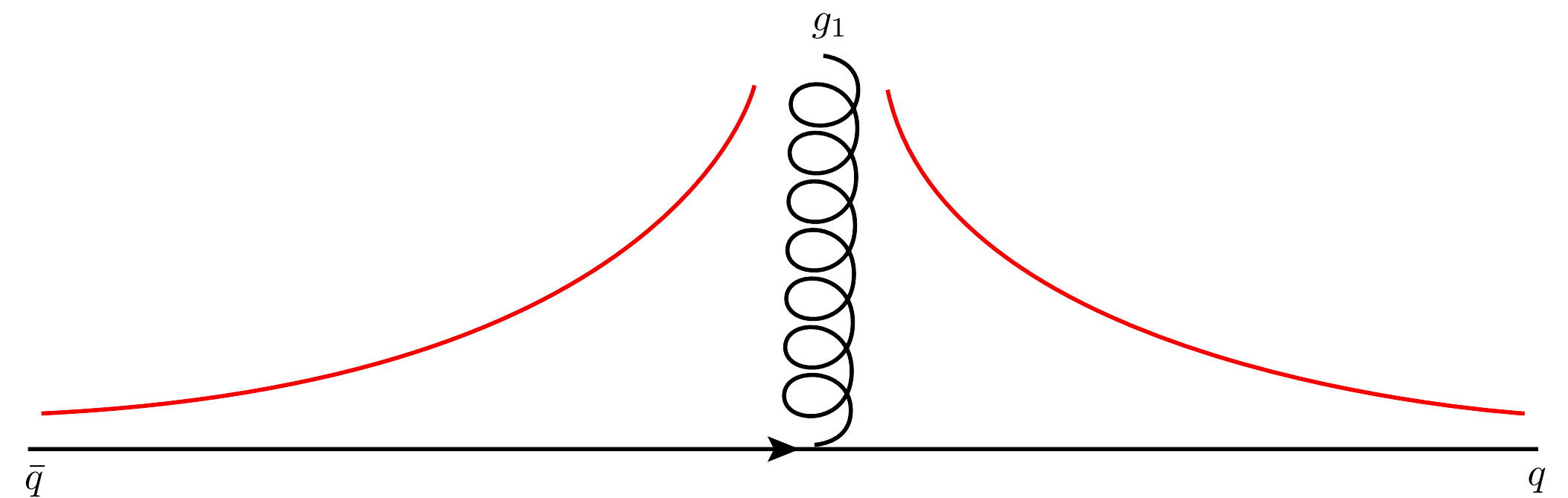
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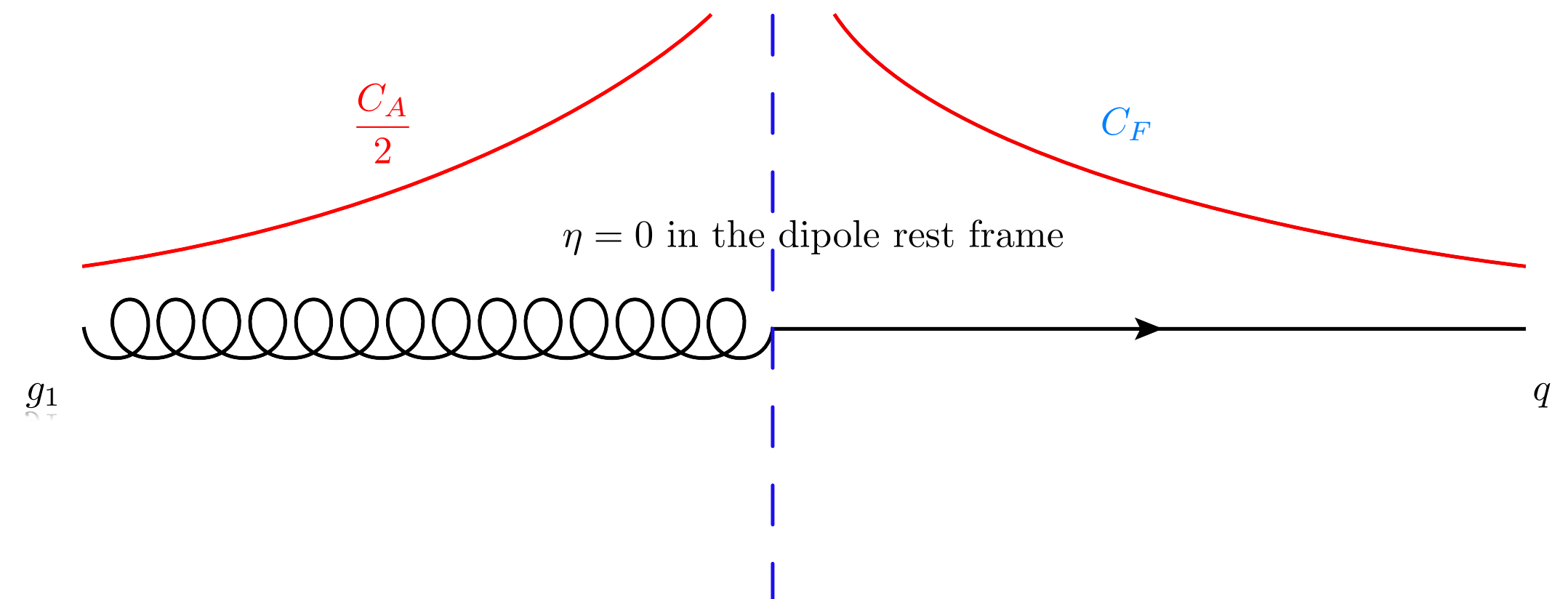
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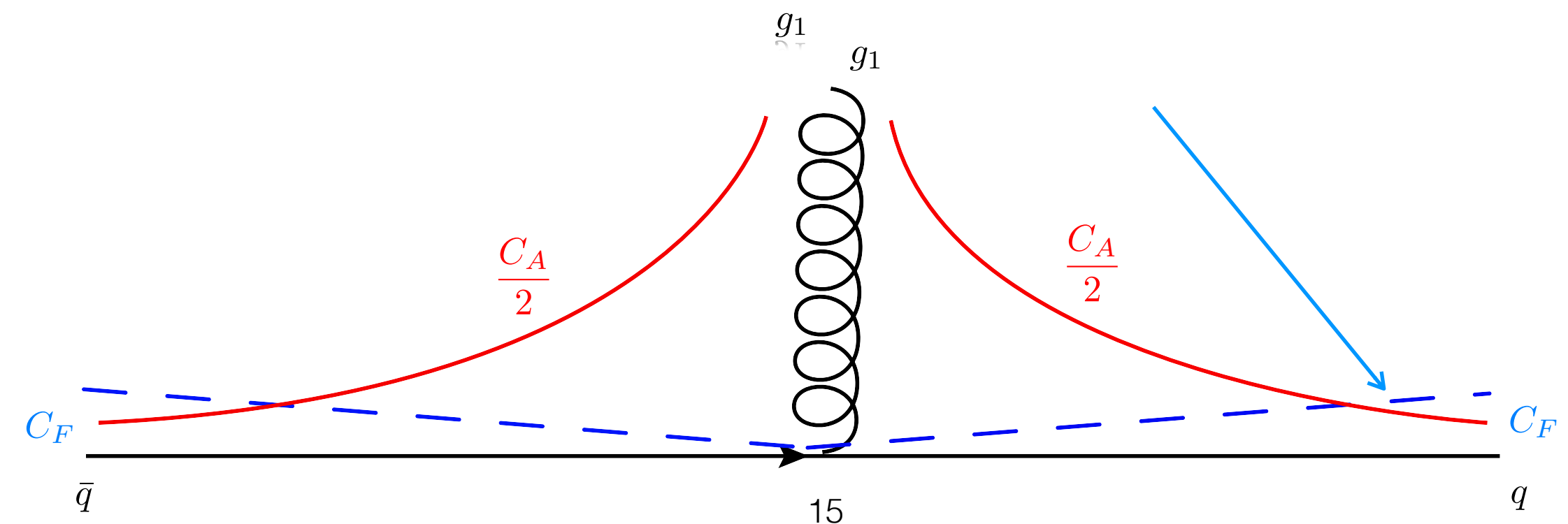
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- Recoil & color taken from g_1 even if the second emission is collinear to the quark: **breakdown of the independent emission picture**

Line of zero rapidity in the dipole's rest frame, boosted into the event frame



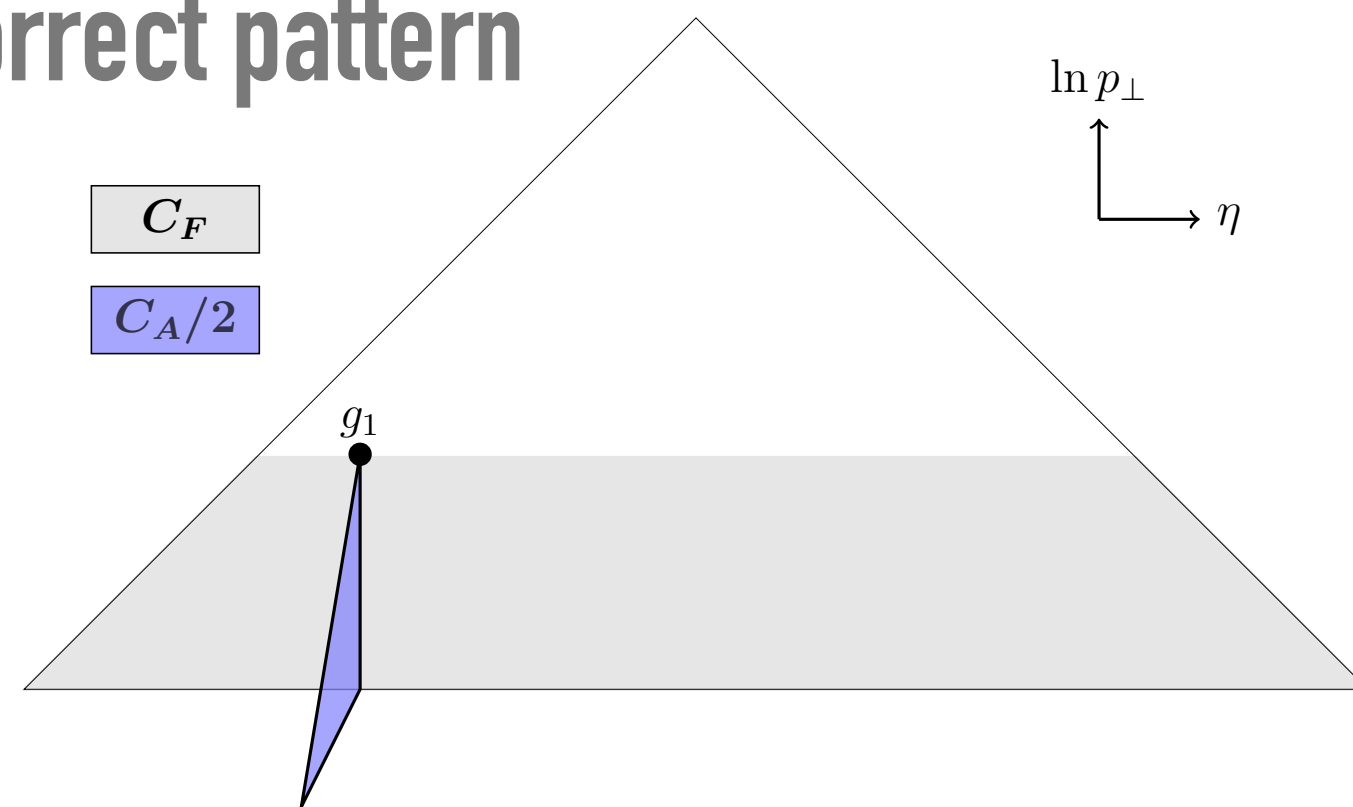
A case study: k_t ordering & local recoil

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '18;
see also Bewick, Ferrario Ravasio, Richardson, Seymour '19]

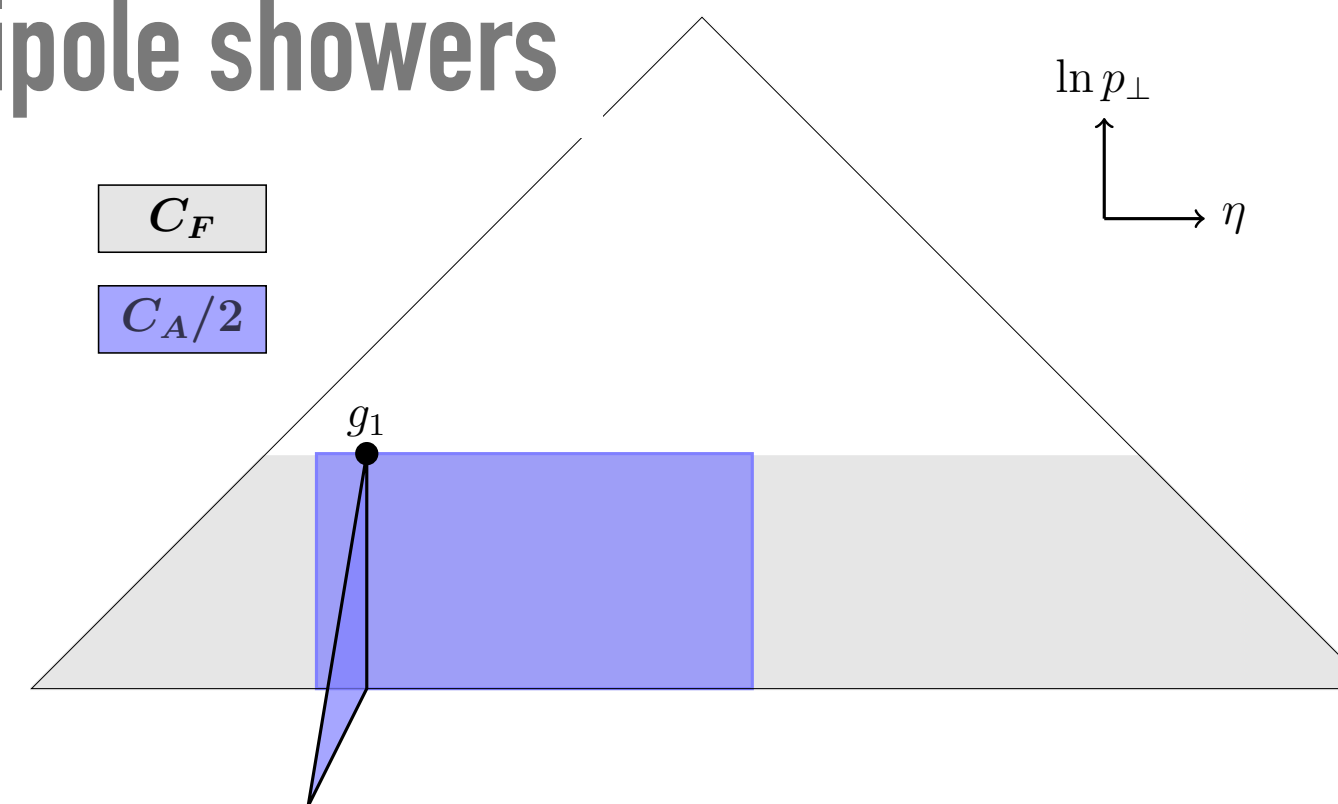
- Problems start at LL: even with strong k_T ordering (i.e. no kinematic recoil), the colour factors are assigned incorrectly, leading to a $1/N_c^2$ - suppressed mistake for specific observables

e.g. Double logarithmic difference from correct result for the Thrust

Correct pattern



Dipole showers

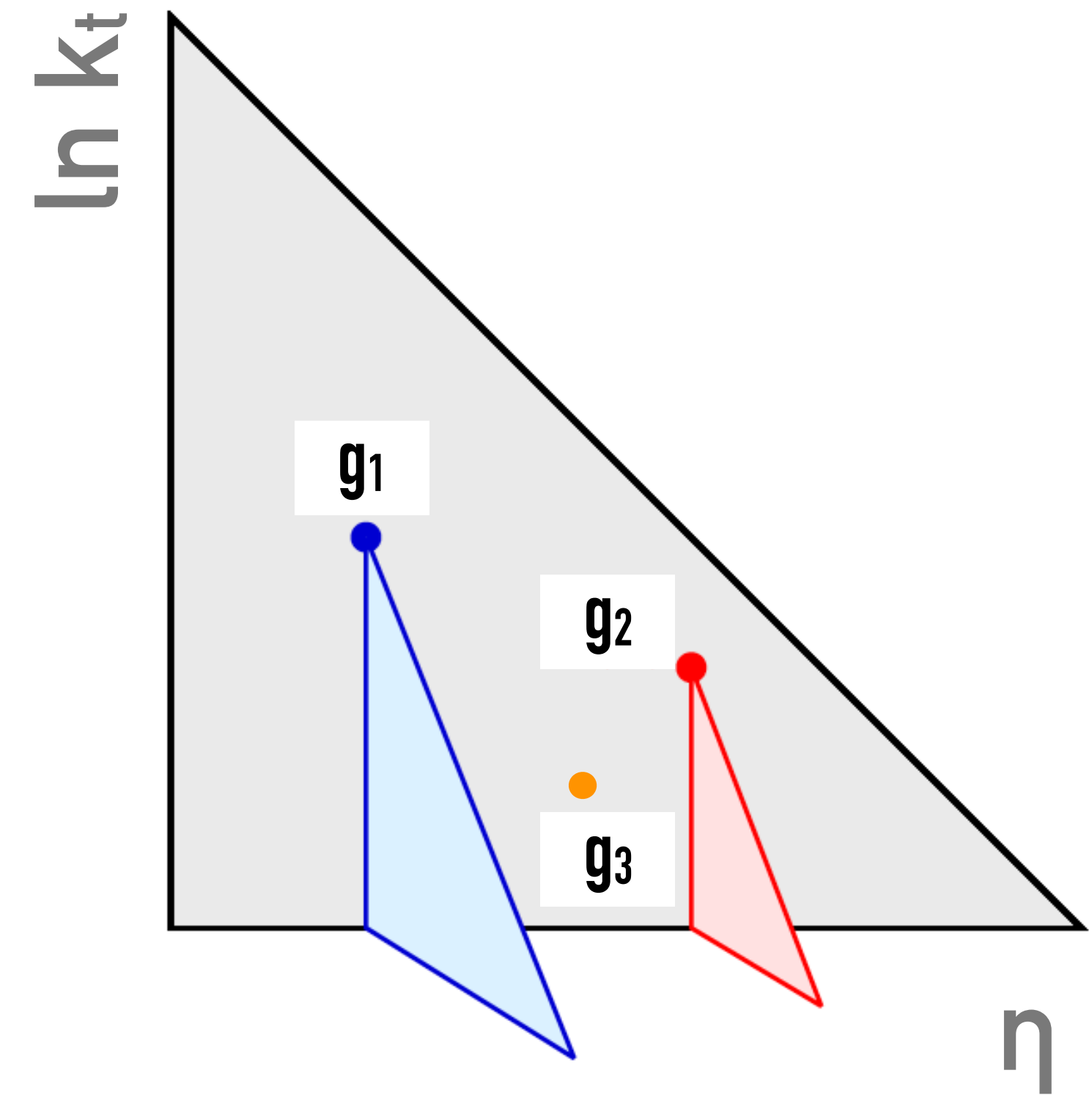
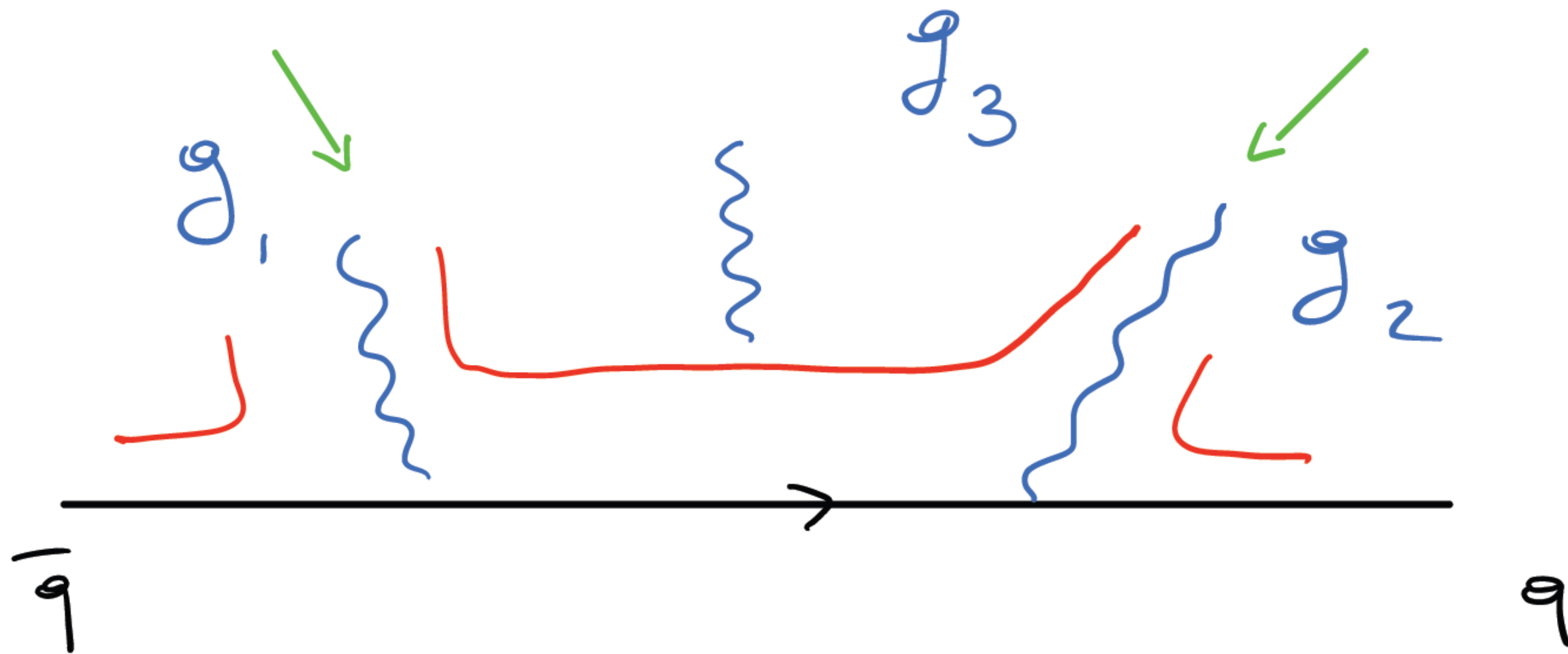


$$\delta\Sigma(L) = -\frac{1}{64} \left(\frac{2\alpha_s C_F}{\pi} \right)^2 L^4 \left(\frac{C_A}{2C_F} - 1 \right)$$

- At NLL the kinematic recoil plays a role, and all global observables formally have a problem originating from the above mechanism: $\alpha_s^n L^n$ terms wrong (see later)

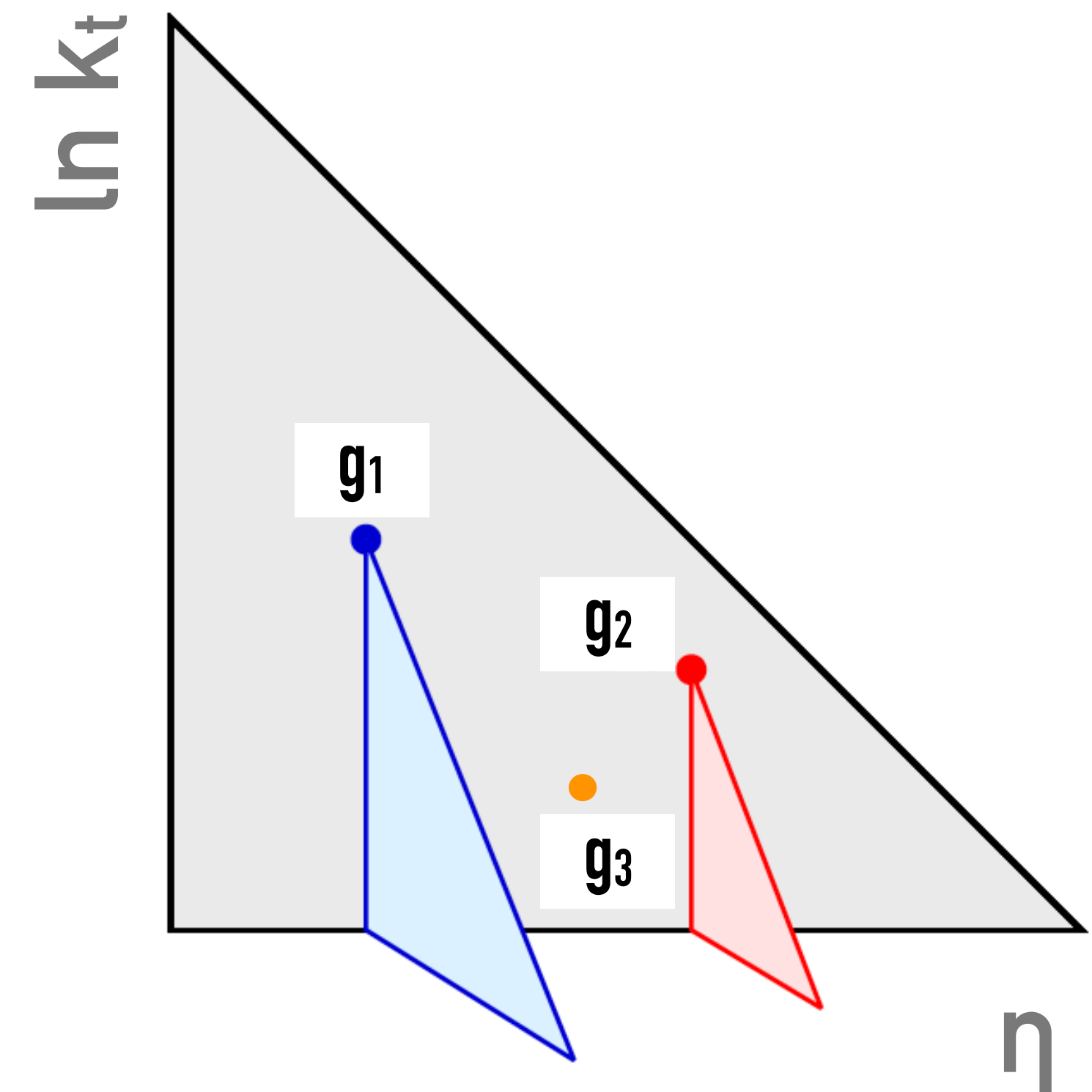
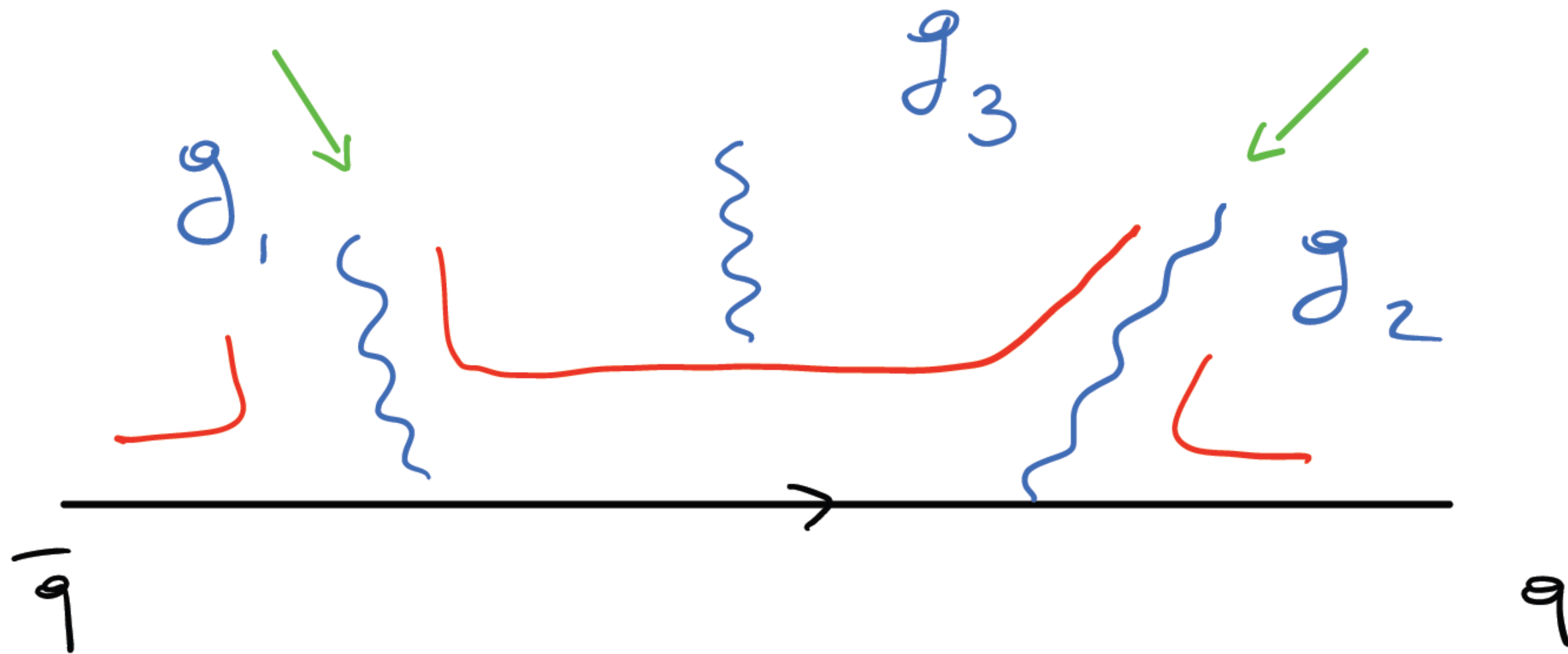
A case study: k_t ordering & local recoil

- ▶ Beyond α_s^2 the independent emission picture is further violated by emissions from soft $\{gg\}$ dipoles, for which the recoil is necessarily taken from one of the two soft ends rather than from the hard fermion line



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- ▶ Additional problems are discovered for classes of observables for which the recoil mechanism leads to a violation of coherence and the appearance of **super leading logarithms (SLL) starting at $O(\alpha_s^3)$ or $O(\alpha_s^4)$** , not predicted by QCD (cf. paper appendix).

Design of NLL parton showers

Some remarks

- ▶ Solving these problem demands a fundamental redesign of the dipole shower
 - ▶ Ultimately, we want to achieve a parton shower that is NLL accurate simultaneously for rIRC safe global, and for non-global observables across many collider processes. Crucial to build a framework to **demonstrate the formal accuracy** in a solid manner
 - ▶ Let's start from a clear theoretical environment: **e^+e^- collisions, large N_c limit**. Also, neglect for now azimuthal (spin) correlations, which are known at this order
- [Collins '88; Knowles '90; Super '08; Richardson & Webster '18]
- ▶ NB: QCD resummation provides us with guidelines, therefore **more than one solution is possible**. Difference between various NLL accurate solutions gives us a way to estimate the size of genuine subleading (NNLL) logarithmic corrections

The PanLocal shower (local recoil)

- ▶ Keep the recoil dipole-local, i.e. for each new emission

$$\text{dipole } \{\tilde{p}_i, \tilde{p}_j\} \longrightarrow \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp, \\ p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_\perp \end{aligned}$$

- ▶ Novel element #1: partitioning of the dipole (at $\bar{\eta} = 0$) occurs at equal angles between the emission and the dipole ends in the event c.o.m. frame

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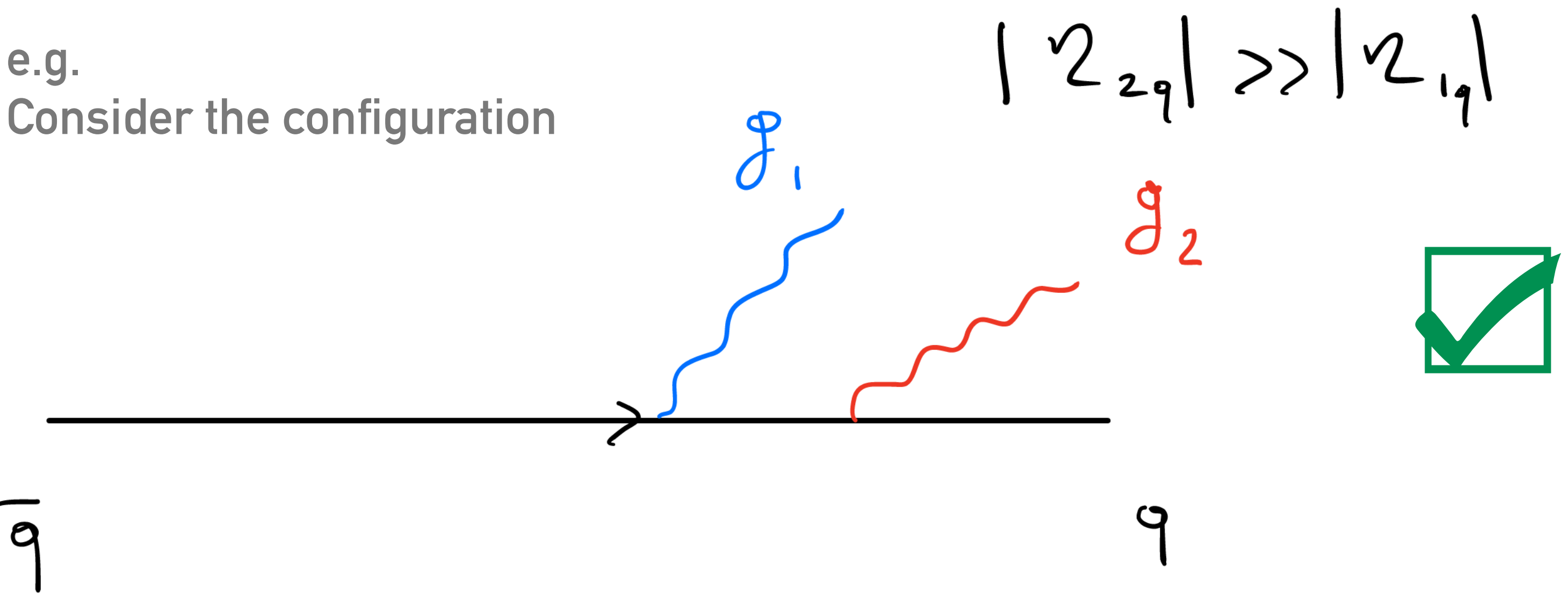
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- Novel element #1:** partitioning of the dipole (at $\bar{\eta} = 0$) occurs at equal angles between the emission and the dipole ends in the event c.o.m. frame

e.g.
Consider the configuration



In the limit of strong angular ordering and commensurate k_T 's, g_2 takes the recoil from the hard quark

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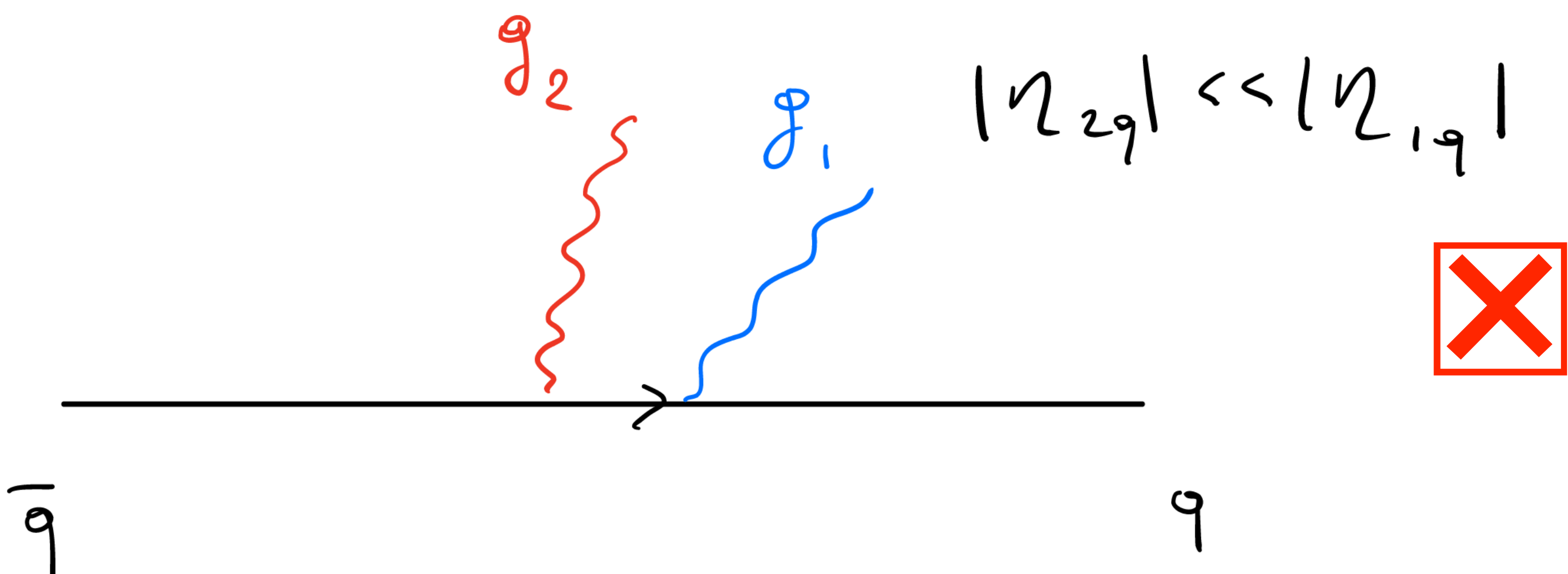
dipole $\{\tilde{p}_i, \tilde{p}_j\}$ \longrightarrow

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp},$$

$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j - f k_{\perp},$$

$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_{\perp}$$

► Novel element #1: partitioning of the dipole (at $\bar{\eta} = 0$) occurs at equal angles between the emission and the dipole ends in the event c.o.m. frame



Instead, if g_2 is produced at larger rapidities than g_1 , and they are both collinear to the quark, the recoil is still taken from g_1 in a logarithmic (NLL) region of phase space

The PanLocal shower (local recoil)

- ▶ Keep the recoil dipole-local, i.e. for each new emission

$$\text{dipole } \{\tilde{p}_i, \tilde{p}_j\} \longrightarrow \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp, \\ p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_\perp \end{aligned}$$

- ▶ Novel element #2: use an evolution variable v defined as ($\beta < 1$)

$$k_t = \rho v e^{\beta |\bar{\eta}|} \sim v e^{\beta |\eta|^{\text{w.r.t. emitter}}|} \quad \rho = \left(\frac{s_{\tilde{i}} s_{\tilde{j}}}{Q^2 s_{\tilde{i}\tilde{j}}} \right)^{\frac{\beta}{2}}$$

- ▶ Choosing $\beta > 0$ effectively reproduces angular ordering in the limit of commensurate k_t 's and strong angular separation (**NB: k_t ordering not allowed in local recoil scheme**)

The PanLocal shower (local recoil)

► Keep the recoil dipole-local, i.e. for each new emission

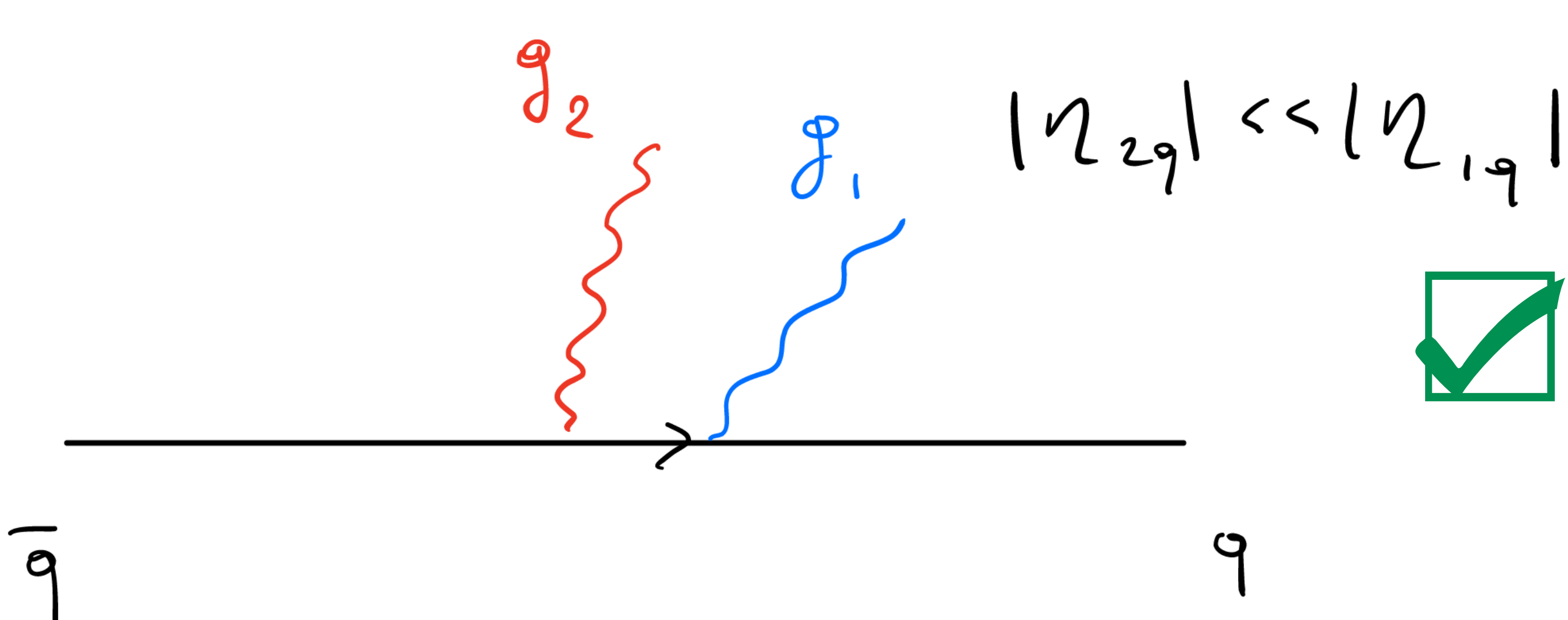
dipole $\{\tilde{p}_i, \tilde{p}_j\}$ \longrightarrow

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp},$$

$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j - f k_{\perp},$$

$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_{\perp}$$

► Novel element #2: use an evolution variable v defined as ($\beta < 1$)



- Ordering in v now implies that $k_{t2} \ll k_{t1}$ [i.e. no recoil]
- The combination of partition \oplus ordering creates a mechanism in which the recoil is always taken from the hard extremities of the dipole chain [correct at NLL]

The PanGlobal shower (local recoil)

- ▶ Longitudinal recoil is kept dipole local

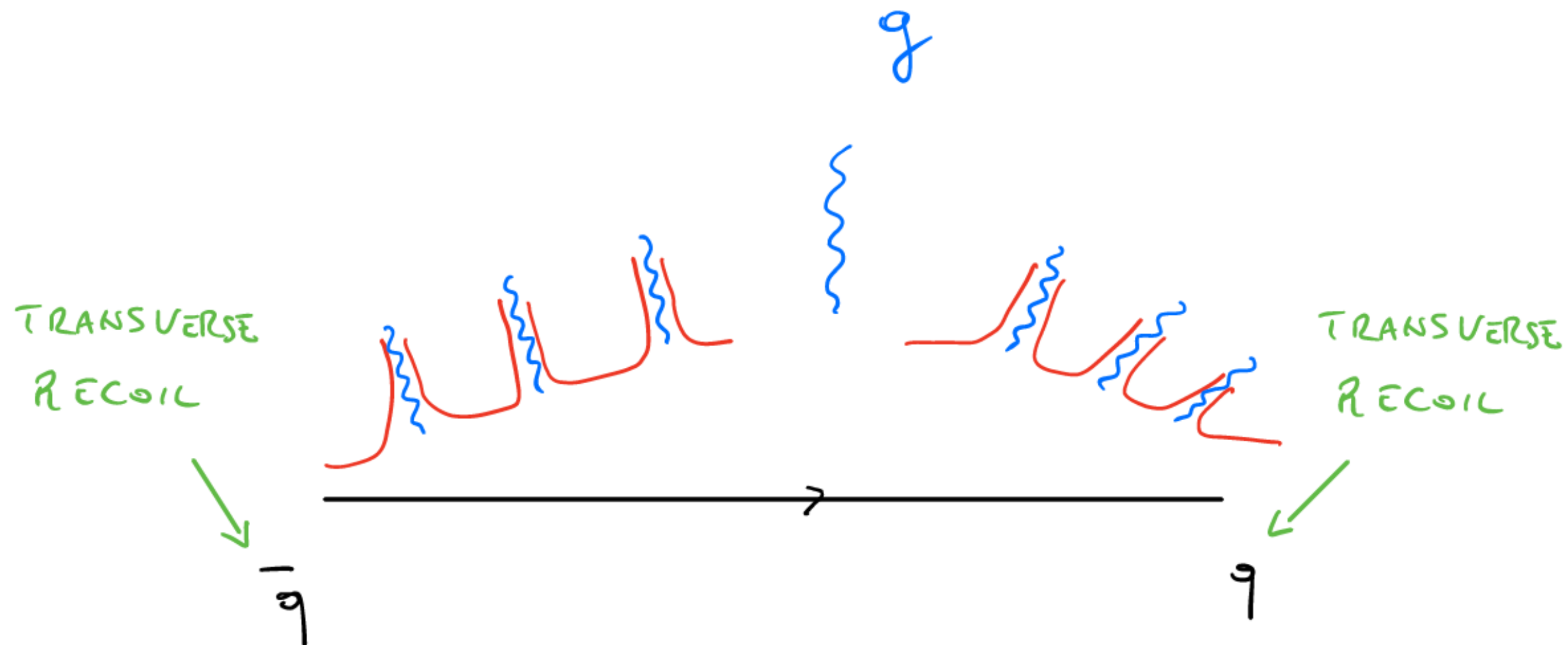
dipole $\{\tilde{p}_i, \tilde{p}_j\}$ \longrightarrow

$$\bar{p}_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp},$$

$$\bar{p}_i = (1 - a_k) \tilde{p}_i,$$

$$\bar{p}_j = (1 - b_k) \tilde{p}_j.$$

- ▶ Transverse recoil is distributed globally across the event via a Lorentz boost + rescaling i.e. recoil is taken from (and shared among) the hard extremities of the dipole string



- ▶ With this scheme, also **k_t ordering is now a viable option**

Testing the logarithmic accuracy of a parton shower

- ▶ Definition of observables (sometimes new) sensitive to different aspects of QCD radiation
- ▶ Formulation of a toy model of each shower algorithm
(soft limit, fixed coupling, simplified kinematics, primary radiation only):
 - ▶ All-order numerical tests against NLL calculation in the same toy model
 - ▶ Fixed order calculations up to $O(\alpha_s^4)$, numerical and analytic.
 - ▶ reveal issues that give small effects when resummed
(e.g. spurious super-leading logarithms)
- ▶ Accuracy tests in the full shower, algorithmic optimisation necessary
 - ↳ **Discussed in the following**

Testing the logarithmic accuracy of a parton shower

- ▶ Tests of logarithmic accuracy in the full shower against NLL resummations:
 - ▶ Consider cumulative distributions for an observable (e.g. jet rate, event shapes, ...) in the limit $\alpha_s |L| \sim 1$, and $|L| \gg 1$

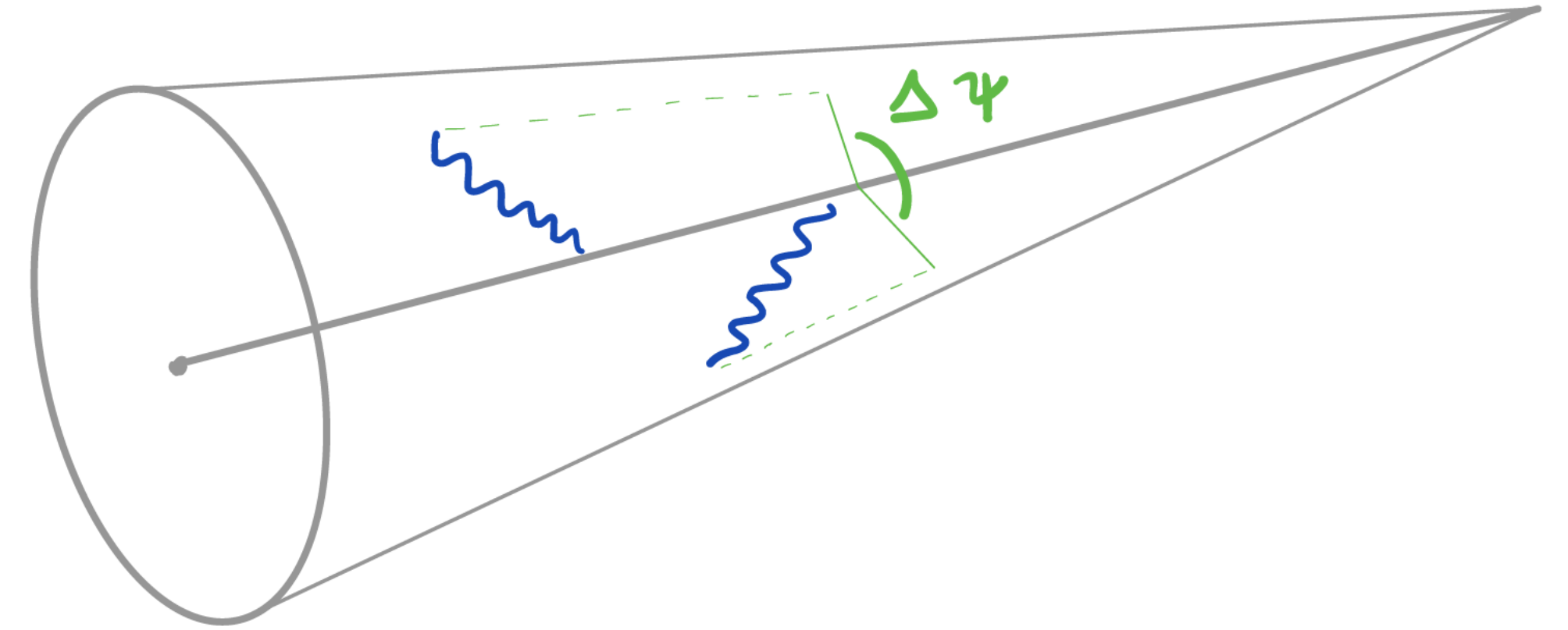
$$\Sigma(\alpha_s, \alpha_s L) = \exp \left[\overset{\text{LL}}{\underset{(\text{=0 sometimes})}{\alpha_s^{-1} g_1(\alpha_s L)}} + \overset{\text{NLL}}{g_2(\alpha_s L)} + \mathcal{O}(\alpha_s^n L^{n-1}) \right]$$

- ▶ Compute the ratio $\frac{\Sigma_{\text{PS}}}{\Sigma_{\text{NLL}}}$

- ▶ PS is LL: Σ_{PS} misses $\mathbf{O(1)}$ corrections, i.e. $\lim_{\alpha_s \rightarrow 0} \frac{\Sigma_{\text{PS}}}{\Sigma_{\text{NLL}}} \neq 1$

- ▶ PS is NLL: Σ_{PS} misses $\mathbf{O(\alpha_s)}$ corrections, i.e. $\lim_{\alpha_s \rightarrow 0} \frac{\Sigma_{\text{PS}}}{\Sigma_{\text{NLL}}} = 1$

An example: azimuthal substructure of jets

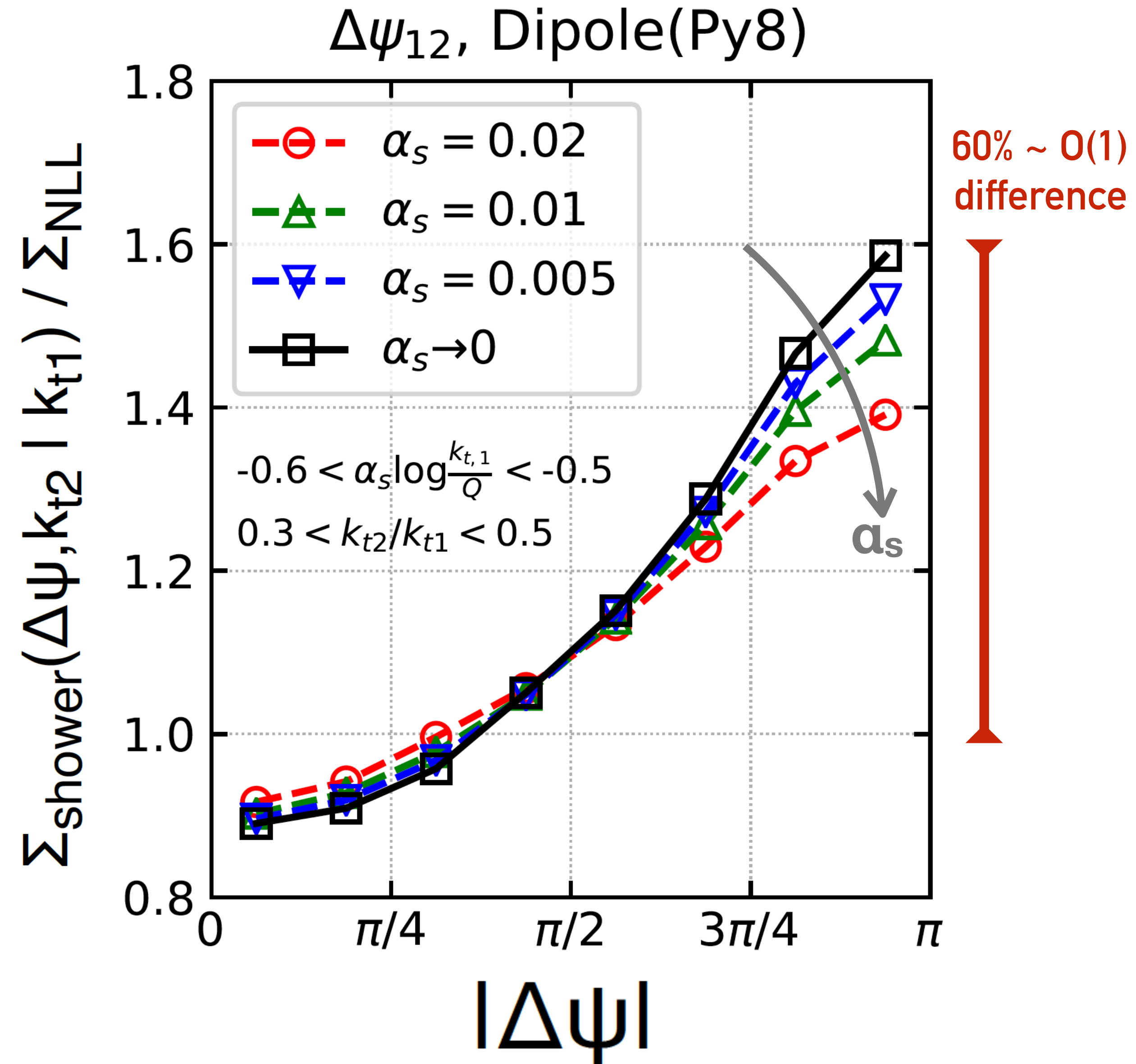


- Definition of Lund Jet Plane (LJP) based azimuthal angle between two leading primary declusterings [actual definition involves a dynamical frame]

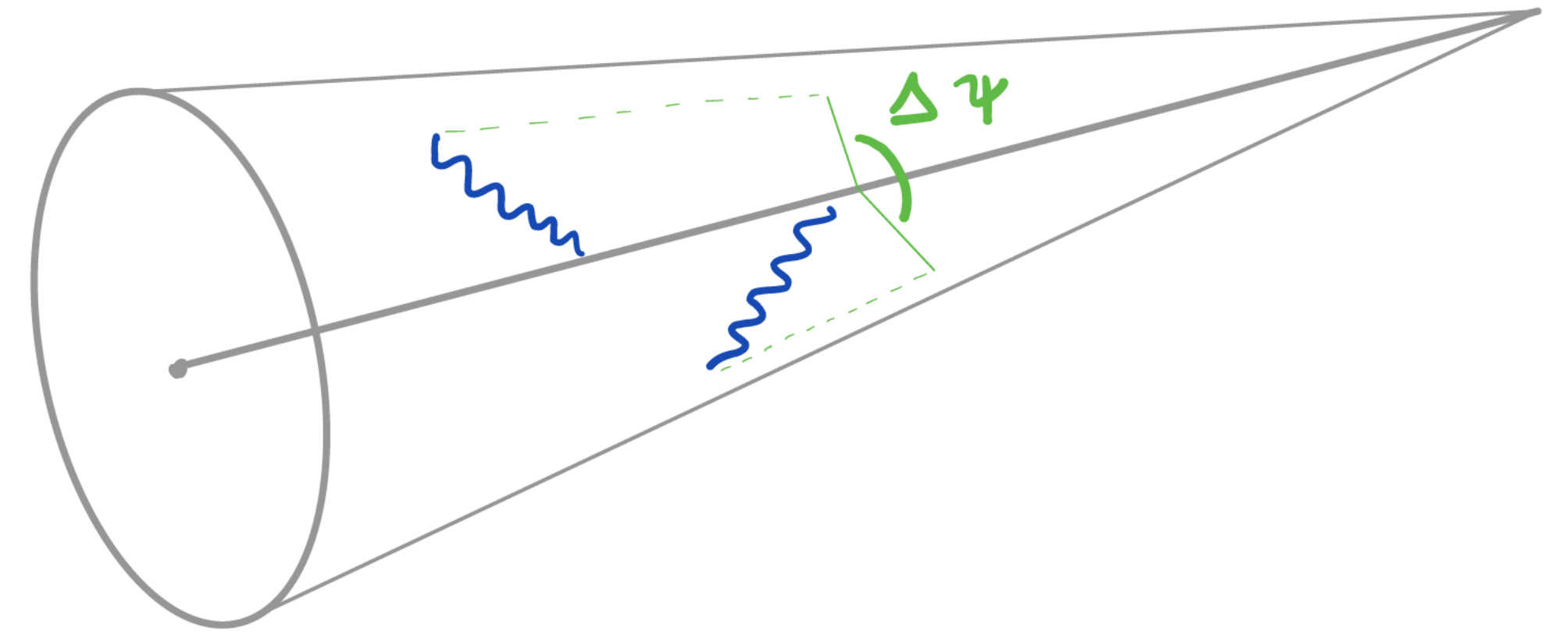
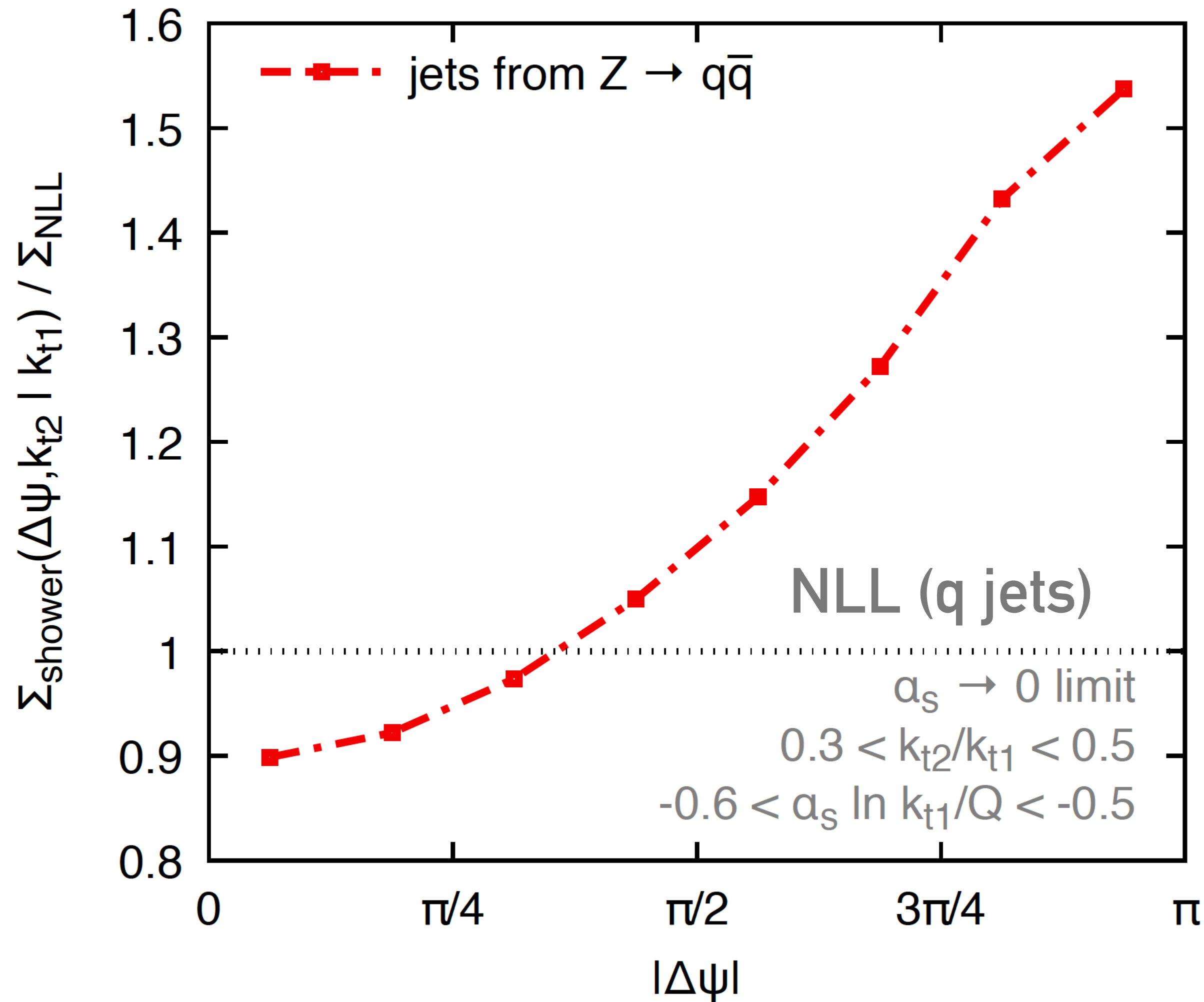
An example: azimuthal substructure of jets

e.g. let's consider the $\Delta\Psi$ distribution given earlier. Ratio to NLL shows a residual & non-trivial shape difference in the limit $\alpha_s \rightarrow 0$.

➔ The observed discrepancy is due to the unphysical features in the (transverse) recoil assignment which fails to reproduce the correct NLL matrix elements. This translates into a breaking of NLL accuracy



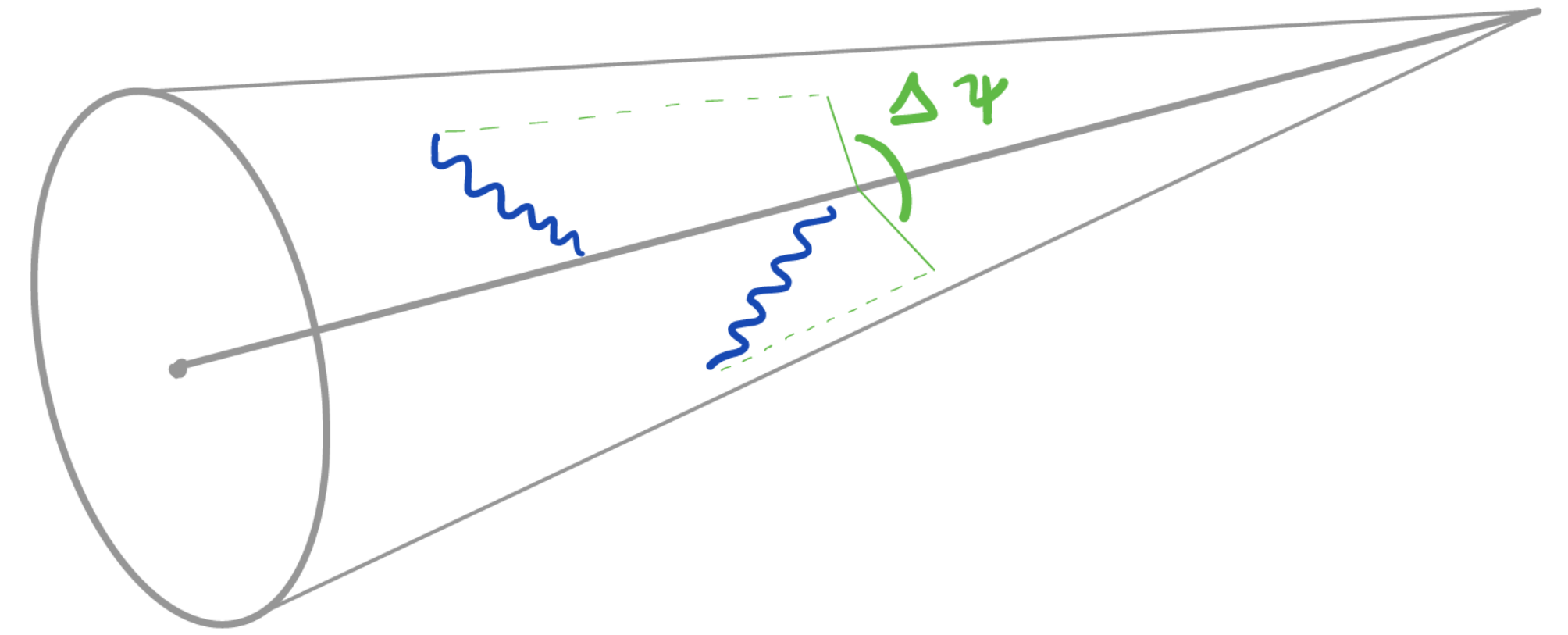
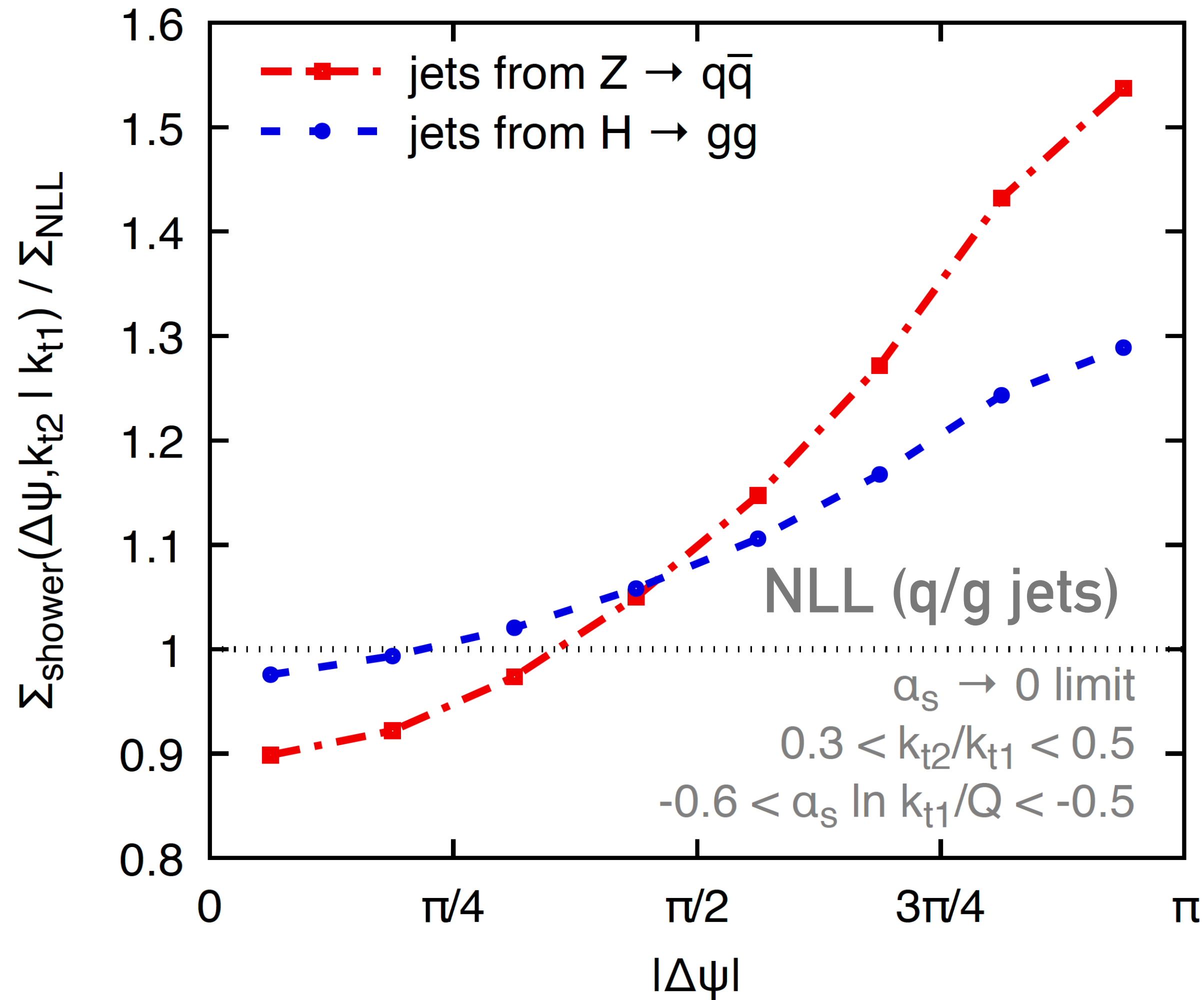
An example: azimuthal substructure of jets



- $\Delta\Psi$ distribution is uniform at NLL, while modern dipole showers (e.g. Pythia8 / Dire) predict a non-trivial shape

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]
 cf. [Dreyer, Salam, Soyez '18] for construction of the Lund Jet Plane

An example: azimuthal substructure of jets



- $\Delta\Psi$ distribution is uniform at NLL, while modern dipole showers (e.g. Pythia8 / Dire) predict a non-trivial shape
- Unphysical dependence on jet flavour, with implications for q/g jet discrimination if Machine Learning tools learn these features.

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]
 cf. [Dreyer, Salam, Soyez '18] for construction of the Lund Jet Plane

An example: azimuthal substructure of jets

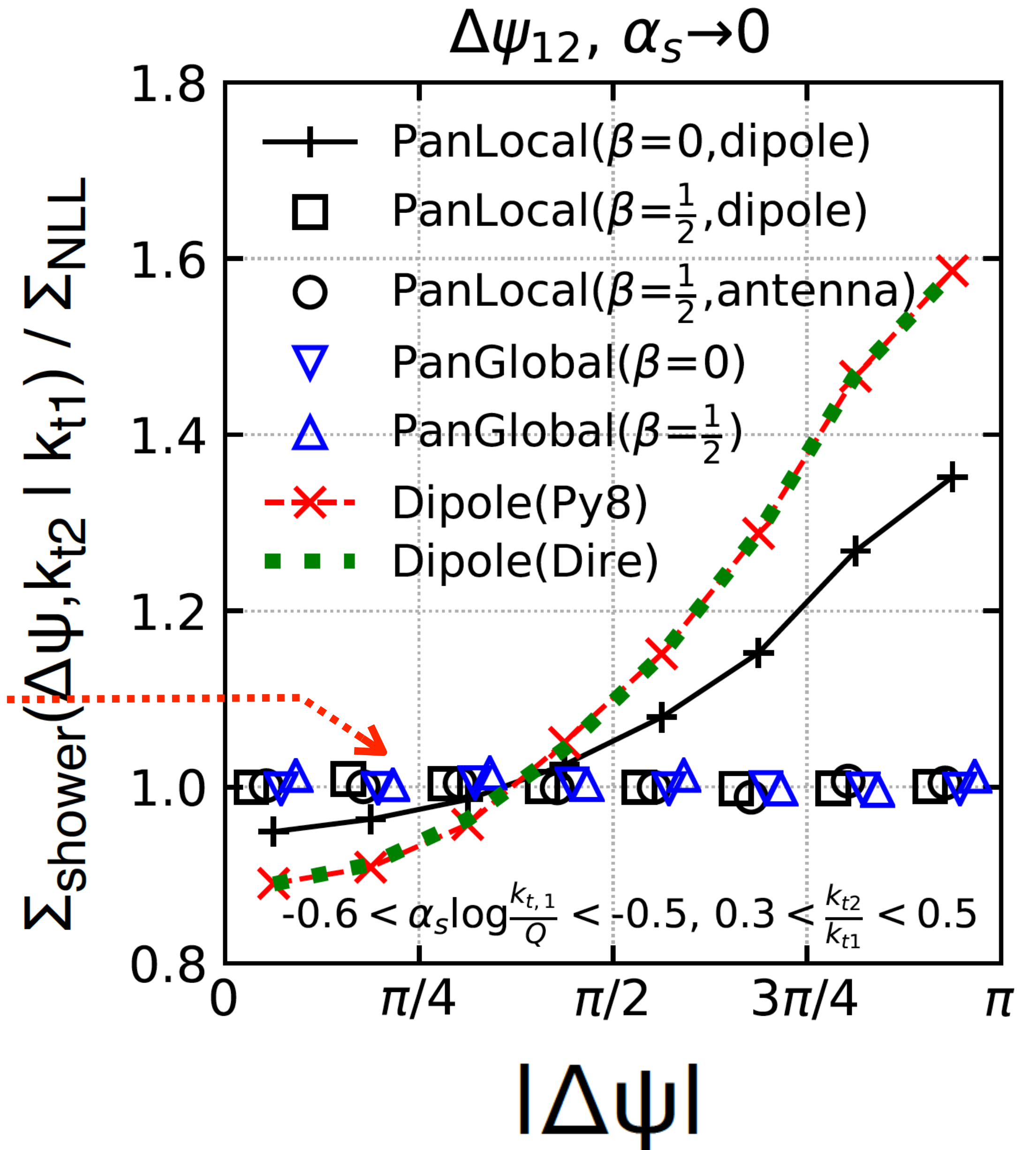
► Use input from NLL resummations to construct a parton shower alg. that constructs the correct multiparton squared amplitudes in the relevant kinematic limits

➡ New classes of NLL shower algorithms (PanLocal = local recoil map; PanGlobal = global recoil map) reproduce correct NLL results as expected

[Dasgupta, Dreyer, Hamilton, PM, Salam '18]

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]

See also [Forhsaw, Holguin, Plaetzer '20]



Accuracy across many observables

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]

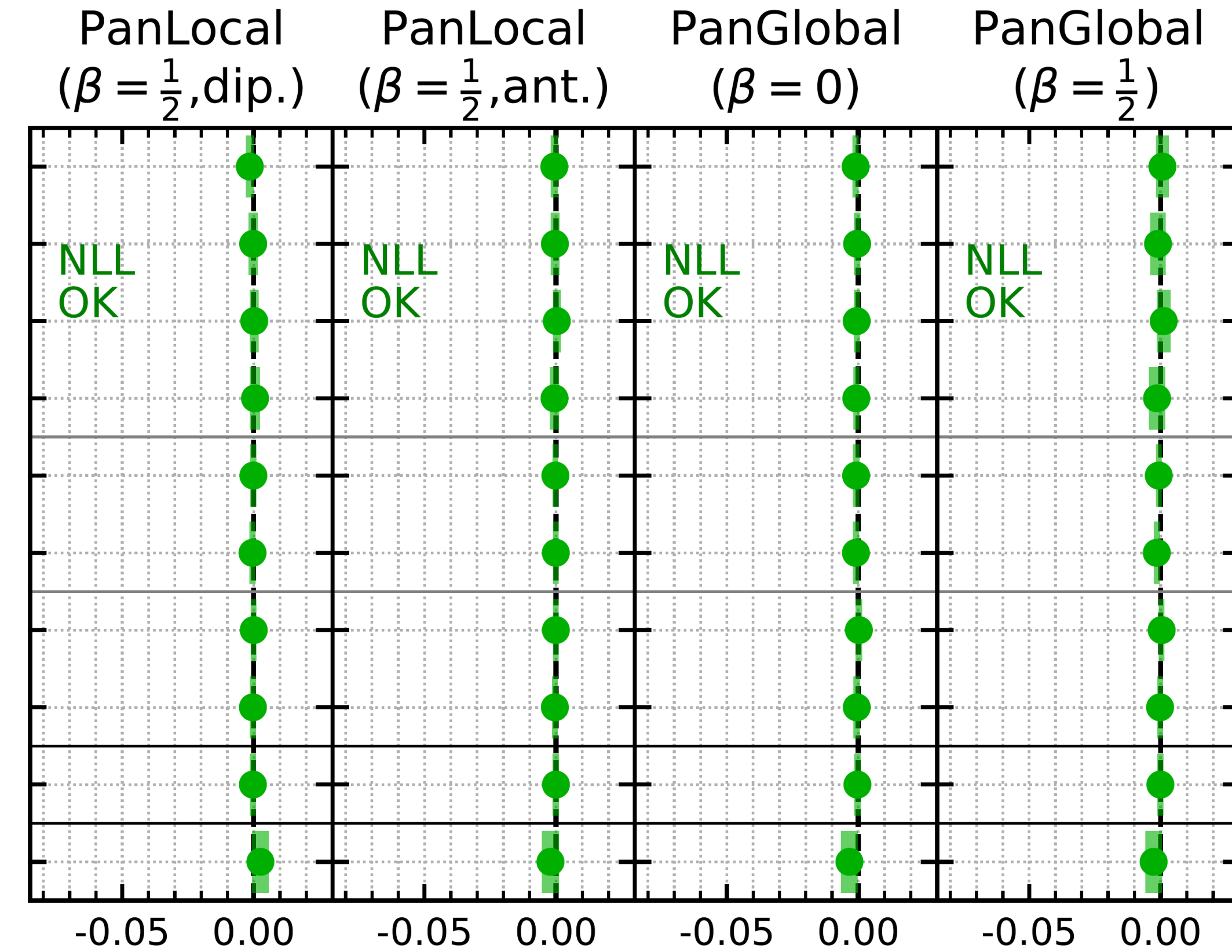
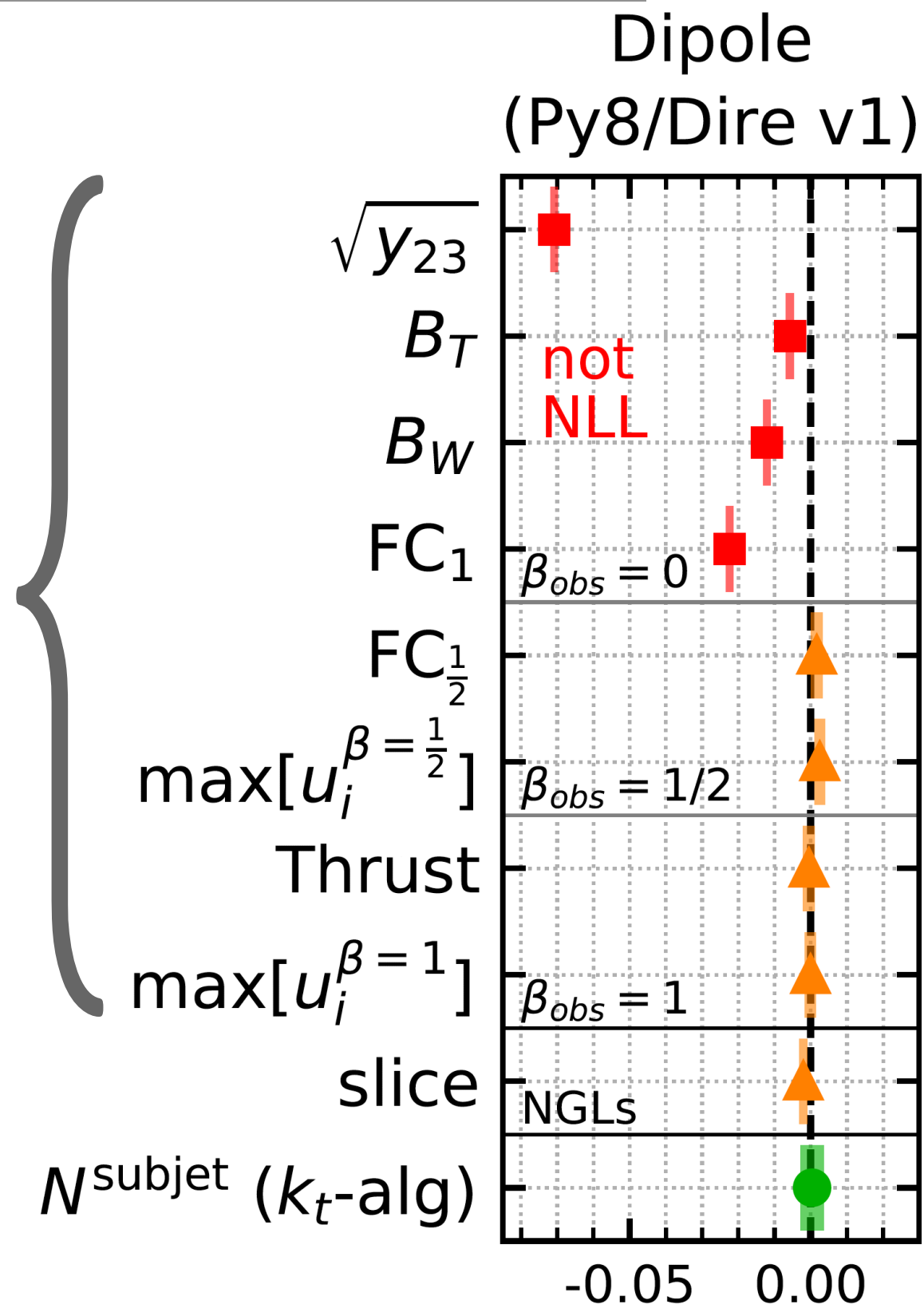
Plots: relative deviation from exact NLL
[in $\alpha_s \rightarrow 0$ limit at fixed $\alpha_s L$]

Global observables:
sensitive to strong η (θ)
separation at NLL

e.g.

$$T = \max_{\hat{n}} \frac{\sum_i |\mathbf{p}_i \cdot \hat{n}|}{\sum_i |\mathbf{p}_i|} \quad [\text{Thrust}]$$

$$FC_x = \sum_{i \neq j} \frac{E_i E_j}{(\sum_i E_i)^2} |\sin \theta_{ij}|^x (1 - |\cos \theta_{ij}|)^{1-x} \Theta((\mathbf{p}_i \cdot \hat{n})(\mathbf{p}_j \cdot \hat{n})) \quad [\text{Moment of Energy-Energy correlation}]$$



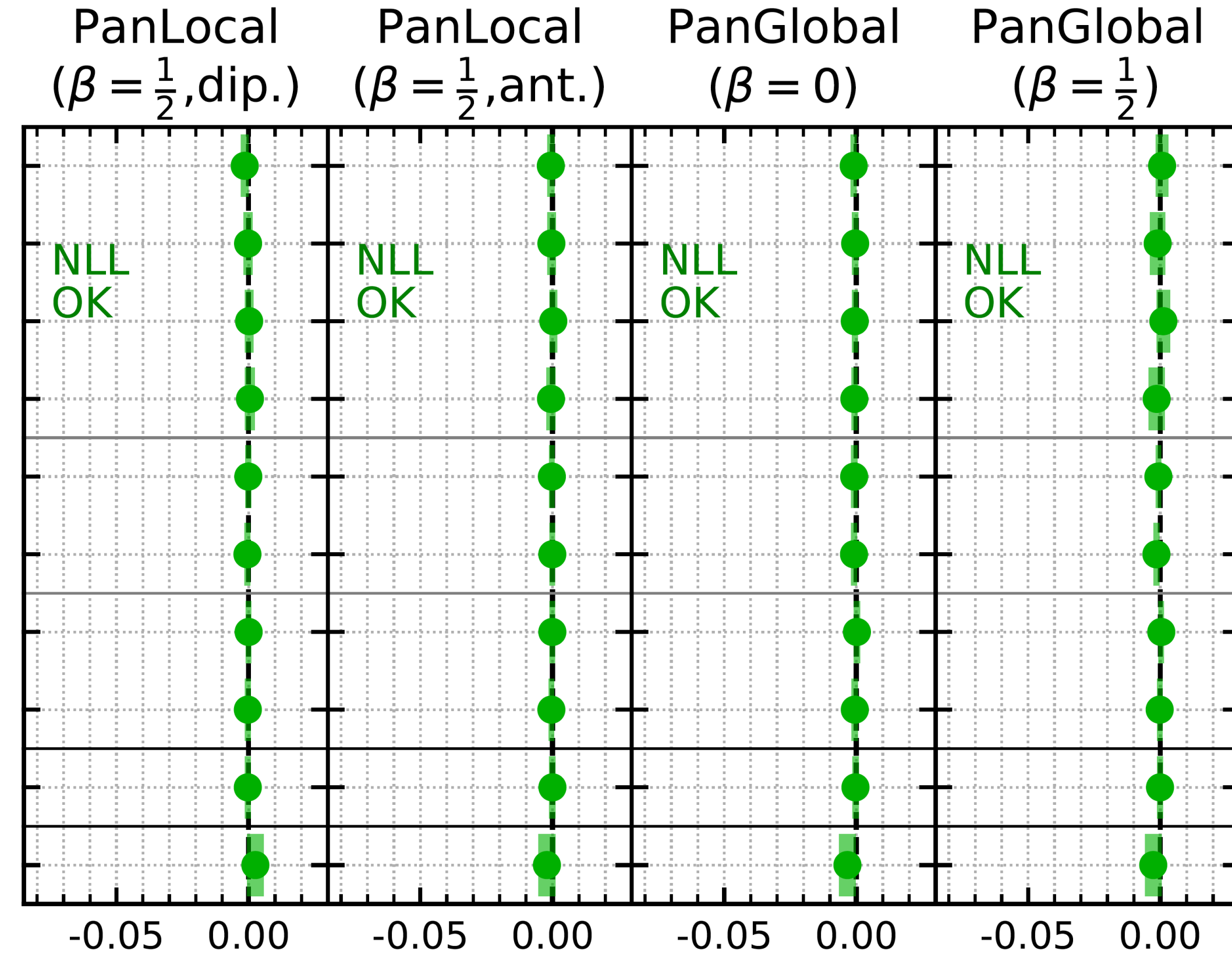
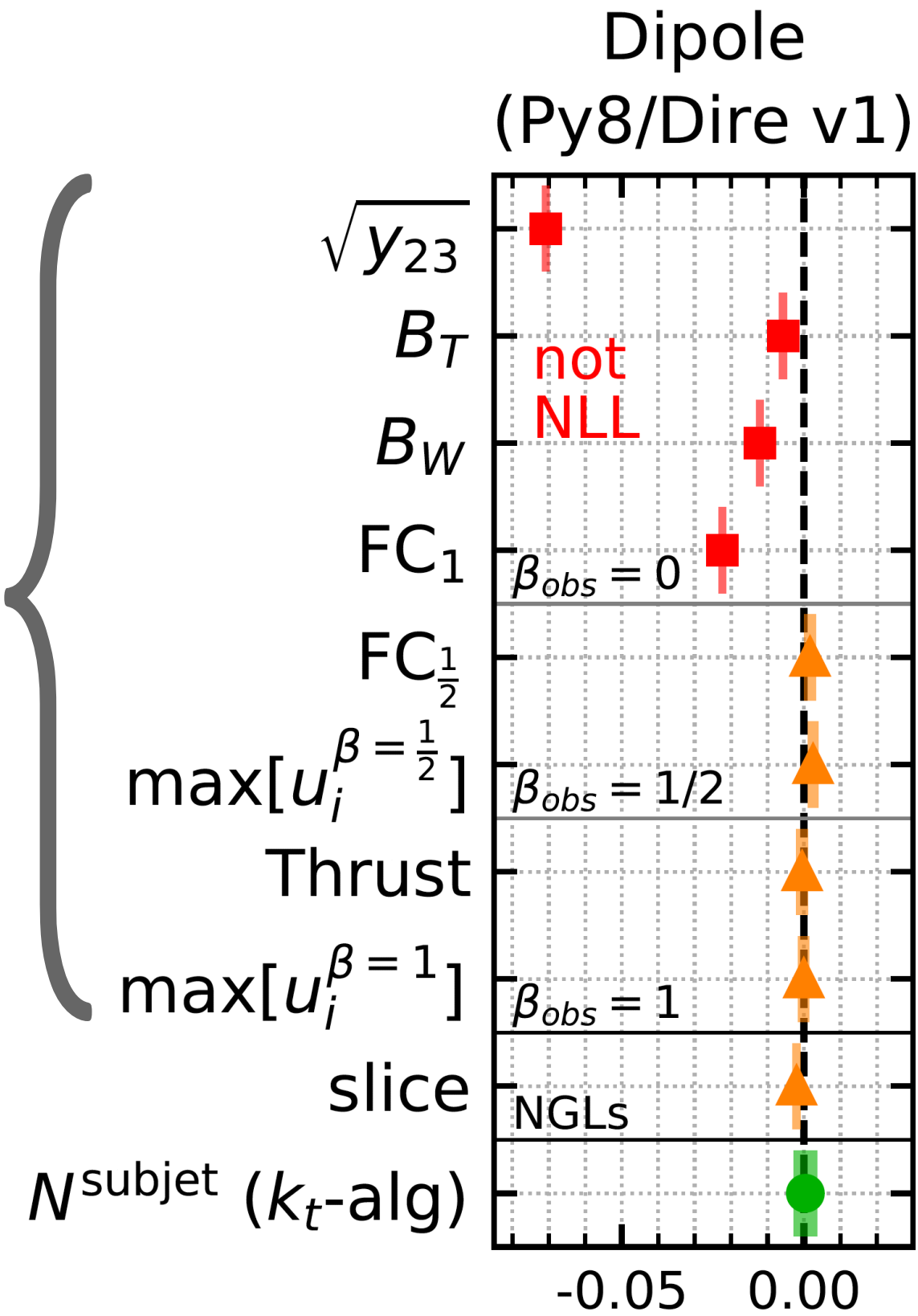
Accuracy across many observables

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]

Plots: relative deviation from exact NLL
[in $\alpha_s \rightarrow 0$ limit at fixed $\alpha_s L$]

↓ [Sjostrand et al. '15]
[Hoeche, Prestel '15]

Global observables:
sensitive to strong η (θ)
separation at NLL



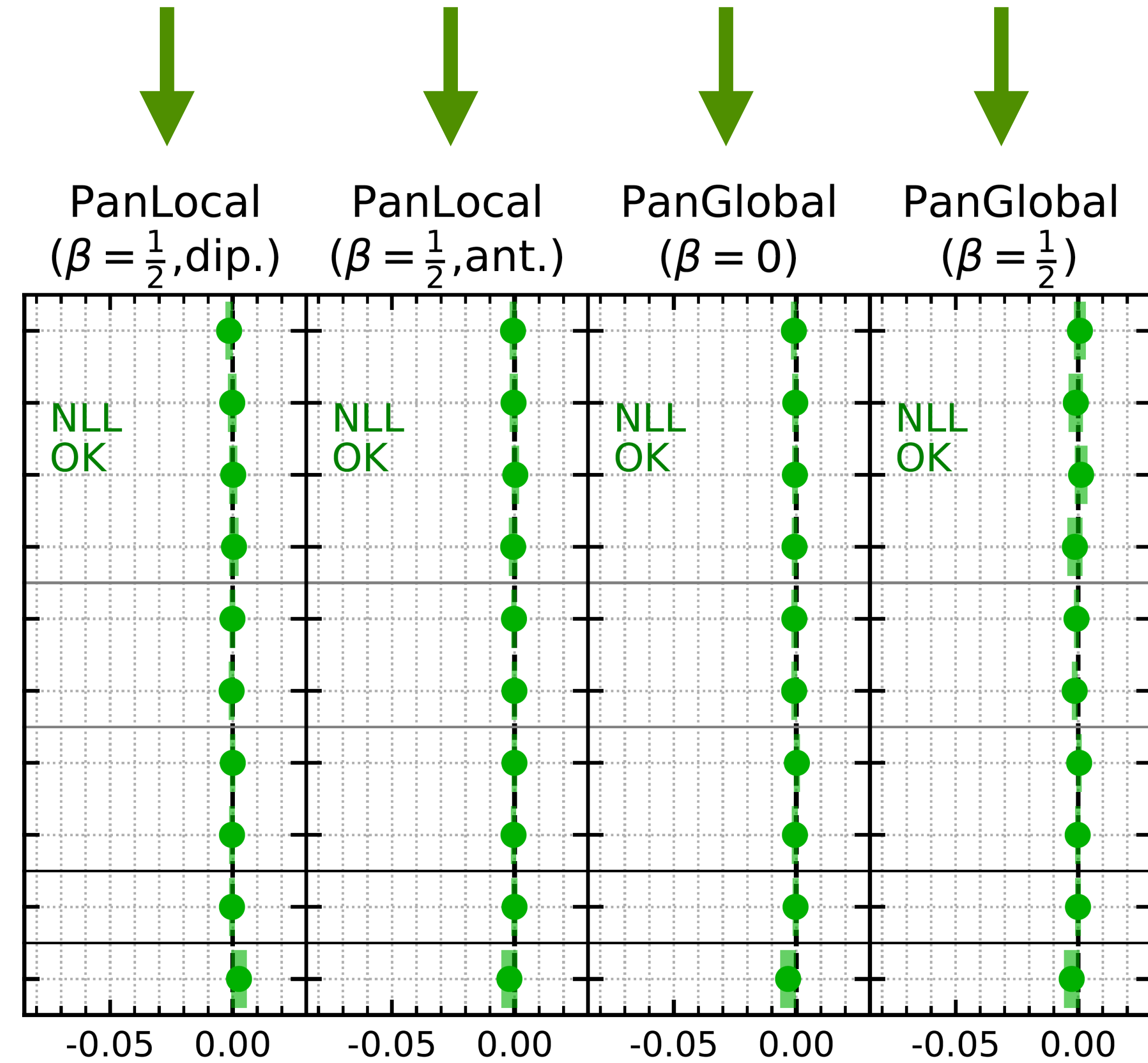
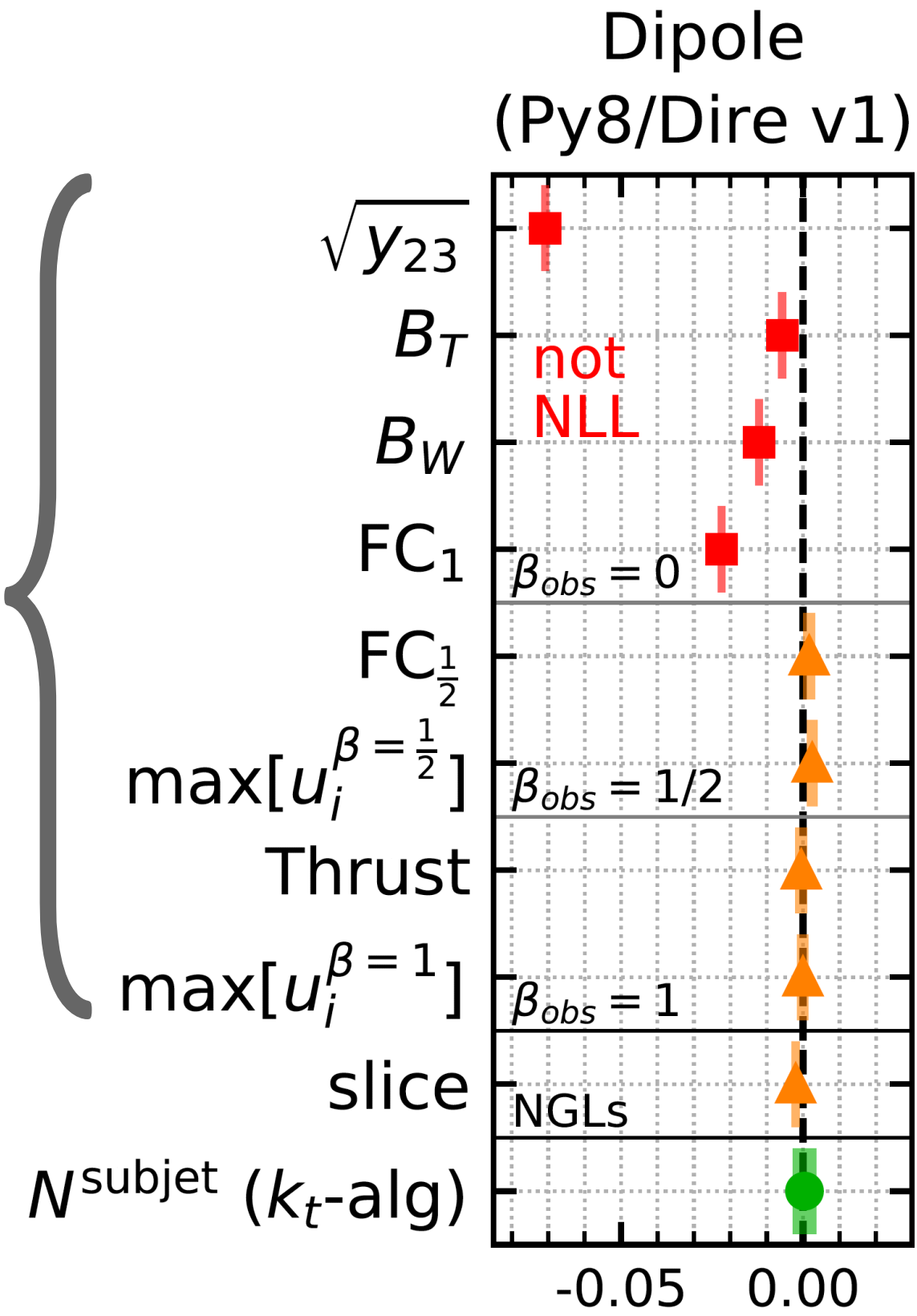
Orange triangles indicate spurious terms (either NLL or SLL) at fixed order, that become small when resummed

Accuracy across many observables

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]

Plots: relative deviation from exact NLL
[in $\alpha_s \rightarrow 0$ limit at fixed $\alpha_s L$]

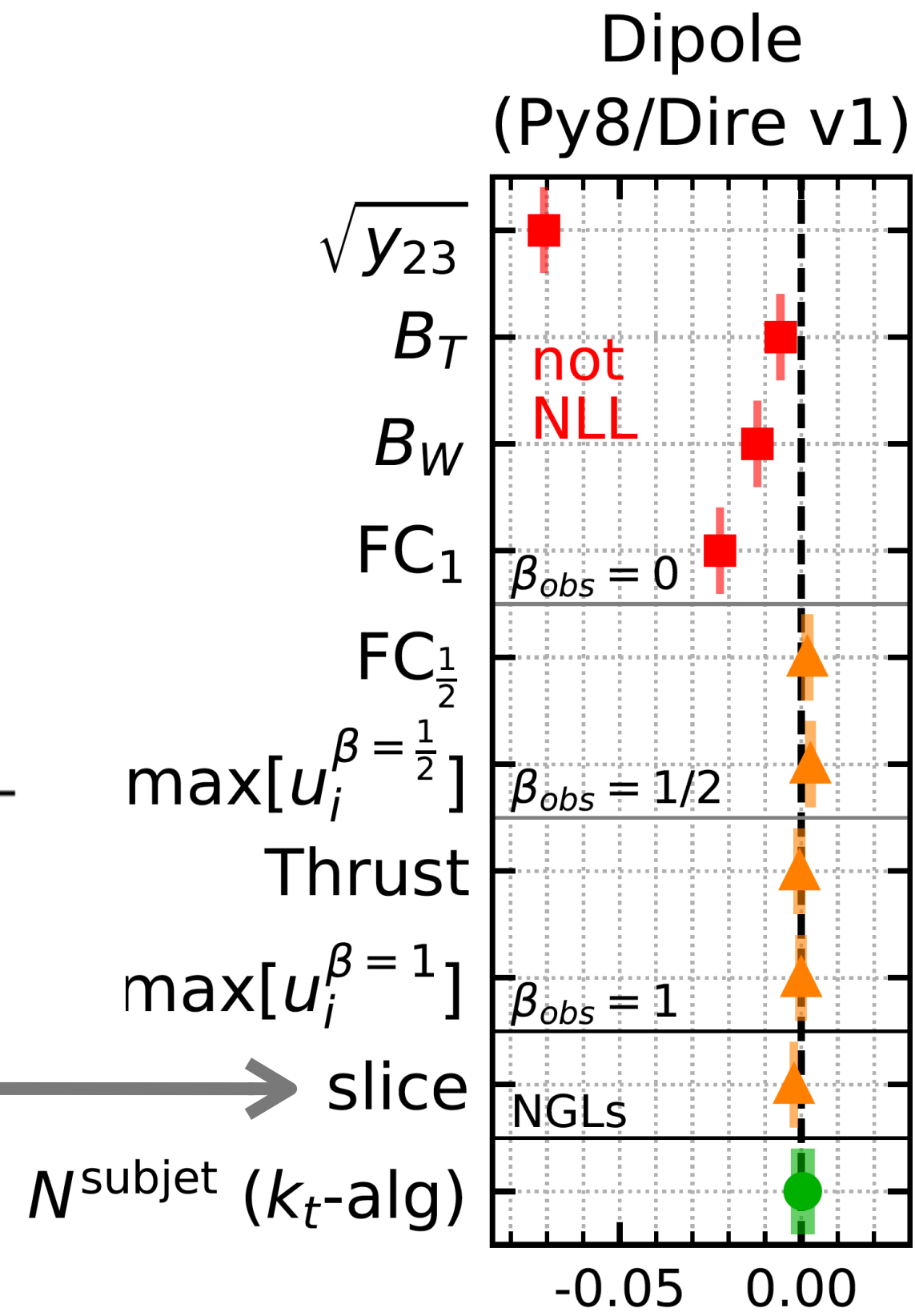
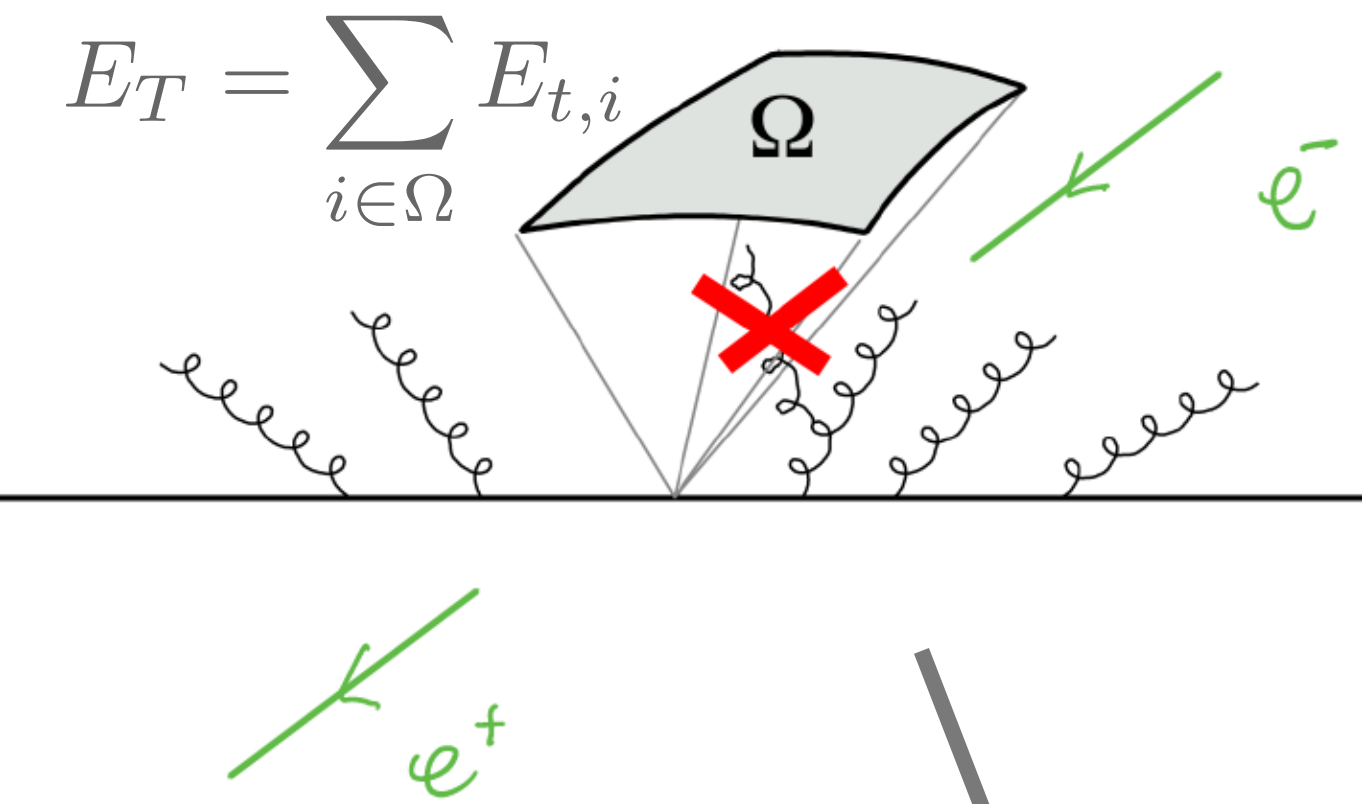
Global observables:
sensitive to strong η (θ)
separation at NLL



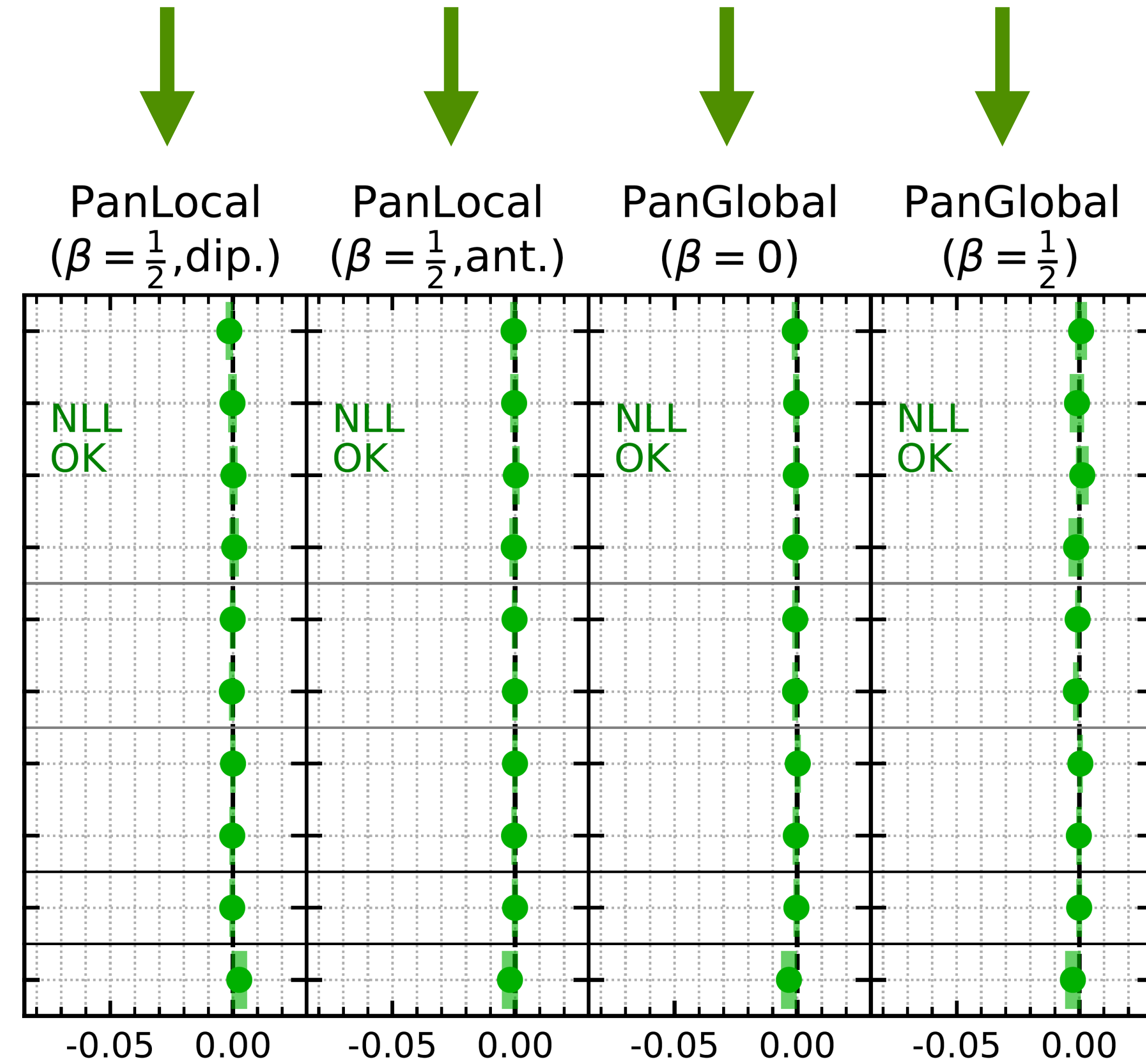
Accuracy across many observables

Non-global observables:
sensitive to strong kt (E)
separation at NLL

e.g.

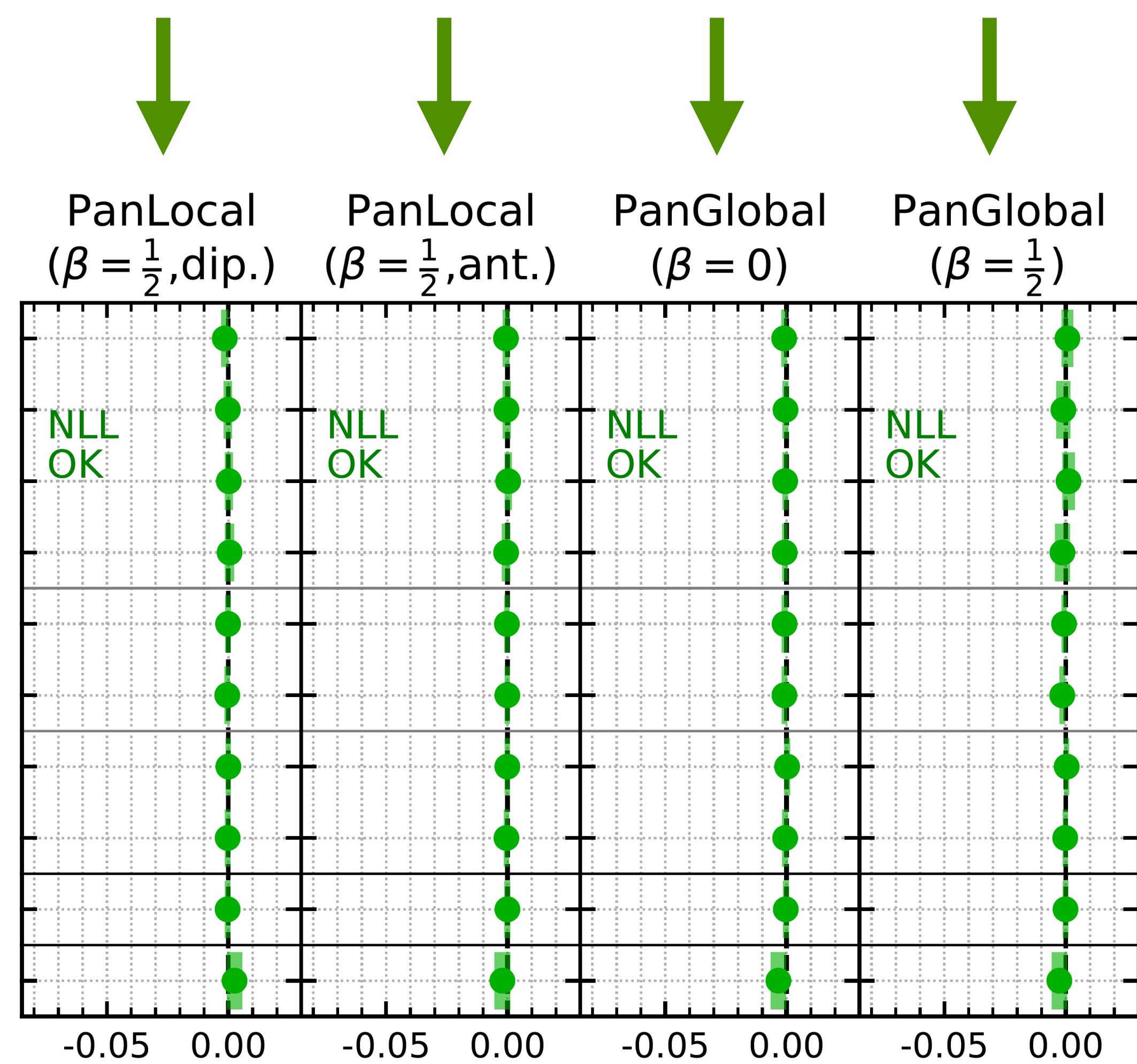
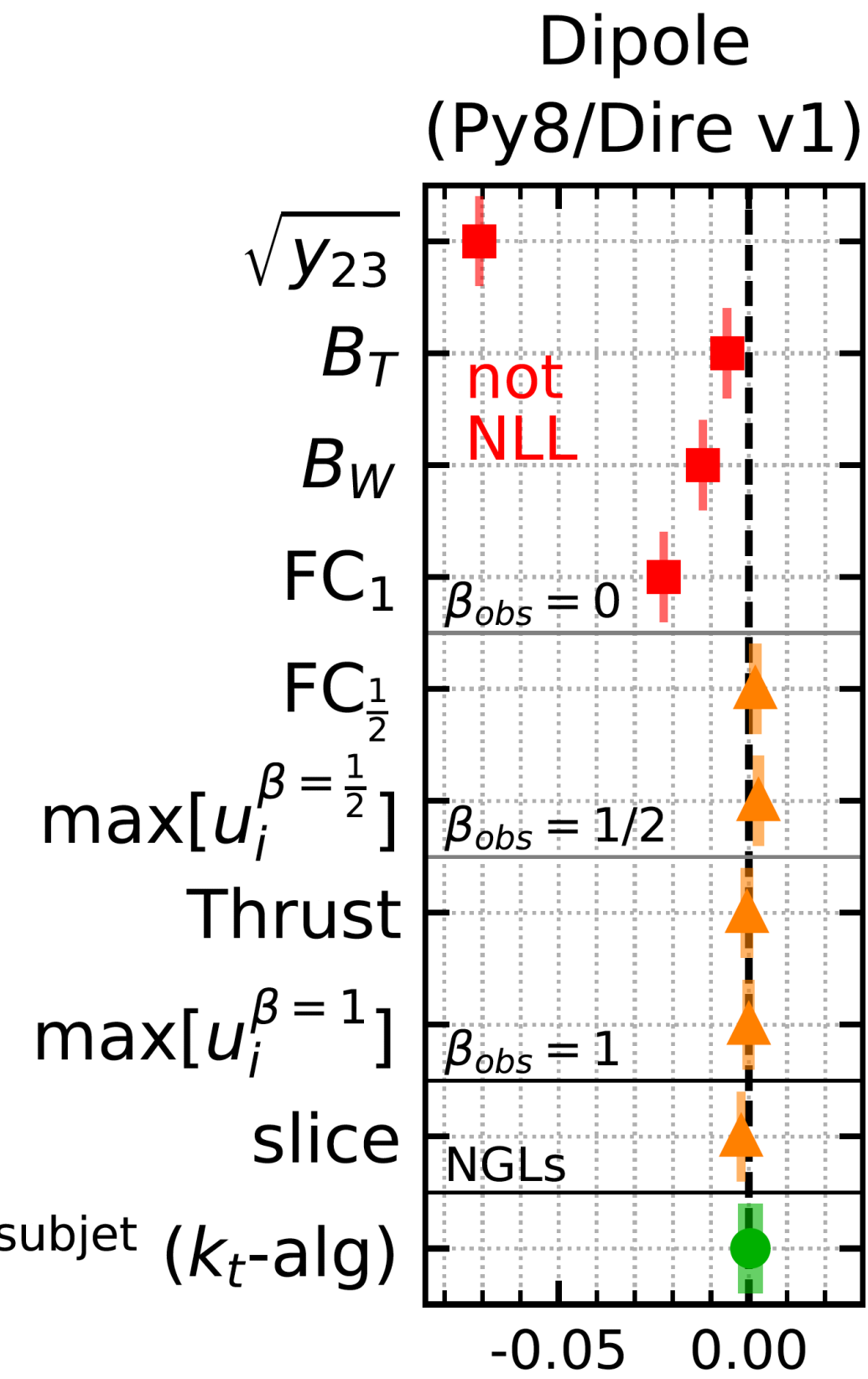


[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]



Accuracy across many observables

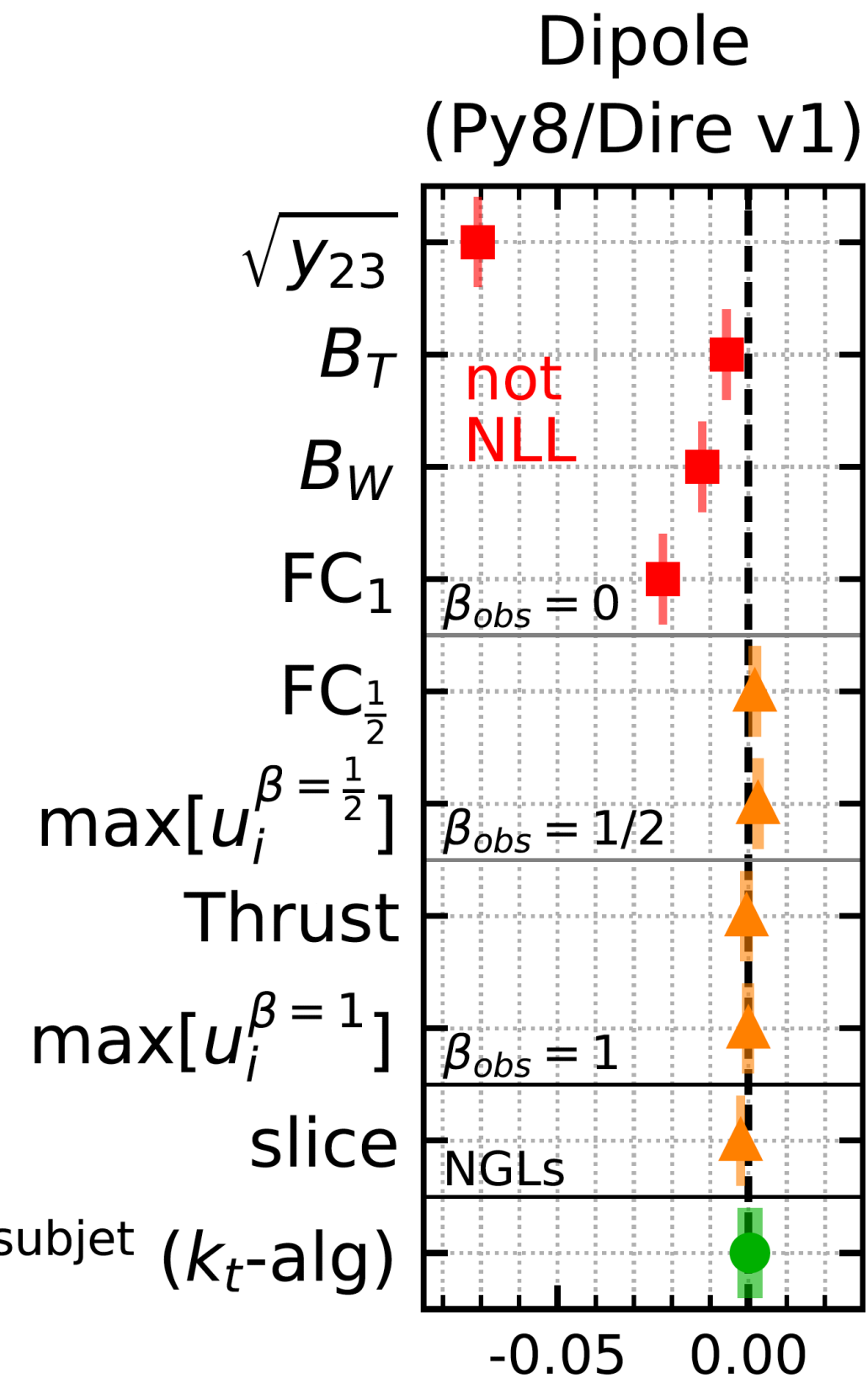
[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]



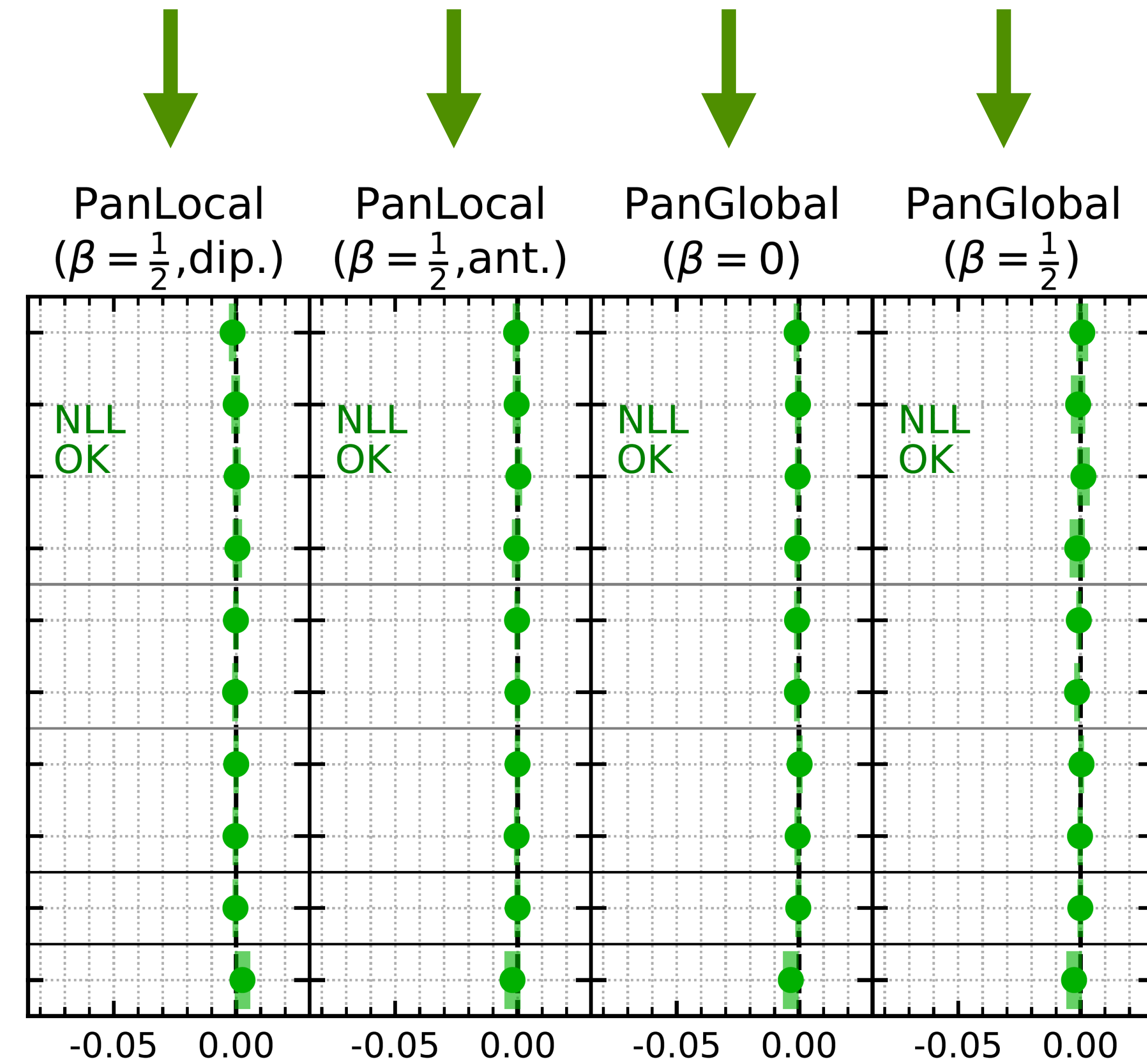
Subjet particle multiplicity in the k_T algorithm: sensitive to full recursive shower structure

Accuracy across many observables

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez '20]



Subject particle multiplicity in the k_T algorithm: sensitive to full recursive shower structure

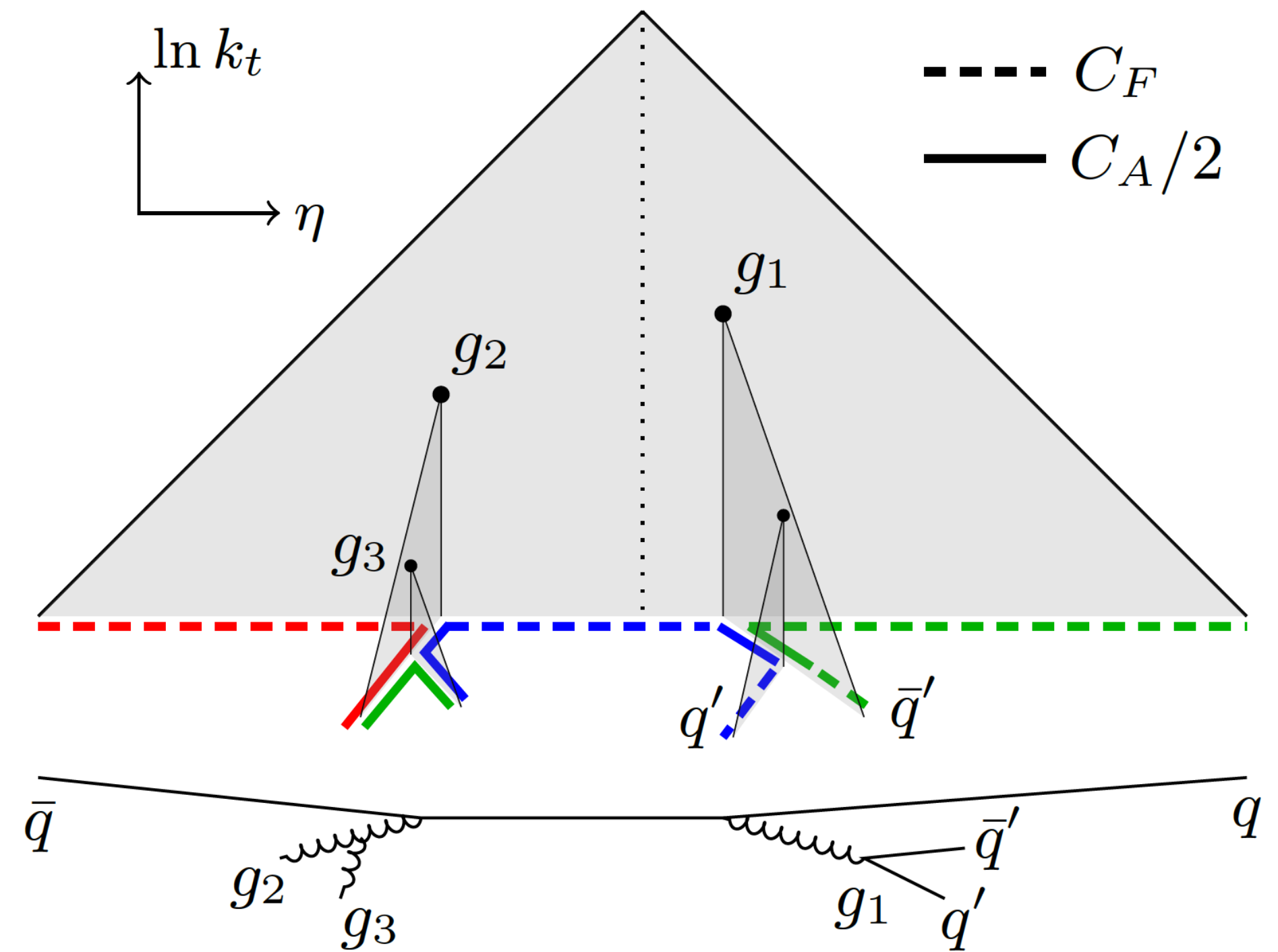
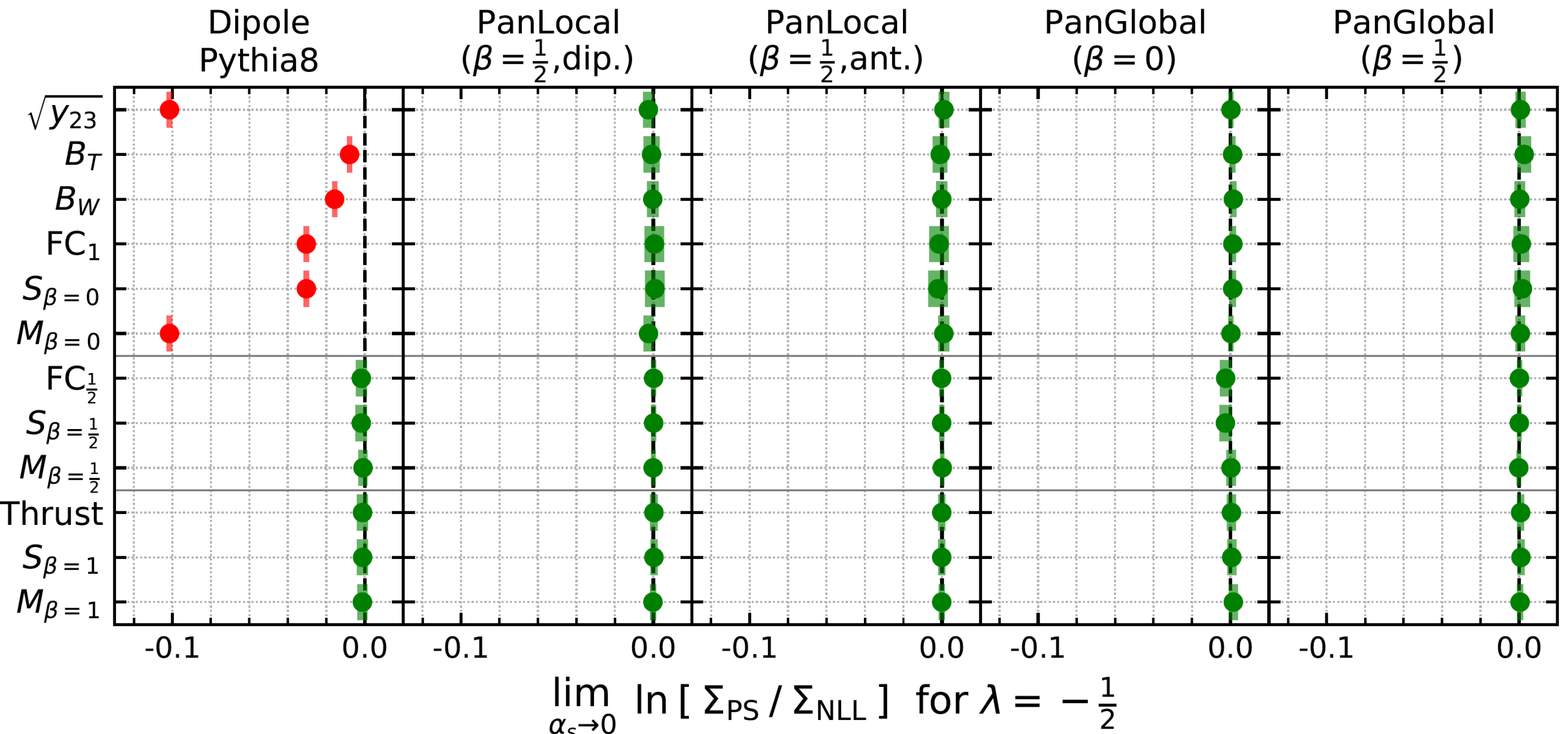


New classes of shower:
NLL for all observables considered
[global & non-global at once]

Beyond the planar limit: subleading N_c

- ▶ Same guiding principles can be used to include some information about subleading colour corrections
- ▶ Full colour accuracy can be achieved for global observables in processes with up to three coloured legs

NLL accuracy test – NODS procedure



[Hamilton, Medves, Salam, Scyboz, Soyez '20]
 see also related work by
 [Plaetzer, Sjo Dahl '12 + Thoren '18; Nagy, Soper '12-'19;
 Hoeche, Reichelt '20; De Angelis, Forshaw, Plaetzer '20;
 Forshaw, Holguin, Plaetzer '20]

Conclusions and Outlook

- ▶ Formulation of accuracy criteria for parton showers guided by principles of QCD resummations
 - ▶ Testing framework for algorithms based on comparison to all order calculations
- ▶ With seemingly simple methods, one can engineer new PS algorithms that are NLL accurate for global & non-global observables at once
 - ▶ Demonstration of NLL accuracy both at fixed order and all orders
- ▶ Some aspects remain to be addressed (initial state radiation, spin correlations) but the proposed algorithms and techniques can incorporate solutions to the above problems
- ▶ This approach offers a powerful avenue to look beyond NLL, and take the first steps towards a new generation of accurate parton shower algorithms

see also related work by

[Hoeche, Prestel, + Krauss '17; Dulat, Hoeche, Prestel '18]