

Paul Scherrer Institute
August 2 2024

The first row of the CKM matrix: puzzles and perspectives

Vincenzo Cirigliano
University of Washington



Outline

- Introduction: Cabibbo universality in the Standard Model and beyond
- 1st row CKM matrix:
 - Paths to V_{ud} & V_{us} and current puzzles
 - Radiative corrections to neutron and nuclear decays — EFT approach
 - Implications for new physics
- Conclusions and outlook

Cabibbo universality

VOLUME 10, NUMBER 12

PHYSICAL REVIEW LETTERS

15 JUNE 1963

UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo

CERN, Geneva, Switzerland

(Received 29 April 1963)

...

Hypothesis that the weak hadronic current has 'unit length' in flavor space.

Ratio of strangeness changing and conserving weak couplings controlled by the Cabibbo angle.

... We want, however, to keep a weaker form of universality, by requiring the following:

(3) J_μ has "unit length," i. e., $a^2 + b^2 = 1$.

We then rewrite J_μ as⁴

$$J_\mu = \cos\theta(j_\mu^{(0)} + g_\mu^{(0)}) + \sin\theta(j_\mu^{(1)} + g_\mu^{(1)}), \quad (2)$$

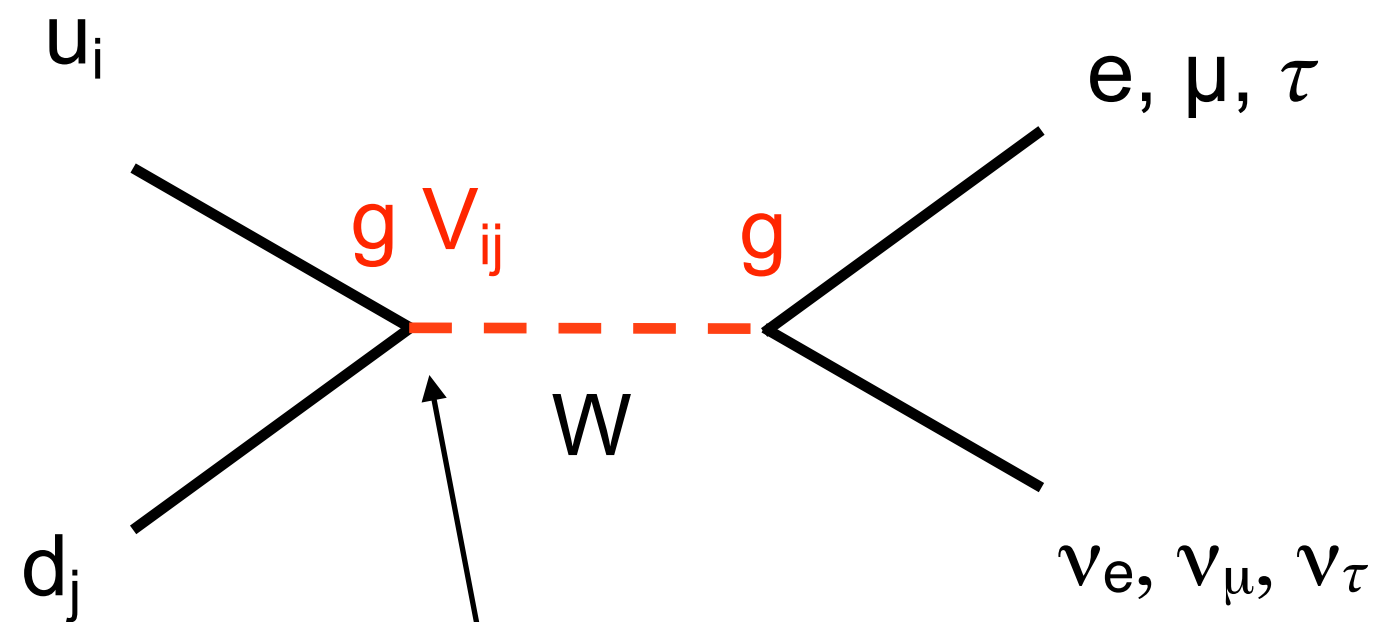
where $\tan\theta = b/a$.

...

Visionary!

Cabibbo universality in the SM and beyond

- In the Standard Model: Cabibbo universality \Leftrightarrow unitarity of the quark mixing matrix (CKM)



$$\mathcal{L} \supset \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L^i V_{ij} \gamma^\mu d_L^j$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashi-Maskawa

~ 0.95
 ~ 0.05
 $\sim 1.5 \times 10^{-5}$

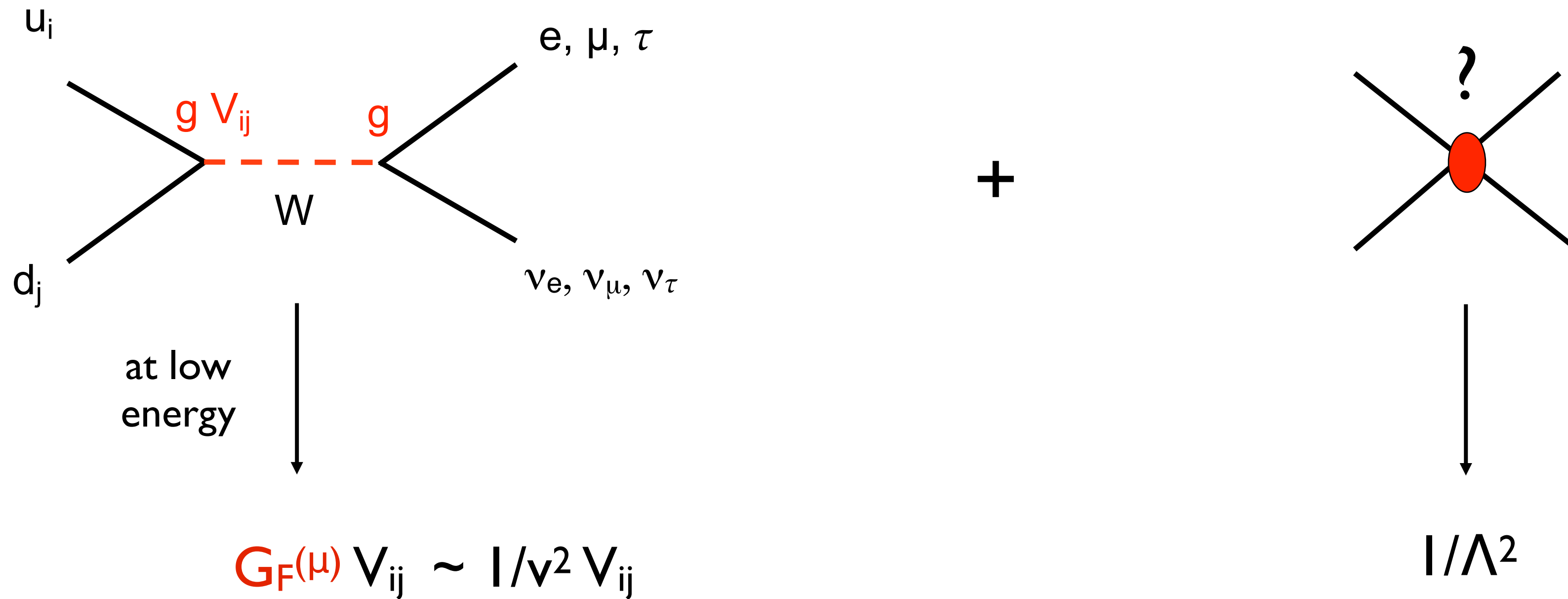
$$|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 1$$

$\delta V_{ud}/V_{ud} \sim 0.03\%$
 $\delta V_{us}/V_{us} \sim 0.2\%$
 $\delta V_{ub}/V_{ub} \sim 5\%$

V_{ud} and V_{us} are the most accurately known elements of the CKM matrix \Rightarrow
 1^{st} row provides the most stringent test of universality & sensitivity to new physics

Cabibbo universality in the SM and beyond

- In the Standard Model: Cabibbo universality \Leftrightarrow unitarity of the quark mixing matrix (CKM)



New physics can spoil universality: $|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 1 + O\left(\frac{v^2}{\Lambda^2}\right)$

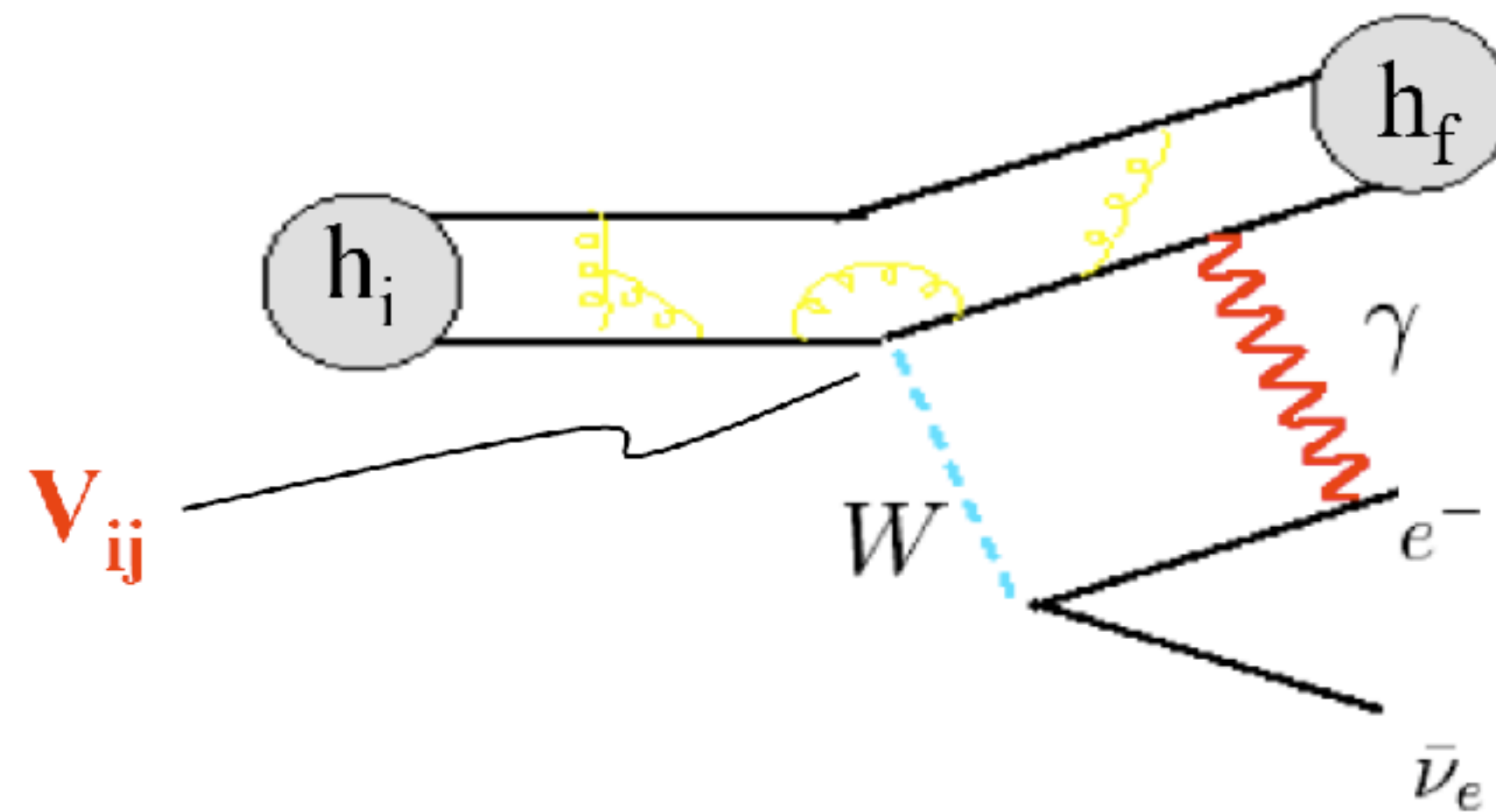
Current precision \Rightarrow probe effective scale $\Lambda \sim 10 \text{ TeV}$

Interesting and timely, but also challenging!

V_{ud} & V_{us} :
status and puzzles

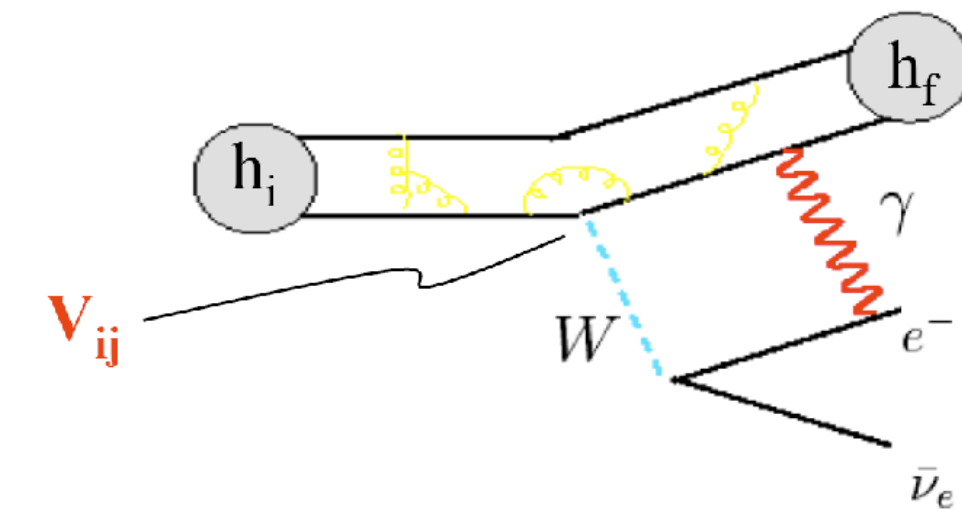
Paths to V_{ud} and V_{us}

	Hadron decays			Lepton decays
V_{ud}	$\pi^\pm \rightarrow \pi^0 e \nu$ Nucl. $0^+ \rightarrow 0^+$	$n \rightarrow p e \bar{\nu}$	$\pi \rightarrow \mu \nu$	$\tau \rightarrow h_{NS} \nu$
V_{us}	$K \rightarrow \pi l \nu$	$\Lambda \rightarrow p e \nu, \dots$	$K \rightarrow \mu \nu$	$\tau \rightarrow h_{S} \nu$



The challenge of CKM precision tests

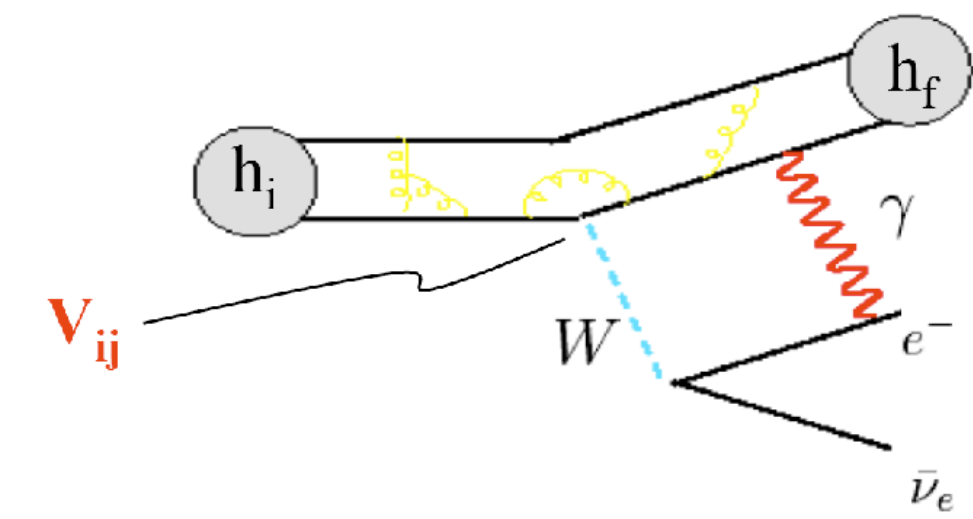
Extract $V_{us} = \sin\theta_C = \lambda$ and $V_{ud} = \cos\theta_C \simeq 1 - \lambda^2/2$
with *sub-percent precision* from decays involving hadrons
(currently $\delta\lambda/\lambda \sim 0.2-0.5\%$)



$$\Gamma = G_F^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \Delta_R) \times F_{\text{kin}}$$

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Lifetimes,
BRs

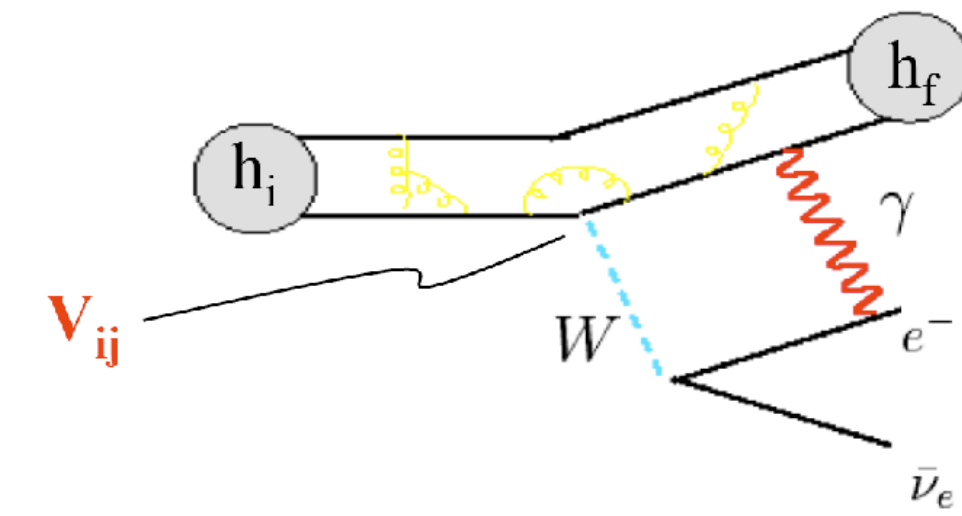
Muon
decay

Experimental input

Q-values, form
factors, ... →
phase space

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$$\Gamma = G_F^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \Delta_R) \times F_{\text{kin}}$$

Theory input

Hadronic / nuclear matrix elements of the weak V-A current,
including small corrections such as those induced by
electromagnetic radiative corrections $[(\alpha/\pi) \sim 2 \times 10^{-3}]$

Experiment

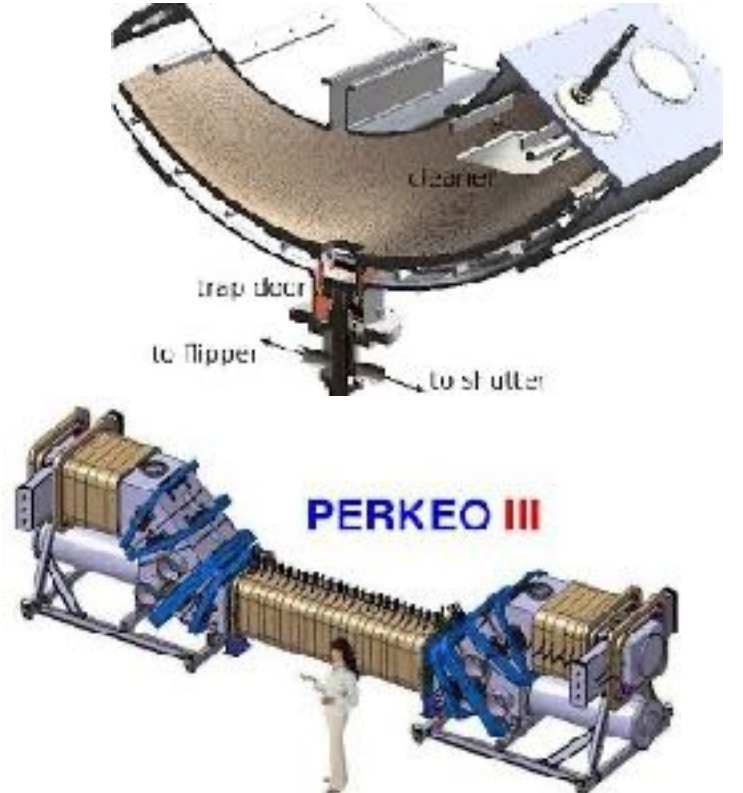
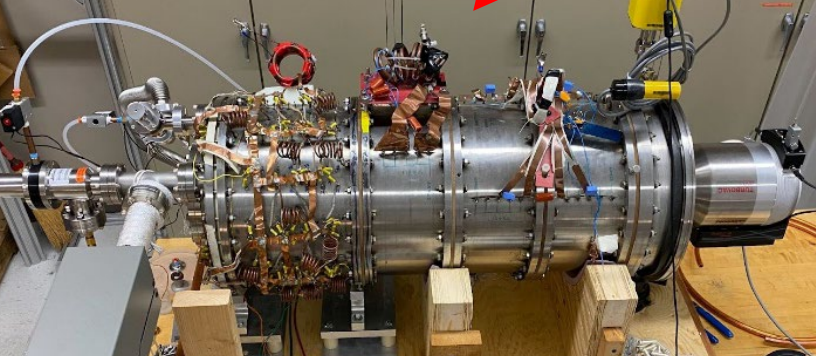
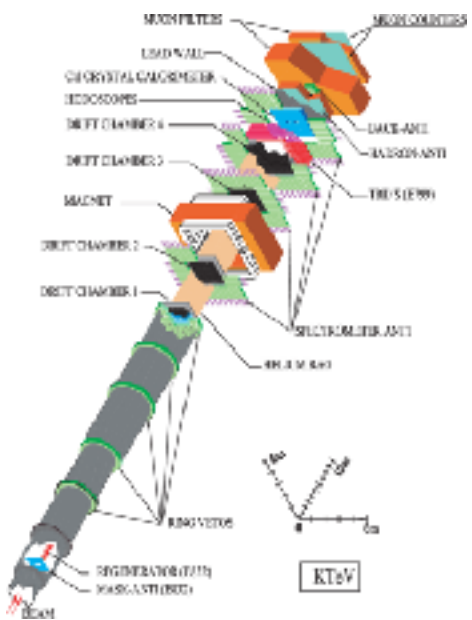
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Experimental input with sub-% precision from broad array of facilities and techniques

K, π , Hyperons:
 Meson factories & fixed target experiments
 (KLOE, KTeV, NA48, ...), with future
 experiment possible at CERN and PSI

Nuclear beta decay experiments
 Cold and Ultra Cold Neutron sources

τ decays:
 LEP (ALEPH, OPAL), Babar, Belle,
 Belle-II, future tau-charm factory



Hadronic matrix elements

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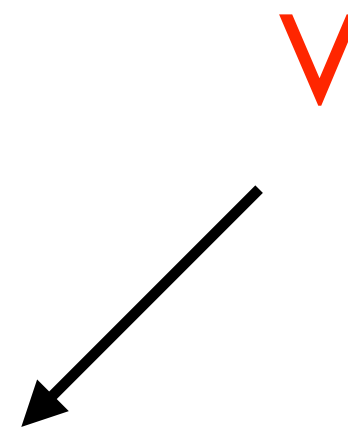
Hadronic matrix elements: 'Vector - Axial' quark current

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Hadronic matrix elements: 'Vector - Axial' quark current

Berhends-Sirlin
Ademollo-Gatto



Traditionally "Golden modes":
 $\langle f | V_\mu | i \rangle$ known in SU(2) [SU(3)] limit
 &
 corrections are 2nd order in
 SU(2) [SU(3)] breaking.
 Computed in lattice QCD for $K \rightarrow \pi$

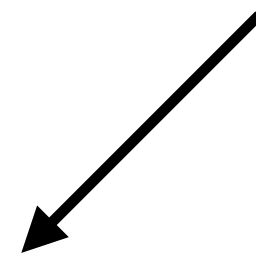
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V, A



Need experimental input on
 $\langle f | A | i \rangle / \langle f | V | i \rangle$
 For neutron and hyperons,
 Lattice QCD catching up but
 not as precise as experiment

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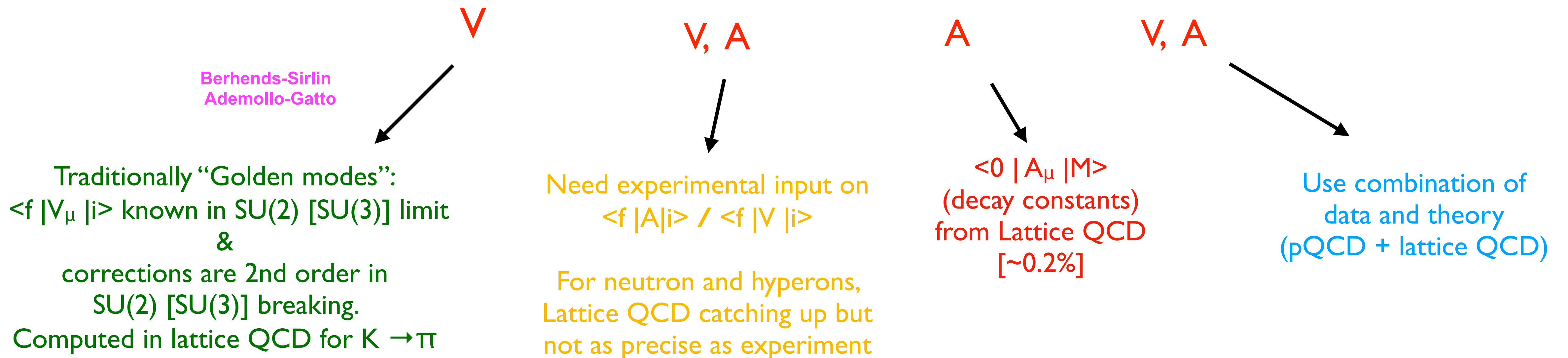
A

$\langle 0 | A_\mu | M \rangle$
 (decay constants)
 from Lattice QCD
 [$\sim 0.2\%$]

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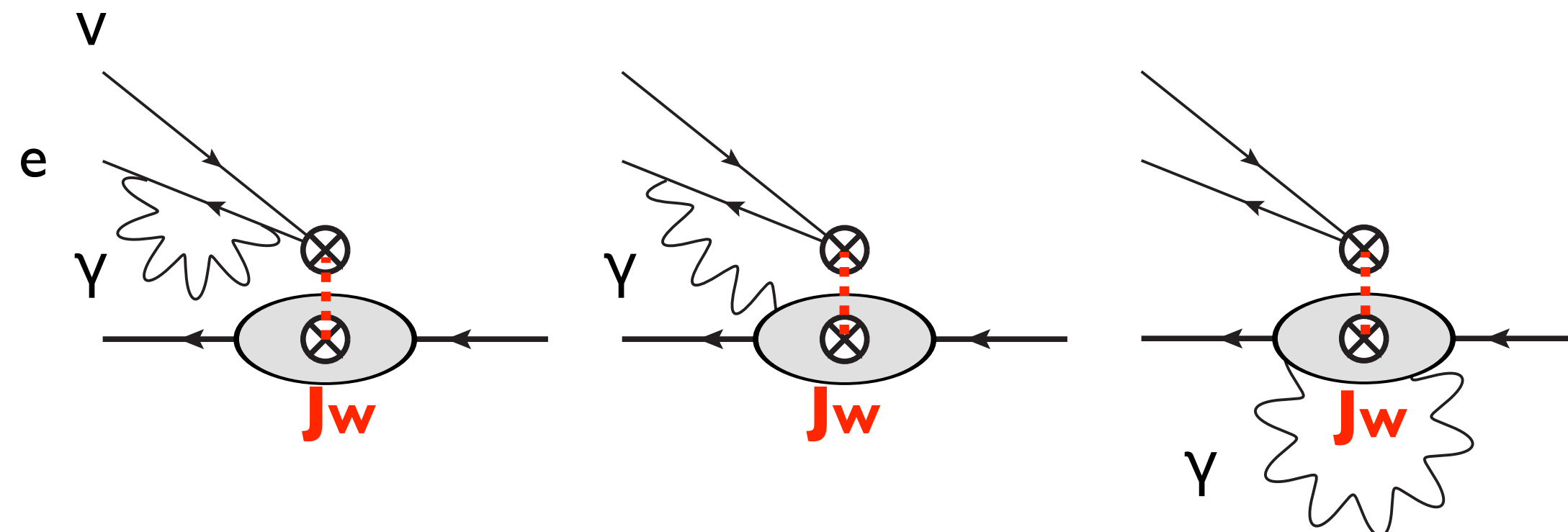
Radiative corrections

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Electroweak radiative corrections

Mesons and neutron:
 well developed frameworks (Sirlin's
 current algebra and Effective Field
 Theory), with non-perturbative
 input from lattice QCD and / or
 dispersive methods
 — systematically improvable

For leptonic meson decays:
 full lattice QCD+QED available



Radiative corrections

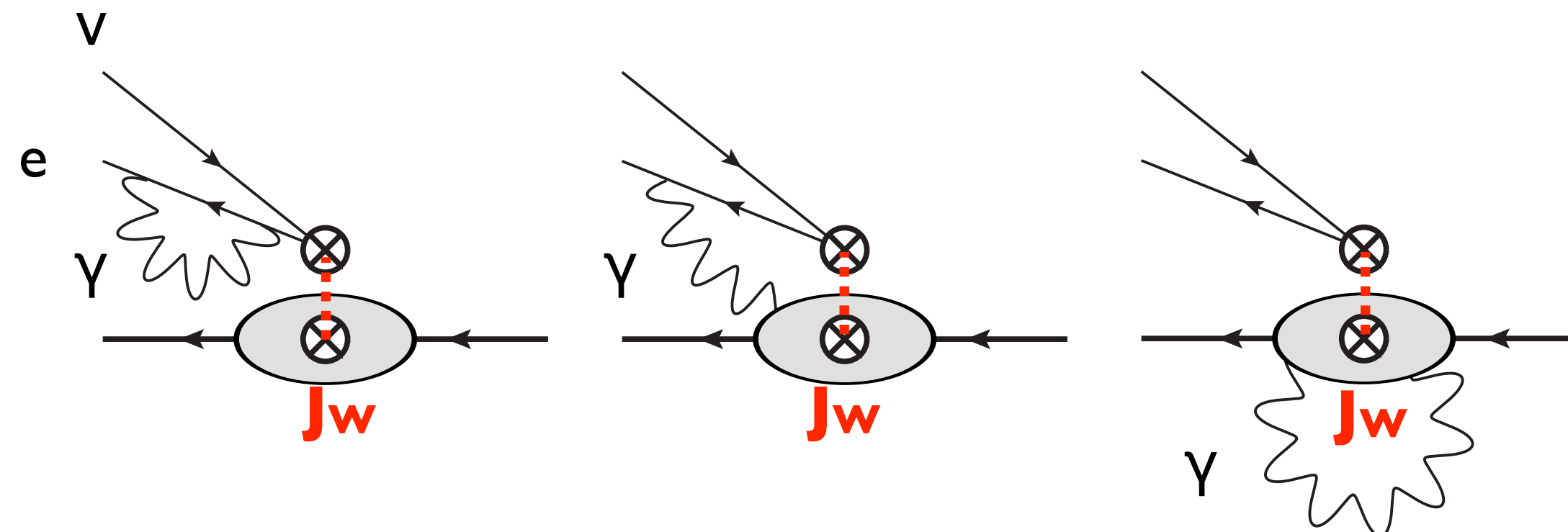
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 Dispersive approach recently developed.
 Recent progress towards multi-nucleon EFT

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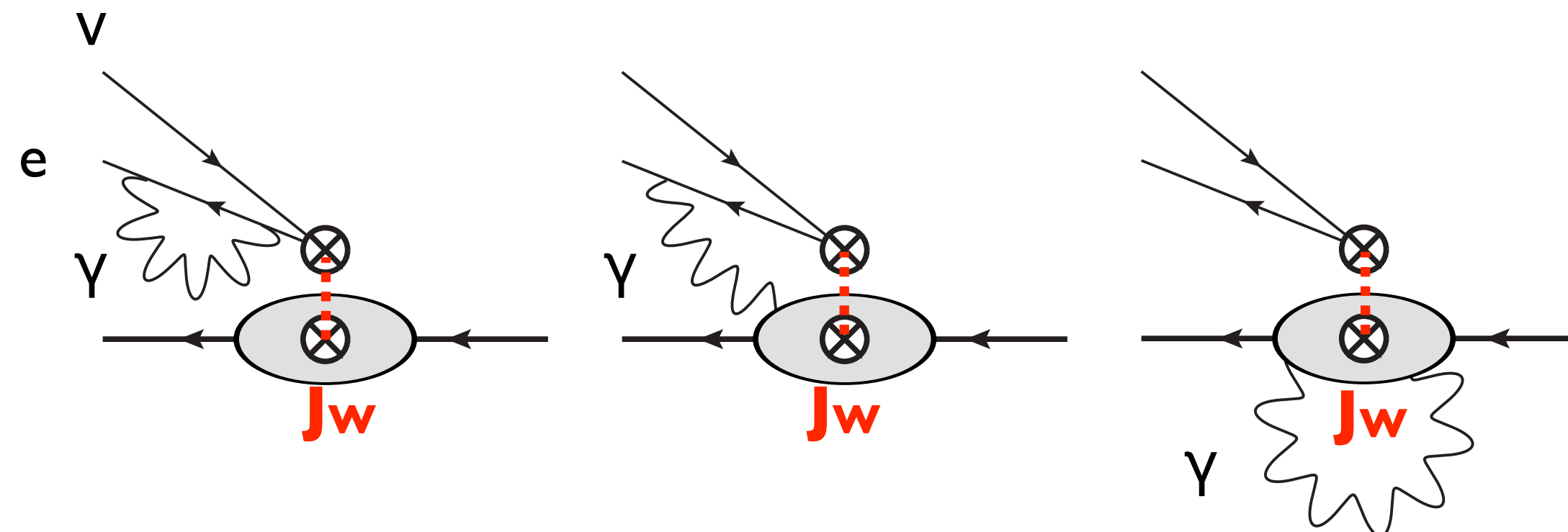
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Recent activity to assess nuclear structure uncertainties:
 Dispersive approach recently developed.
 Recent progress towards multi-nucleon EFT

For exclusive channels, difficult
 to estimate the hadronic
 structure-dependent effects.
 The only way may be Lattice
 QCD+QED?

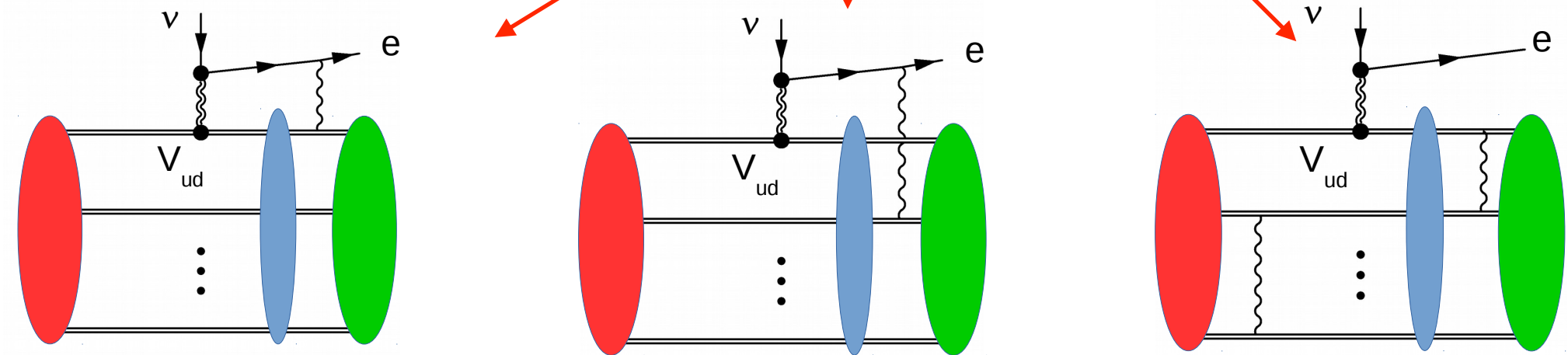
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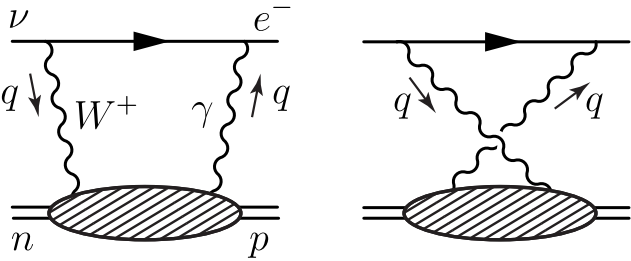
V_{ud} from nuclear 0⁺ → 0⁺ beta decays

$$|V_{ud}|^2 = \frac{2984.432(3) s}{ft \left(1 + \delta'_R + \delta_{NS} - \delta_C + \Delta_R^V \right)}$$

Point-like nucleus
'outer corrections'
(depend on Z, (E_e)^{max})



Quark-level
corrections &
single nucleon
'γ-W box'

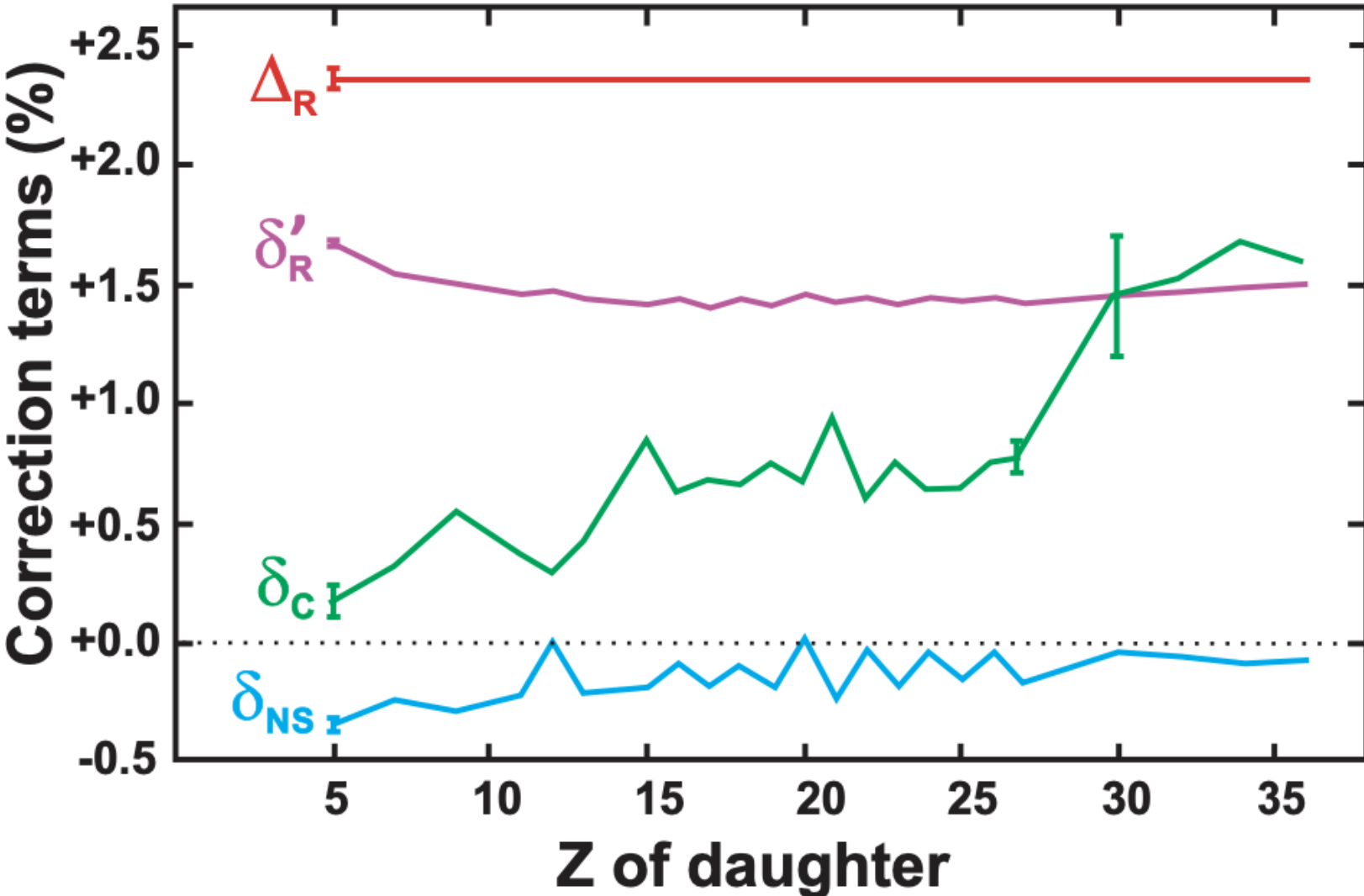


Hardy-Towner, PRC 2020

Seng et al. 1807.10197, Czarnecki et al, 1907.06737,
Shiells et al. 2012.01580
Hayen 2010.07262, Gorchtein-Seng 2106.09185

$$V_{ud}^{0^+ \rightarrow 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_R^V} (27)_{\text{NS}} [32]_{\text{total}}$$

For a review see
Gorchtein, Seng 2311.00044
and references therein



V_{ud} from neutron decay

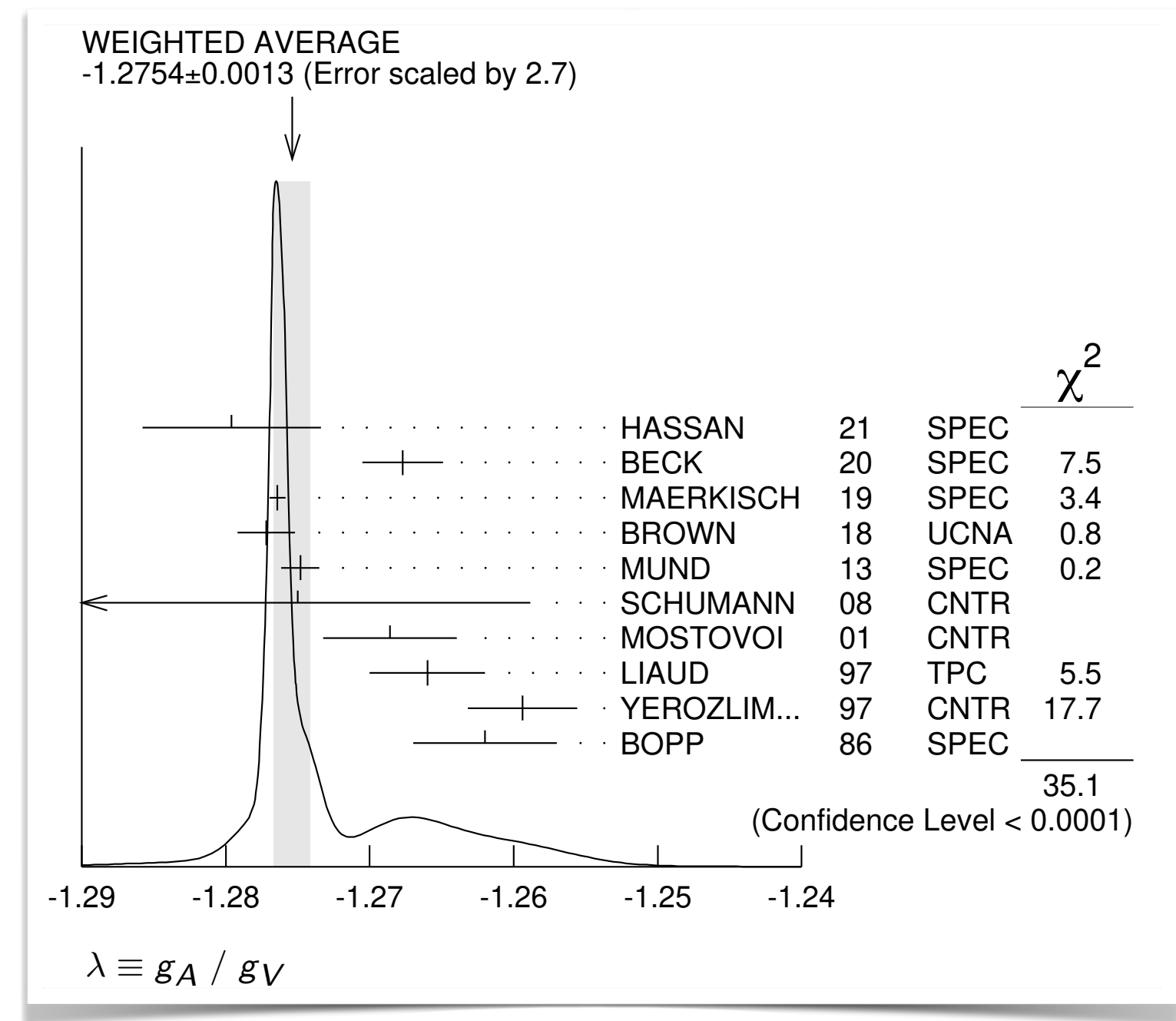
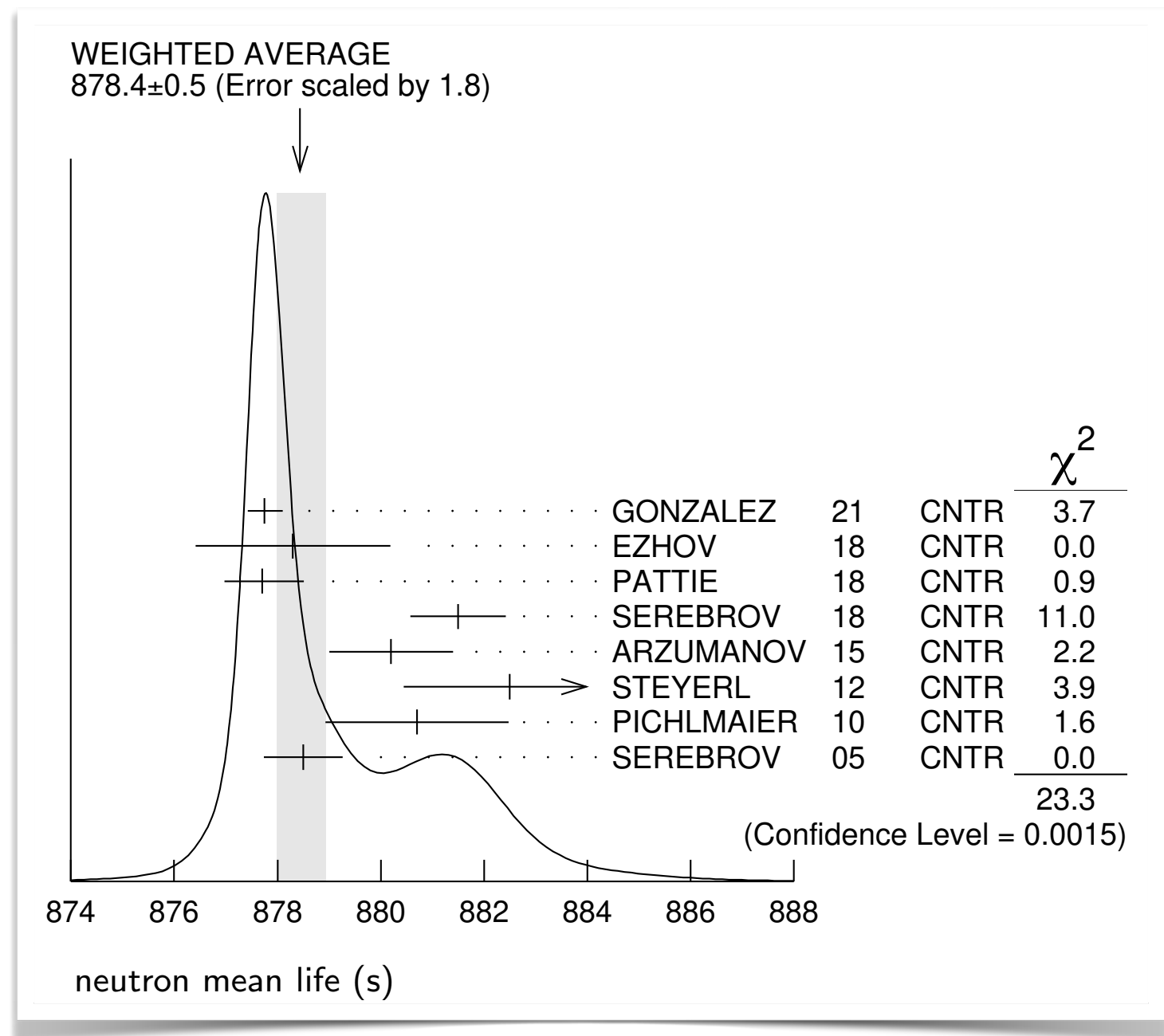
$\lambda = g_A / g_V$

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda^2) \cdot f_0 \cdot (1 + \Delta_f) \cdot (1 + \Delta_R),$$

$\Delta_R = 4.044(27)\%$
 $\Delta_f = 3.573(5)\%$

VC, W. Dekens, E. Mereghetti,
 O. Tomalak, 2306.03138
 and references therein

- Radiative corrections: NLL setup + LECs in terms of 'γ-W box' (dispersive & Lattice QCD)
- Experimental input: PDG averages include large scale factor, particularly for g_A / g_V



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Single most precise
measurements of lifetime
and λ imply very
competitive V_{ud} !

Maerkish et al,
1812.04666

Gonzalez et al,
2106.10375

$$V_{ud}^{n, \text{PDG}} = 0.97430(2)_{\Delta_f} (13)_{\Delta_R} (82)_{\lambda} (28)_{\tau_n} [88]_{\text{total}}$$

$$V_{ud}^{n, \text{best}} = 0.97402(2)_{\Delta_f} (13)_{\Delta_R} (35)_{\lambda} (20)_{\tau_n} [42]_{\text{total}}$$

Need improvements in lifetime
and g_A / g_V .
Within reach in next 5 years

V_{ud} from pion β decay

$$\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 |V_{ud}|^2 m_{\pi^+}^5 |f_+^\pi(0)|^2}{64\pi^3} (1 + RC_\pi) I_\pi,$$

- Vector form factor

$$f_+(0) = 1 - \frac{1}{(4\pi F_\pi)^2} \frac{(M_{K^+}^2 - M_{K^0}^2)_{\text{QCD}}^2}{24M_K^2} = 1 + O\left(\frac{m_u - m_d}{\Lambda_{\text{QCD}}}\right)^2$$

- Radiative corrections

$$RC_\pi = 0.0342(10) \quad (\text{ChPT})$$

VC-Neufeld-Pichl 2002

Box diagram

$$RC_\pi = 0.0332(1)_{\gamma W}(3)_{HO} \quad (\text{LQCD})$$

Feng, Gorchtein, Jin, Ma, Seng, 2003.09798



$$V_{ud}^{(\pi\beta)} = 0.97386 \text{ (281)}_{BR} \text{ (9)}_{\tau_\pi} \text{ (14)}_{RC} \text{ (28)}_{I_\pi} [283]_{\text{total}}$$

Theory in great shape.
0.3% total error on V_{ud}
dominated by
BR = $1.036(6) \times 10^{-8}$
[PIBETA, hep-ex/0312030]

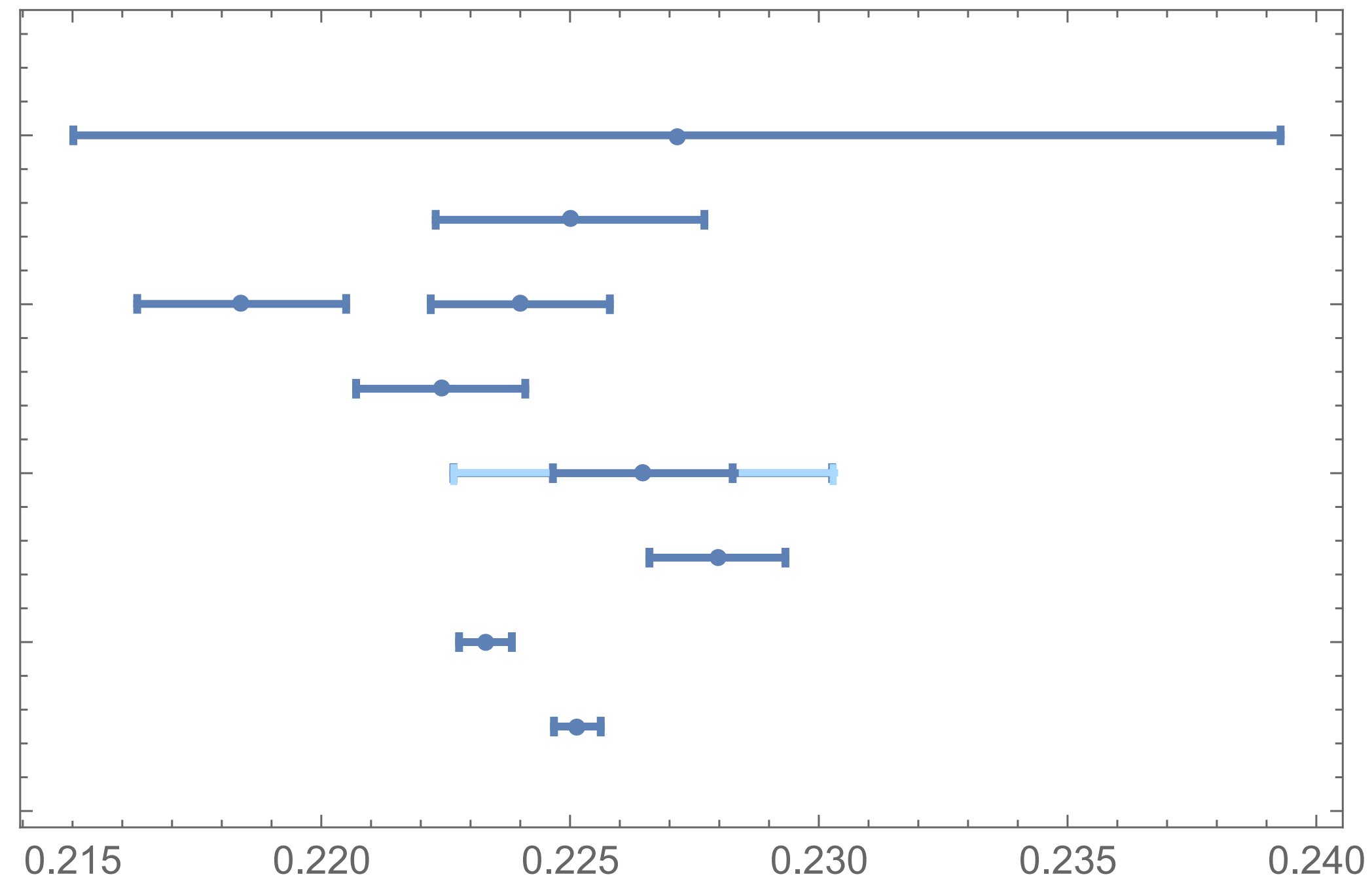
Experiment needs order-of-
magnitude improvement in
precision to be competitive →
PIONEER @ PSI

2203.01908

The Cabibbo angle — global view

Convert V_{ud} to V_{us} via unitarity

$\pi^\pm \rightarrow \pi^0 e \nu$
 Hyperons
 τ inclusive
 τ exclusive
 $n \rightarrow p e \nu$
 $0^+ \rightarrow 0^+$
 $K \rightarrow \pi l \nu$
 $K \rightarrow \mu \nu / \pi \rightarrow \mu \nu$



V_{us}

Fractional uncertainty

5.3%

1.2% + ?

0.8% + ?

0.8%

0.8% (1.7%) PDG

0.6%

0.24%

0.21%

Largest uncertainty

EXP

EXP + TH

EXP + TH

EXP + TH

EXP

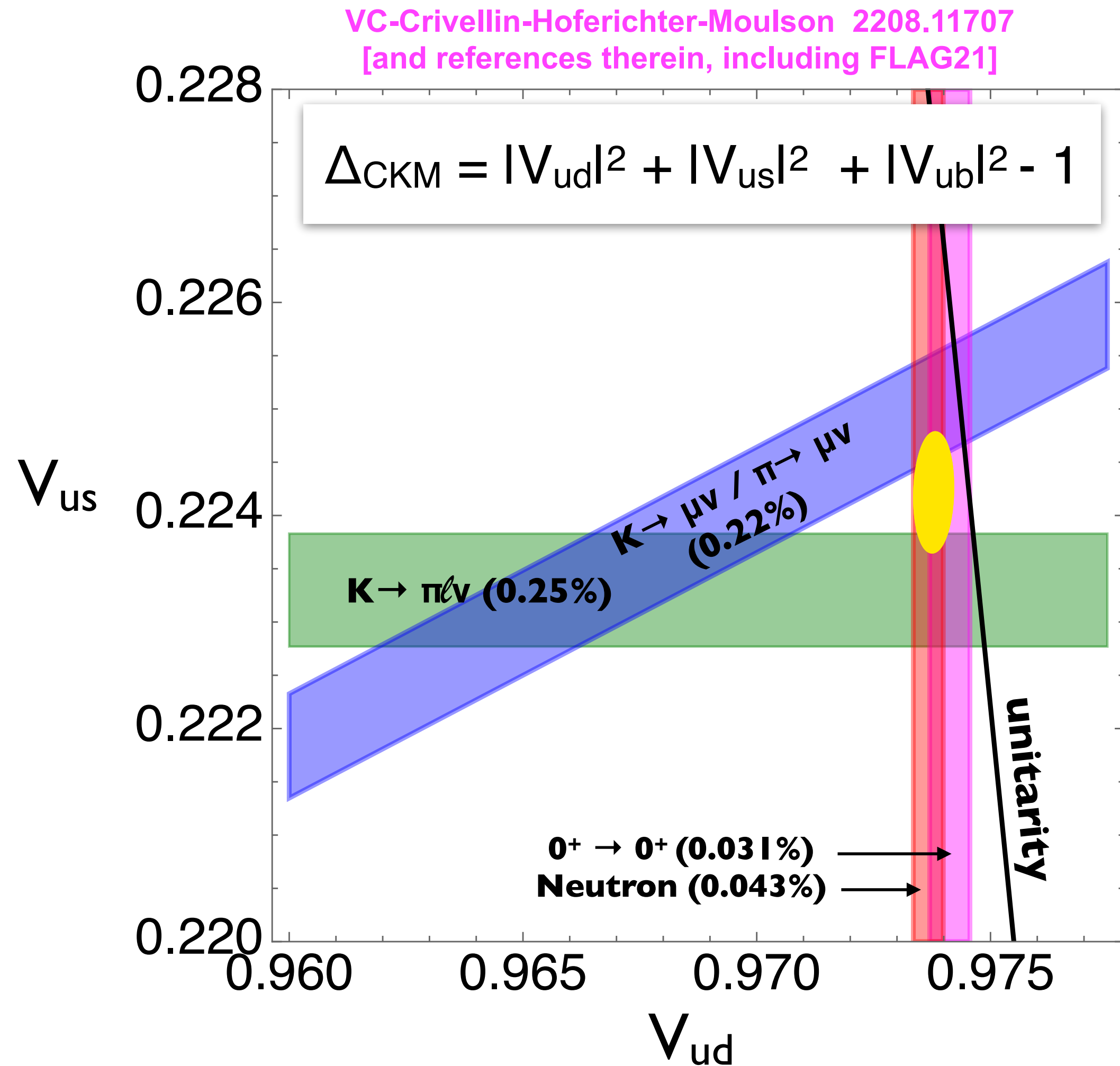
TH

EXP + TH

TH

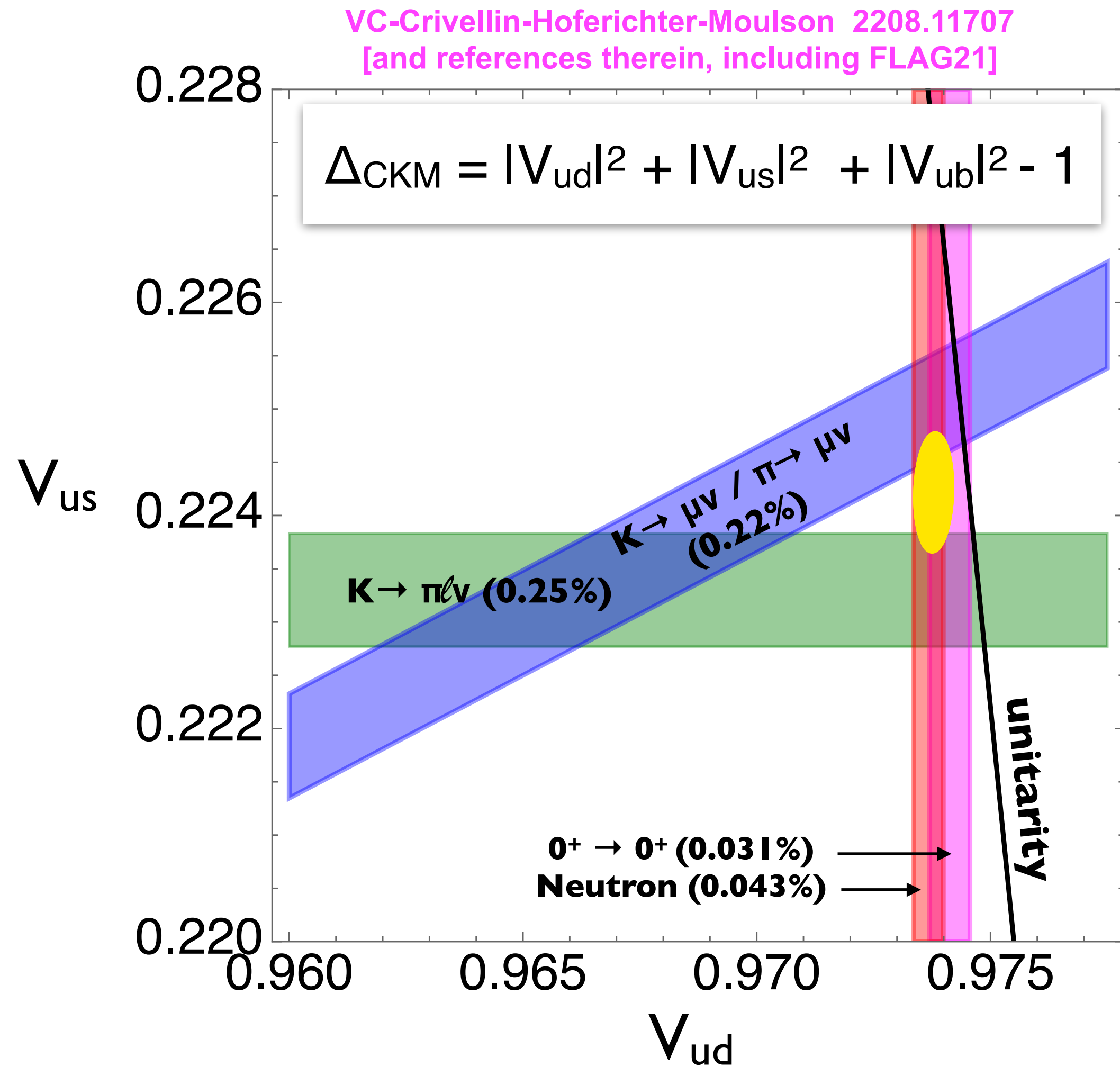
Tension among the most precise determinations

Tensions in the V_{ud} - V_{us} plane



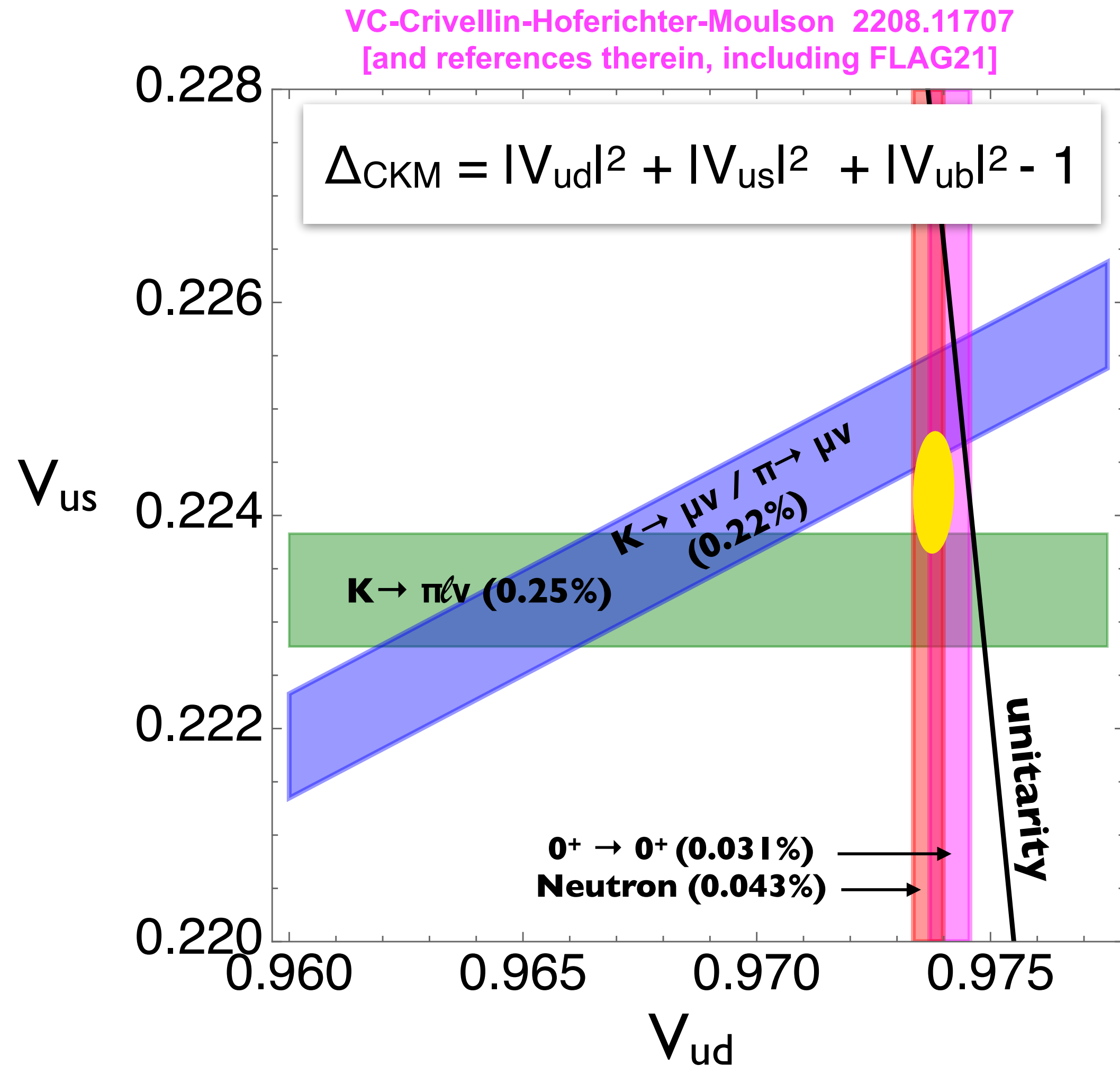
- Bands don't intersect in the same region on the unitarity circle
- $\sim 3\sigma$ effect in global fit ($\Delta_{\text{CKM}} = -1.48(53) \times 10^{-3}$)

Tensions in the V_{ud} - V_{us} plane



- **Expected experimental improvements:**
 - neutron decay (will match nominal nuclear uncertainty)
 - pion beta decay (3x to 10x at PIONEER phases II, III)
 - new $K_{\mu 3}/K_{\mu 2}$ BR measurement at NA62 (ongoing)
- **Further theoretical scrutiny**
 - Lattice: $K \rightarrow \pi$ vector f.f. and rad. corr. for $Kl3$
 - EFT for neutron and nuclei, with goal $\delta\Delta_R \sim 2 \times 10^{-4}$
 - ...
- **Possible BSM explanations:** EFT & specific models

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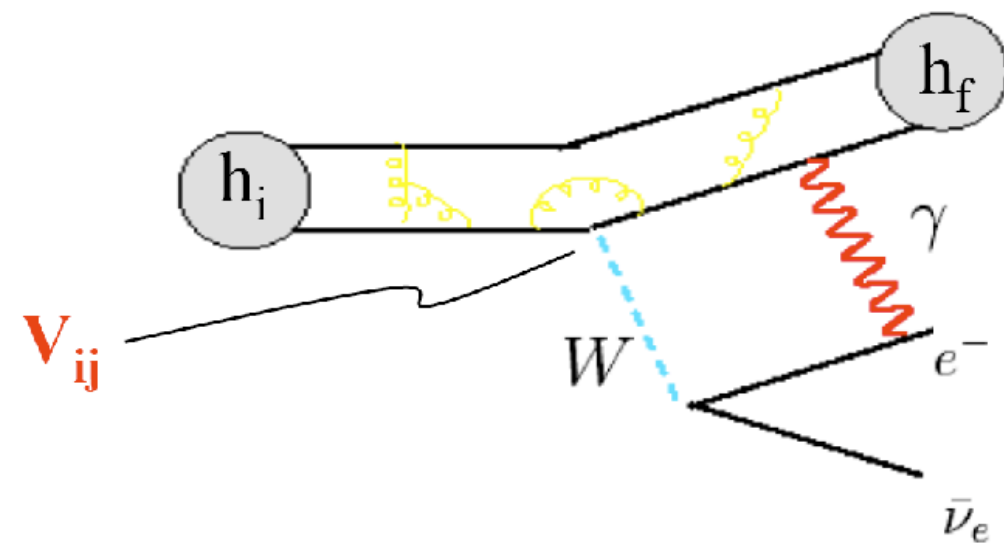
Will discuss in this talk

EFT approach to radiative corrections

(neutron and nuclear beta decays)

EFT for radiative corrections: why?

- Widely separated mass scales play a role in neutron and nuclear beta decays



$$M_{W,Z}$$

$$\gg \Lambda_\chi \sim m_N \sim 4\pi F_\pi \sim 1 \text{ GeV}$$

$$\gg m_\pi \sim 140 \text{ MeV}$$

$$\gg q_{\text{ext}} \sim m_n - m_p \sim m_e \sim 1 \text{ MeV}$$

Weak scale

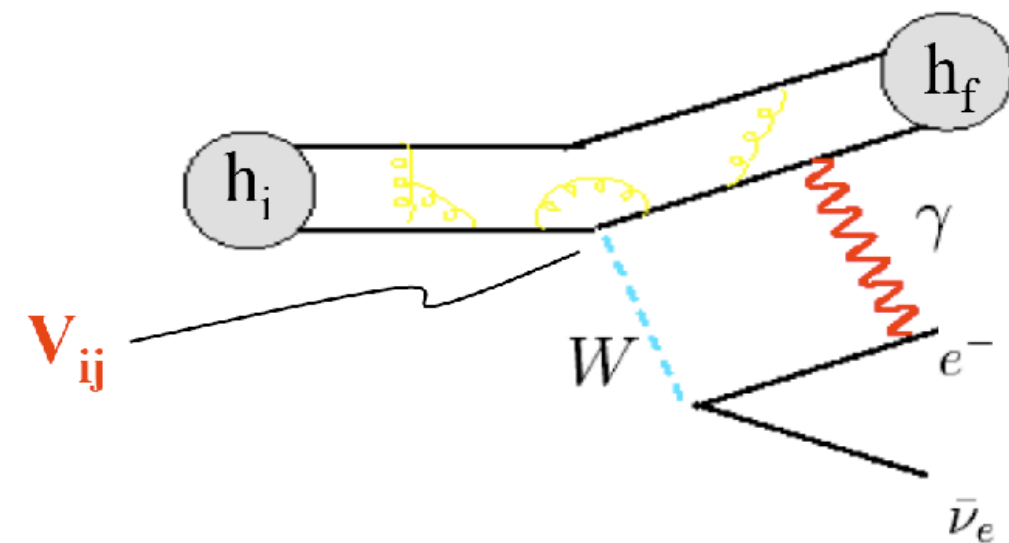
χ SB & nucleon mass scale

Pion mass / hadronic structure

Q value, nuclear excitations

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Weak scale

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- Small ratios appear as expansion parameters and arguments of logarithms

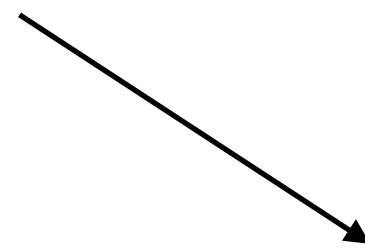
$$\epsilon_W = \Lambda_\chi / M_W \sim 10^{-2} \quad \epsilon_\chi = m_\pi / \Lambda_\chi \sim 0.1 \quad \epsilon_{\text{recoil}} = q_{\text{ext}} / \Lambda_\chi \sim 10^{-3} \sim \alpha / \pi \quad \epsilon_{\pi} = q_{\text{ext}} / m_\pi \sim 10^{-2}$$

- At the required precision ($\sim 10^{-4}$), need to keep terms of $\mathcal{O}(G_F \alpha)$, $\mathcal{O}(G_F \alpha \epsilon_\chi)$, $\mathcal{O}(G_F \epsilon_{\text{recoil}})$, along with leading logarithms (LL $\sim (\alpha \ln(\epsilon))^n$) and next-to-leading logarithms (NLL $\sim \alpha (\alpha_s \ln(\epsilon_W))^n$, $\alpha (\alpha \ln(\epsilon))^n$)

Multi-step strategy

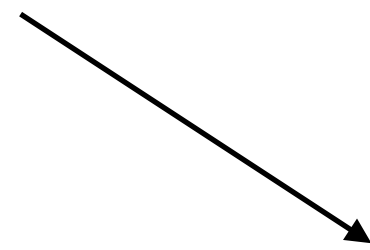
- Matching and running in a tower of EFTs: $SM \rightarrow LEFT \rightarrow ChPT \rightarrow \not{t}EFT, \chi EFT$

One nucleon



VC, J. de Vries, L. Hayen, E. Mereghetti, A. Walker-Loud 2202.10439, PRL
VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306.03138, PRD

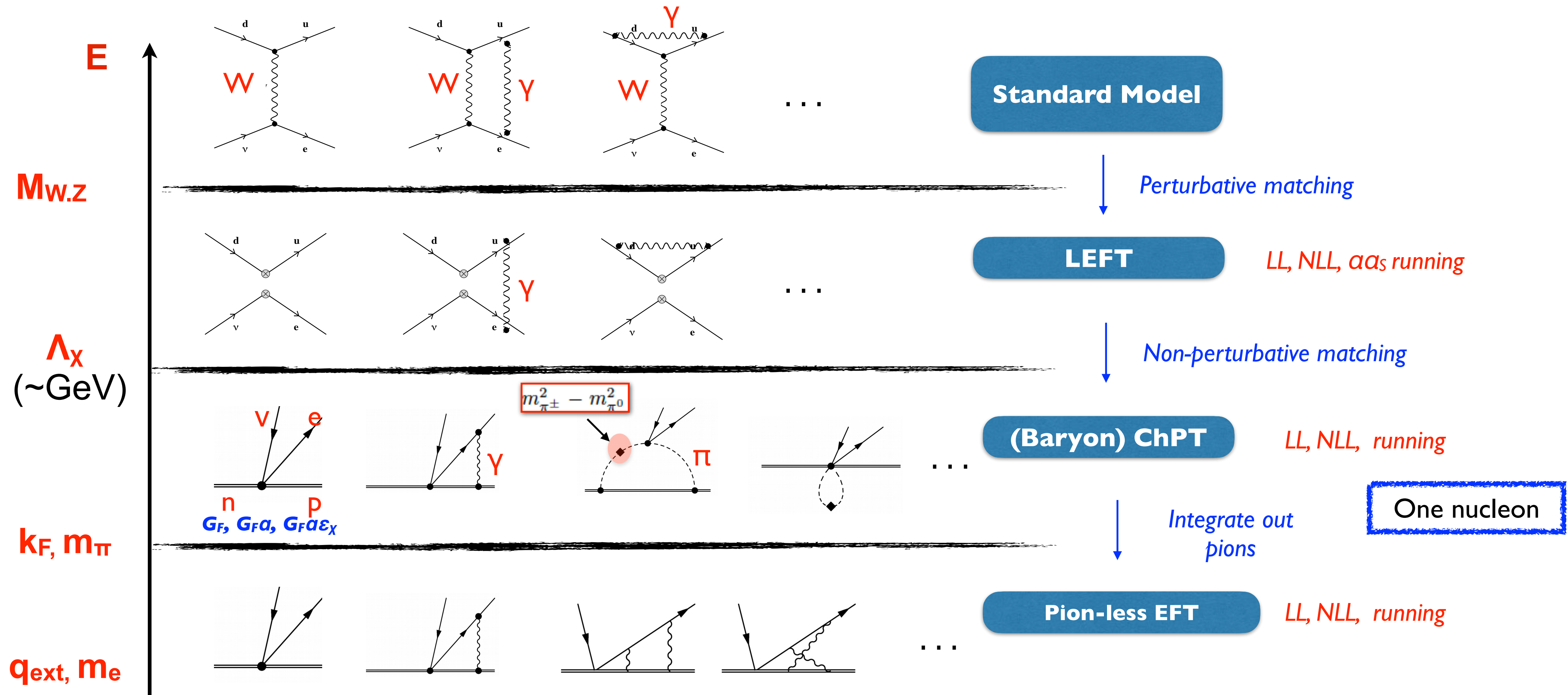
Multi nucleons



VC, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, E. Mereghetti, 2405.18469 & 2405.18464

Multi-step strategy

- Matching and running in a tower of EFTs: SM \rightarrow LEFT \rightarrow ChPT \rightarrow ν EFT, χ EFT



Standard Model → LEFT matching

$$\mathcal{L}_{\text{LEFT}} = -2\sqrt{2}G_F \bar{e}_L \gamma_\rho \mu_L \bar{\nu}_{\mu L} \gamma^\rho \nu_{eL} - 2\sqrt{2}G_F V_{ud} C_\beta^r(a, \mu) \bar{e}_L \gamma_\rho \nu_{eL} \bar{u}_L \gamma^\rho d_L + \text{h.c.} + \dots$$

Muon decay

Beta decays

$$C_\beta^r(a, \mu) = 1 + \frac{\alpha}{\pi} \ln \frac{M_Z}{\mu} + \frac{\alpha}{\pi} B(a) - \frac{\alpha \alpha_s}{4\pi^2} \ln \frac{M_W}{\mu} + \mathcal{O}(\alpha \alpha_s) + \mathcal{O}(\alpha^2)$$

$$B(a) = \frac{a}{6} - \frac{3}{4}$$

Finite piece depends on γ_5 scheme (use NDR) and evanescent scheme

$$\gamma^\alpha \gamma^\rho \gamma^\beta P_L \otimes \gamma_\beta \gamma_\rho \gamma_\alpha P_L = 4 [1 + a(4 - d)] \gamma^\rho P_L \otimes \gamma_\rho P_L + E(a)$$

RGEs in the LEFT

$$\mu \frac{dC_\beta^r(a, \mu)}{d\mu} = \gamma(\alpha, \alpha_s) C_\beta^r(a, \mu),$$

$$\gamma(\alpha, \alpha_s) = \gamma_0 \frac{\alpha}{\pi} + \gamma_1 \left(\frac{\alpha}{\pi}\right)^2 + \gamma_{se} \frac{\alpha}{\pi} \frac{\alpha_s}{4\pi} + \dots$$

$$\gamma_0 = -1$$

$$\gamma_1^{NDR}(a)$$

$$= \frac{\tilde{n}}{18} (2a + 1),$$

$$\tilde{n} = \sum_f n_f Q_f^2$$

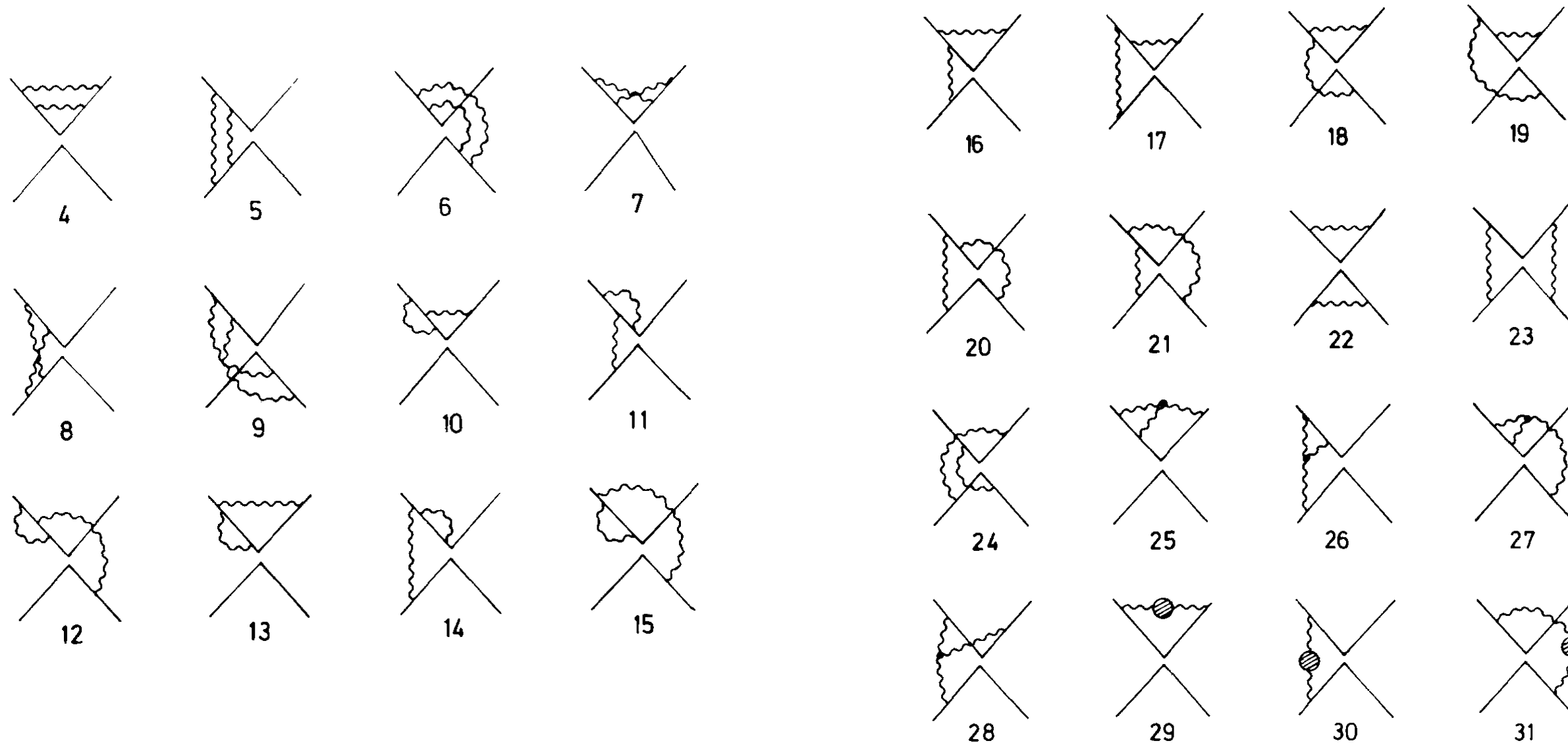
$$\gamma_{se} = +1$$

A. Sirlin 1982



Adapt results from Buras-Weisz
Nucl. Phys. B 333, 66 (1990)

Disagree with Czarnecki-Marciano-Sirlin 2004 on γ_1 , implying a -0.011% decrease in Δ_R



LEFT → ChPT matching

- Require that functional derivatives of the LEFT / ChPT generating functionals w.r.t. spurions ($q_{L,R}$ & q_W) coincide
- Typical matching condition

Kernel involving photon / lepton propagator, etc.

$$LEC^{(r)}(\mu_\chi, \mu) = -f_{\text{ChPT}}^{(\text{loop})}(\mu_\chi) + \int \frac{d^d q}{(2\pi)^d} K(q) T_{JJ\dots}(q)$$

Correlation function of QCD currents in the nucleon.

UV behavior controlled by the Operator Product Expansion (OPE) for $T_{JJ\dots}$

- Add and subtract OPE for $T_{JJ\dots}$ to isolate divergent & finite terms. OPE result in dim-reg introduces scheme dependence that cancels the one in the Wilson Coefficient!

UV finite — requires non-perturbative input on $T_{JJ\dots}(q)$

$$LEC^{(r)}(\mu_\chi, \mu) = -f_{\text{ChPT}}^{(\text{loop})}(\mu_\chi) + f_{\text{LEFT}}^{(\text{OPE})}(\mu, a) + \int \frac{d^4 q}{(2\pi)^4} K(q) \bar{T}_{JJ\dots}(q)$$

$$\bar{T}_{JJ\dots} = T_{JJ\dots} - T_{JJ\dots}^{\text{OPE}}$$

ChPT \rightarrow π EFT matching

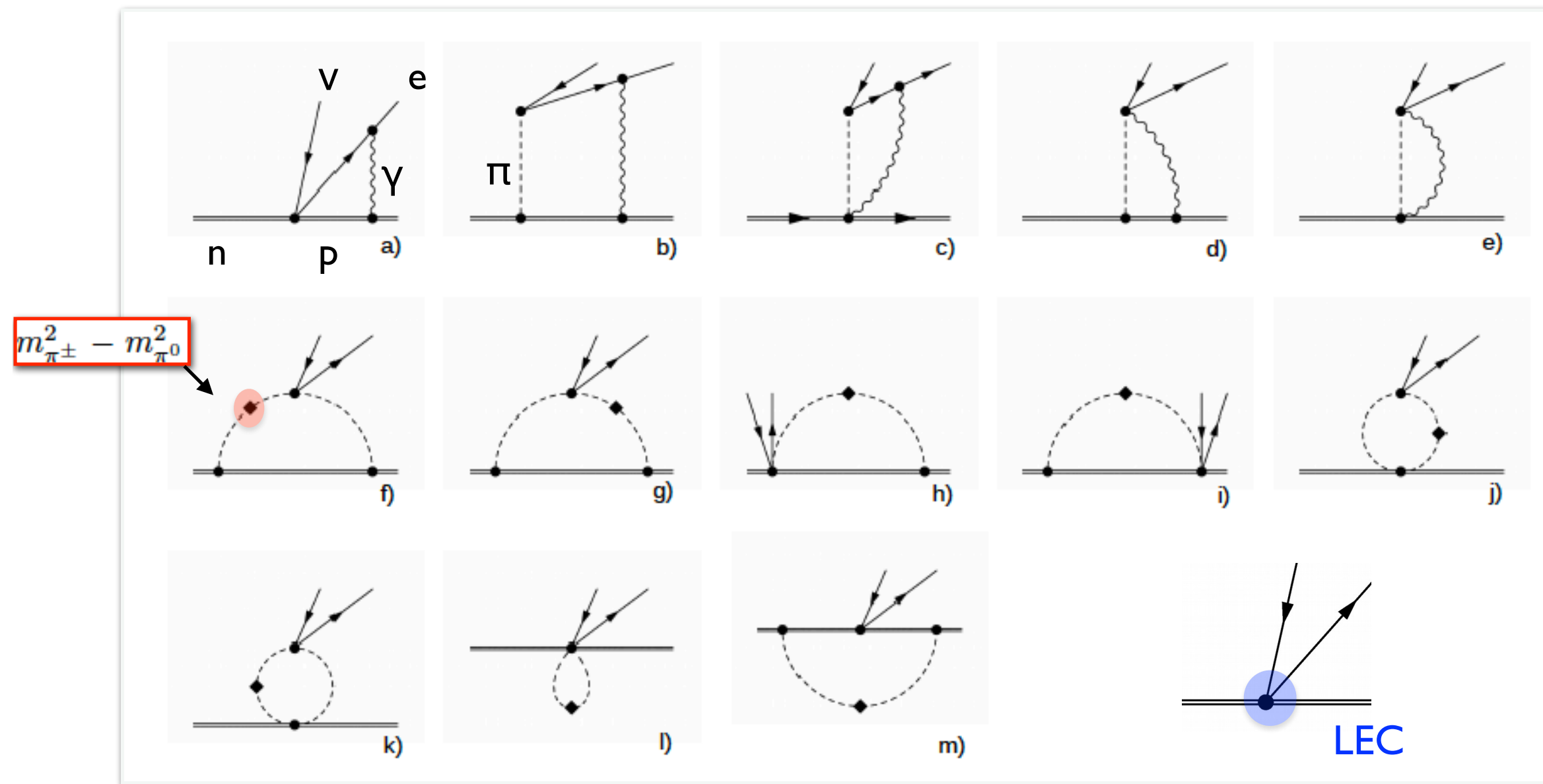
- Identify LECs of π EFT (g_V, g_A, \dots) in terms of
 - ChPT LECs themselves \rightarrow contribute to both g_V and g_A
 - ChPT loops involving pions (“integrate out pions”) \rightarrow contribute only to g_A

$$\mathcal{L}_\pi = -\sqrt{2}G_F V_{ud} \bar{e} \gamma_\mu P_L \nu_e \bar{N} (g_V v_\mu - 2g_A S_\mu) \tau^+ N + \dots$$

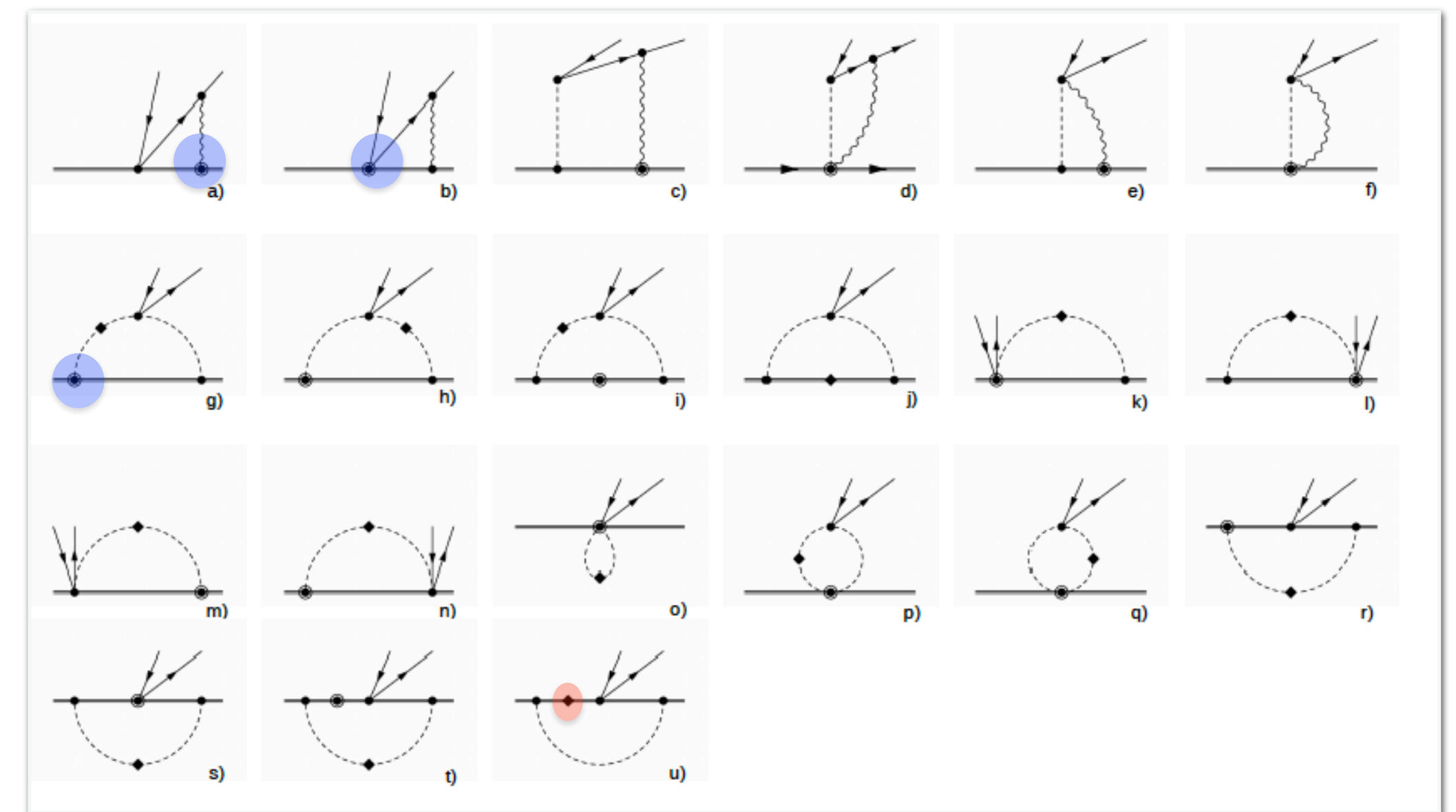
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IPI diagrams contributing to $O(G_F \Omega)$



IPI diagrams contributing to $O(G_F \Omega \epsilon_X)$



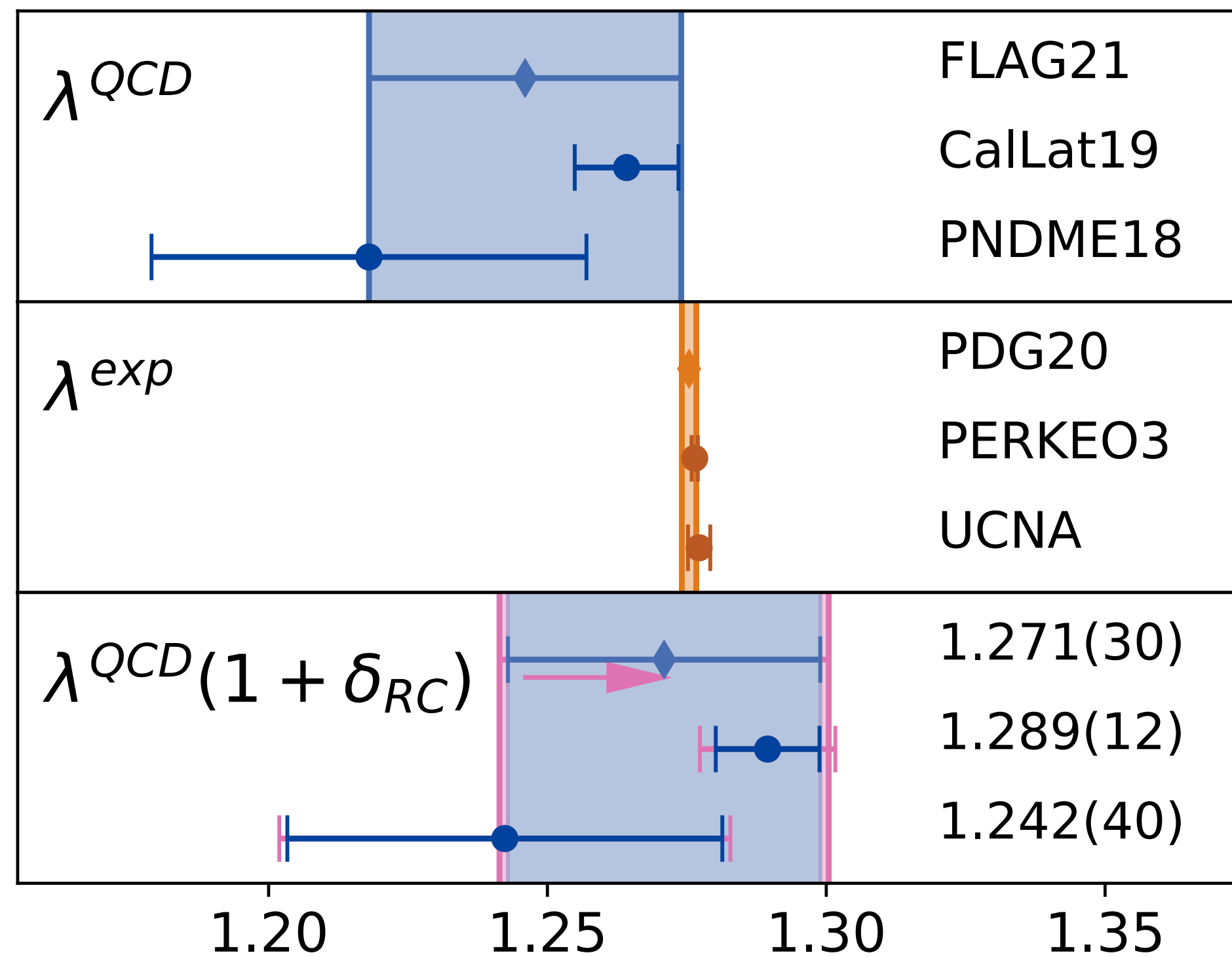
Circled dots: vertices from NLO pion-nucleon Lagrangian

Diamonds on nucleon line: vertices from $\mathcal{L}_{\pi N}^{e^2 p^0}$

g_A/g_V to $O(\alpha)$ and $O(\alpha\varepsilon_\chi)$

VC, J. de Vries, L. Hayen, E. Mereghetti, A. Walker-Loud 2202.10439

- (g_A/g_V) gets %-level corrections proportional to the pion EM mass splitting (100x larger than previous estimates)
- Radiative corrections generally improve agreement between data (neutron decay) and lattice QCD calculations



$$\lambda \equiv \frac{g_A}{g_V}$$

$$\frac{\lambda^{exp}}{\lambda^{QCD}} = 1 + \delta_{RC}$$

$$\delta_{RC} \simeq (2.0 \pm 0.6 \pm ??)\%$$

Scale variation +
knownLECs

Unknown LECs

To further sharpen the test,
need higher precision in $(g_A)^{QCD}$ and δ_{RC}

Vector coupling g_V

VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306.03138, PRD

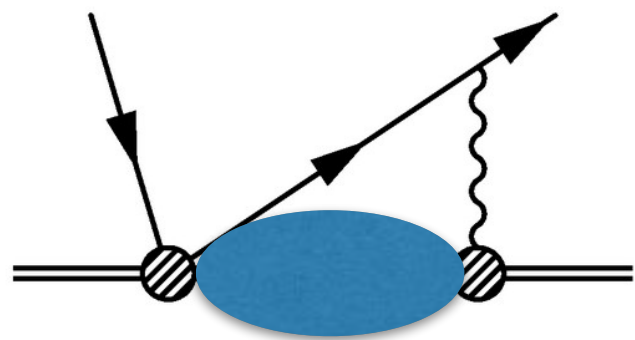
$$g_V(\mu_\chi) = \bar{C}_\beta^r(\mu) \left[1 + \bar{\square}_{\text{Had}}^V(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi} \left(\frac{5}{8} + \frac{3}{4} \ln \frac{\mu_\chi^2}{\mu_0^2} + \left(1 - \frac{\alpha_s}{4\pi} \right) \ln \frac{\mu_0^2}{\mu^2} \right) \right]$$

$\bar{\square}_{\text{Had}}^V(\mu_0)$ is the usual 'box' up to $Q^2 \sim (\mu_0)^2$

$$\bar{\square}_{\text{Had}}^V(\mu_0) = -e^2 \int \frac{id^4q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \left[\frac{T_3(\nu, Q^2)}{2m_N \nu} - \frac{2}{3} \frac{1}{Q^2 + \mu_0^2} \left(1 - \frac{\alpha_s(\mu_0^2)}{\pi} \right) \right]$$

$$Q^2 = -q^2$$

$$\nu = v \cdot q$$



$$T_{VA,0}^{\mu\nu} = i\varepsilon^{\mu\nu\sigma\rho} q_\rho v_\sigma \frac{T_3}{4m_N \nu} + \dots \quad T_{VV(A),0}^{\mu\nu}(q, v) = \frac{\tau_{ij}^a \delta^{\sigma'\sigma}}{12} \frac{i}{6} \int d^d x e^{iq \cdot x} \langle N(k, \sigma', j) | T [\bar{q} \gamma^\mu q(x) \bar{q} \gamma^\nu (\gamma_5) \tau^a q(0)] | N(k, \sigma, i) \rangle$$

Vector coupling g_V

VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306.03138, PRD

$$g_V(\mu_\chi) = \overline{C}_\beta^r(\mu) \left[1 + \overline{\square}_{\text{Had}}^V(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi} \left(\frac{5}{8} + \frac{3}{4} \ln \frac{\mu_\chi^2}{\mu_0^2} + \left(1 - \frac{\alpha_s}{4\pi} \right) \ln \frac{\mu_0^2}{\mu^2} \right) \right]$$

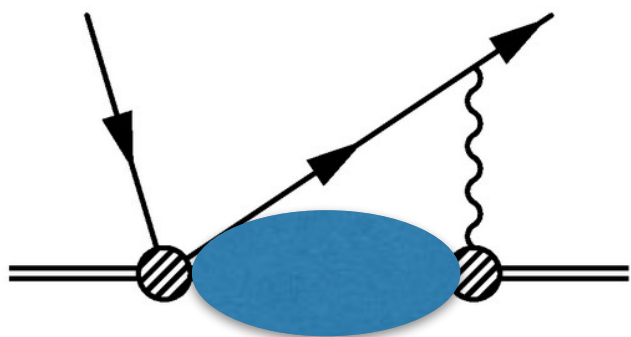
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$$Q^2 = -q^2$$

$$\nu = v \cdot q$$

- Use non-perturbative input on T_3 from dispersive analysis or LQCD Seng et al. 1807.10197, 2308.16755
- No dependence on scheme, μ and μ_0 (up to higher perturbative orders)
- For $\mu_\chi \sim \mu \sim \mu_0 \sim 1 \text{ GeV}$ all large logs are in the NLO Wilson coefficient $\overline{C}_\beta^r(\mu)$
- Dependence on μ_χ canceled by loops in pion-less EFT



Evolution of g_V from Λ_χ to m_e

$$\mu_\chi \frac{dg_V(\mu_\chi)}{d\mu_\chi} = \gamma(\alpha) g_V(\mu_\chi),$$

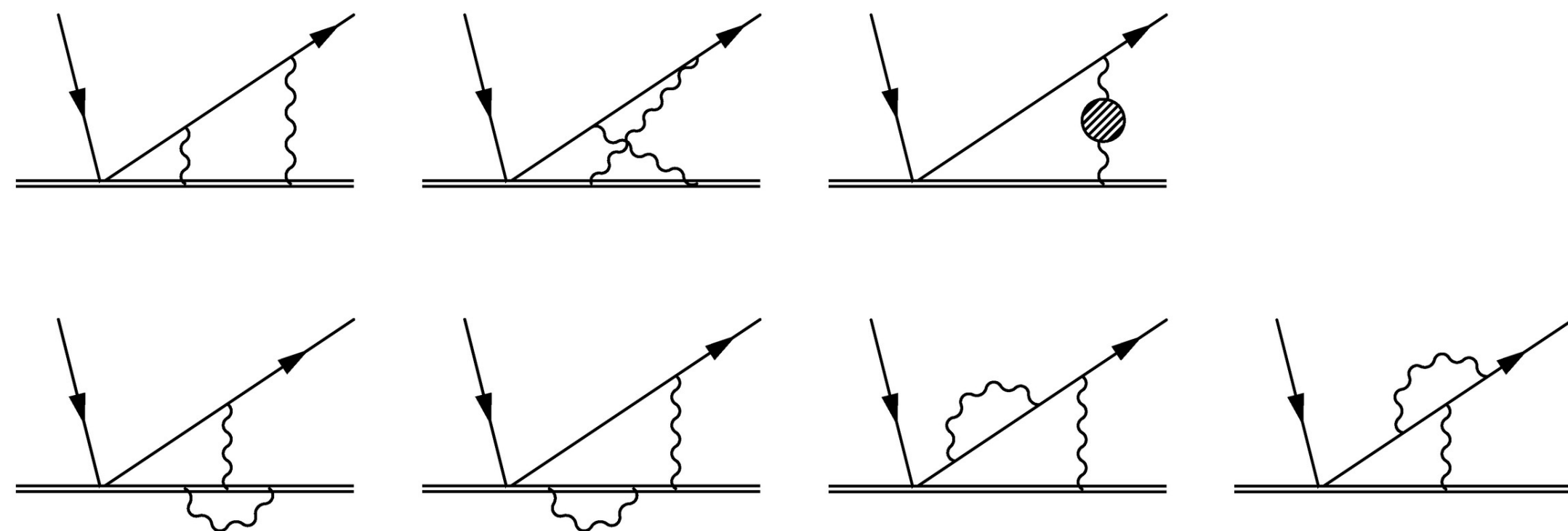
$$\gamma(\alpha) = \tilde{\gamma}_0 \frac{\alpha}{\pi} + \tilde{\gamma}_1 \left(\frac{\alpha}{\pi}\right)^2 + \dots$$

$$\tilde{\gamma}_0 = -\frac{3}{4}$$

$$\tilde{\gamma}_1 = \frac{5\tilde{n}}{24} + \frac{5}{32} - \frac{\pi^2}{6}$$



Adapt results from
 V. Gimenez Nucl. Phys. B 375, 582 (1992),
 Ji, Ramsey-Musolf Phys. Lett. B 257, 409 (1991)



Parametrically large 2-loop anomalous dimension

$$\left. \frac{g_V(m_e)}{g_V(m_p)} \right|_{\text{LO}} = 1.01308,$$

$$\left. \frac{g_V(m_e)}{g_V(m_p)} \right|_{\text{LL}} = 1.01325,$$

$$\left. \frac{g_V(m_e)}{g_V(m_p)} \right|_{\text{NLL}} = 1.01330.$$

NLL (α^2) RGEs induces shift to Δ_R of + 0.01%

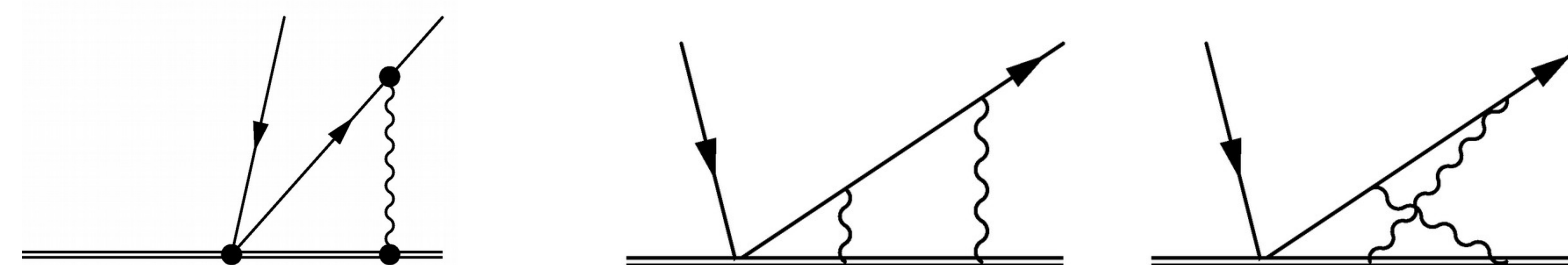
Corrections to neutron decay rate

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda^2) \cdot f_0 \cdot (1 + \Delta_f) \cdot (1 + \Delta_R), \quad \lambda = g_A/g_V$$

$\lambda = g_A/g_V$ taken from experiment.
It includes electromagnetic shift
to g_V and g_A from $E > m_\pi$

Δ_f : Coulomb corrections (photon
loops with \mathcal{L}_π) & $O(\epsilon_{\text{recoil}})$

Δ_R : proportional to $(g_V)^2$
 $\times (1 + O(\alpha))$ virtual and
real effects from \mathcal{L}_π



Fermi function, obtained by re-summing the series in $(\pi\alpha/\beta)$
induced by exchange of 'potential photons'

$$k_0 \sim m_e \beta^2 \ll |\vec{k}| \sim m_e \beta$$

Corrections to neutron decay rate

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$$\Delta_f = 3.573(5)\%$$

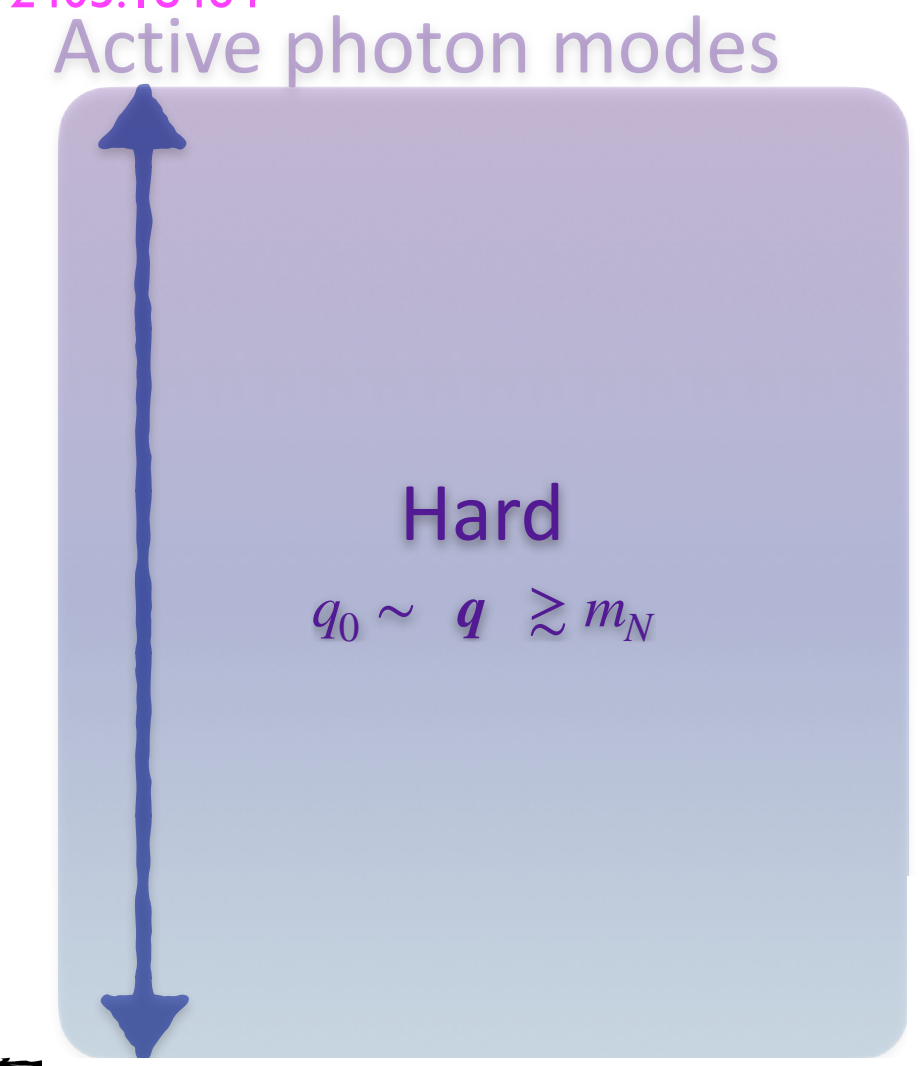
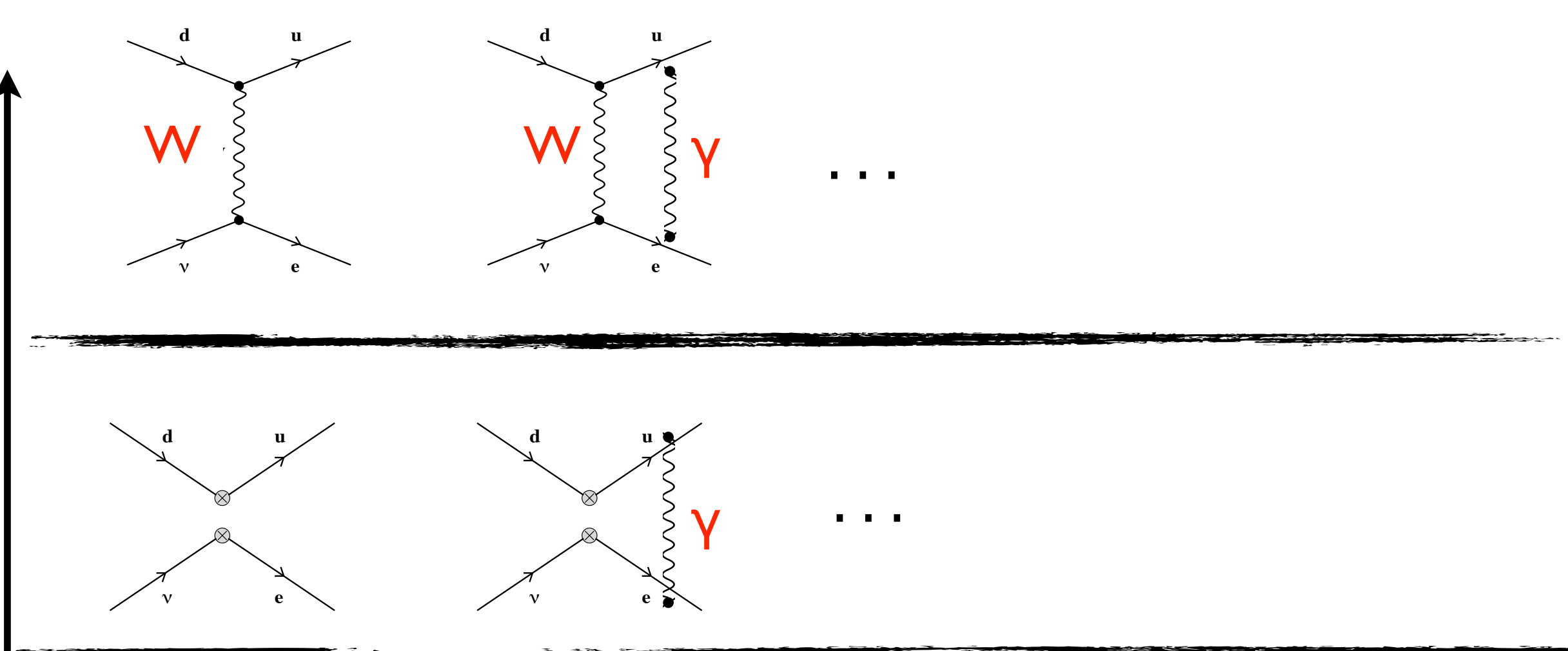
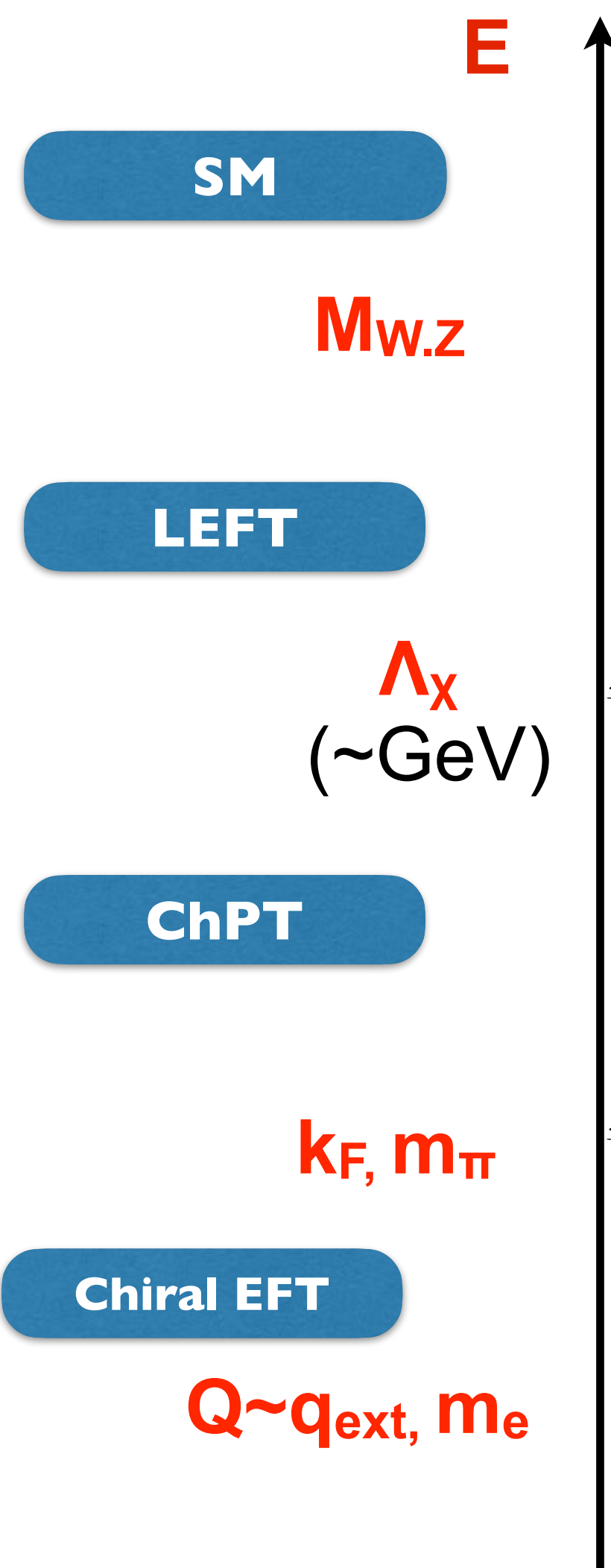
$$\Delta_R = 4.044(24)_{\text{Had}}(8)_{\alpha\alpha_s^2}(7)_{\alpha\epsilon_\chi^2}(5)_{\mu_\chi} [27]_{\text{total}} \%$$

+0.026% shift in total radiative correction to neutron decay compared to previous literature.
Related to the treatment of NLL corrections in the hadronic EFT ($\alpha^2 \text{Log}(m_N/m_e)$)

Overall shift of -0.013% in V_{ud} (neutron) compared to previous literature

EFT for nuclear decays (I)

VC, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, E. Mereghetti, 2405.18469, 2405.18464



EFT for nuclear decays (I)

VC, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, E. Mereghetti, 2405.18469, 2405.18464

E

SM

$M_{W,Z}$

LEFT

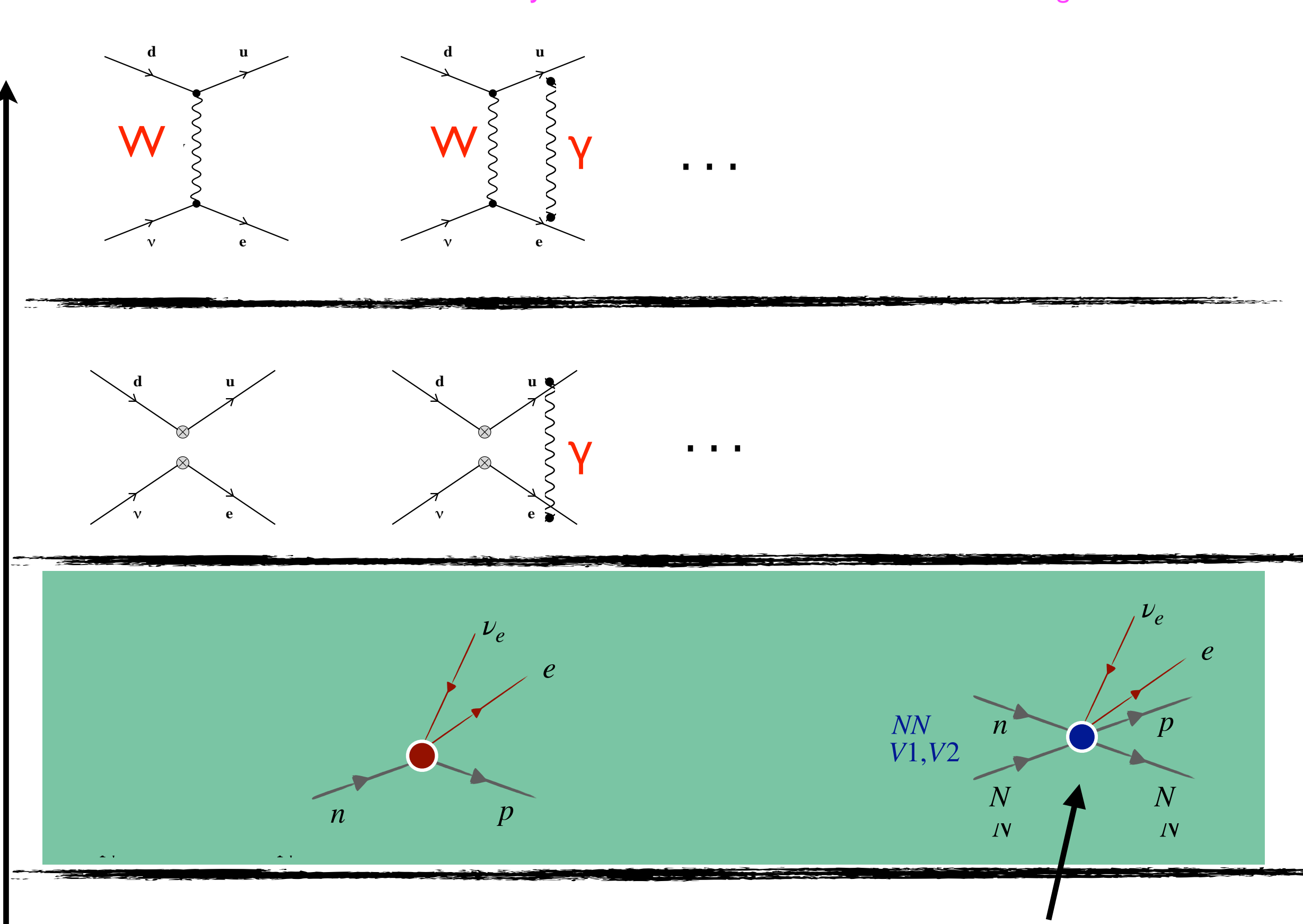
**Λ_χ
(\sim GeV)**

ChPT

k_F, m_π

Chiral EFT

$Q \sim q_{\text{ext}}, m_e$



Active photon modes

Hard
 $q_0 \sim q \gtrsim m_N$

Non-perturbative matching
LQCD/Dispersive methods
Generate 1- and 2-nucleon operators at $O(G_F)$

$$\mathcal{L}_{\chi PT} = -\sqrt{2}G_F V_{ud} \bar{e}_L \gamma_0 \nu_L \bar{N} \tau^+ N [e^2 g_{V1}^{NN} \bar{N} N + e^2 g_{V2}^{NN} \bar{N} \tau_3 N]$$

EFT for nuclear decays (I)

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E

SM

$M_{W,Z}$

LEFT

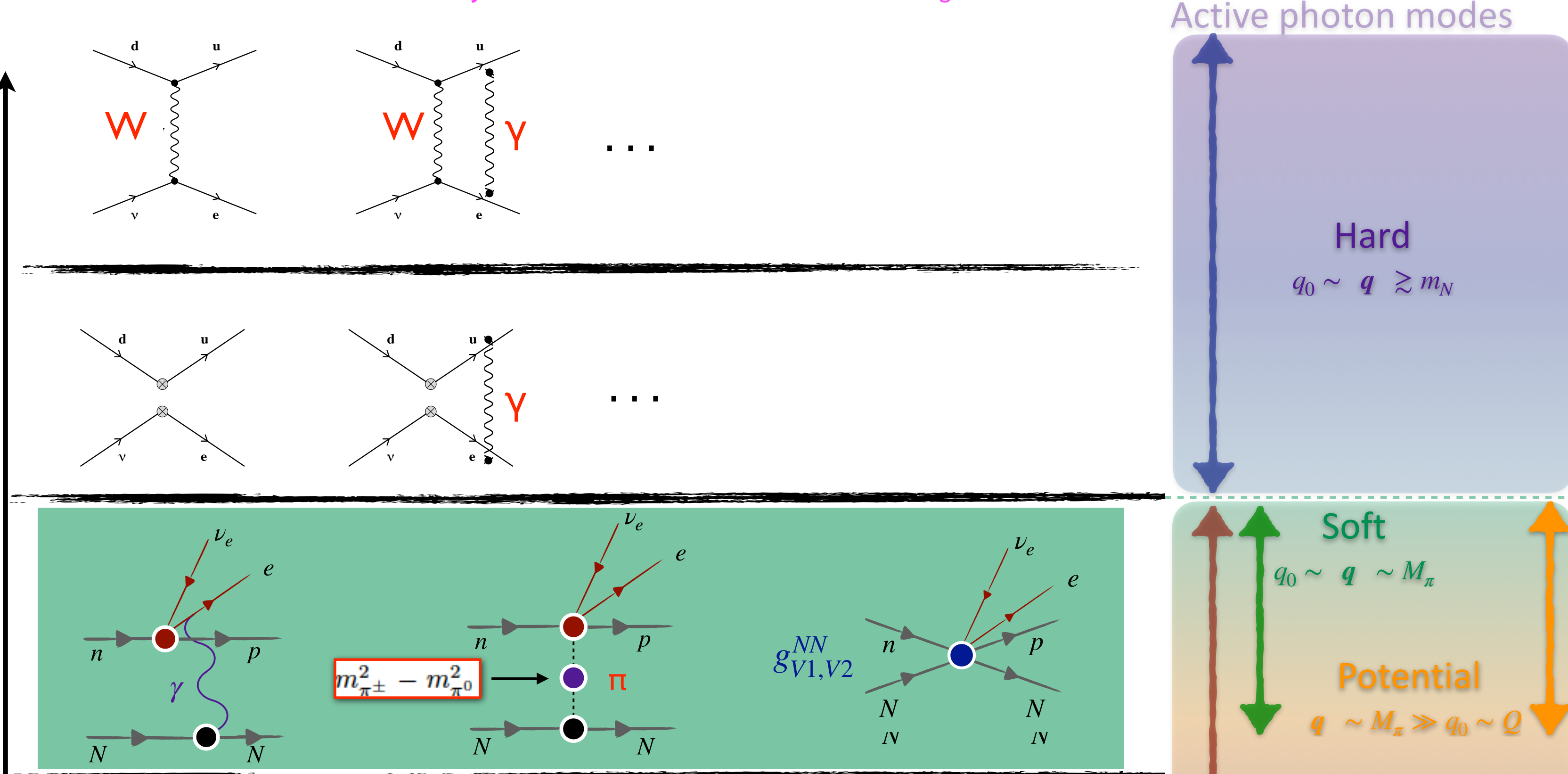
Λ_χ
($\sim \text{GeV}$)

ChPT

k_F, m_π

Chiral EFT

$Q \sim q_{\text{ext}}, m_e$

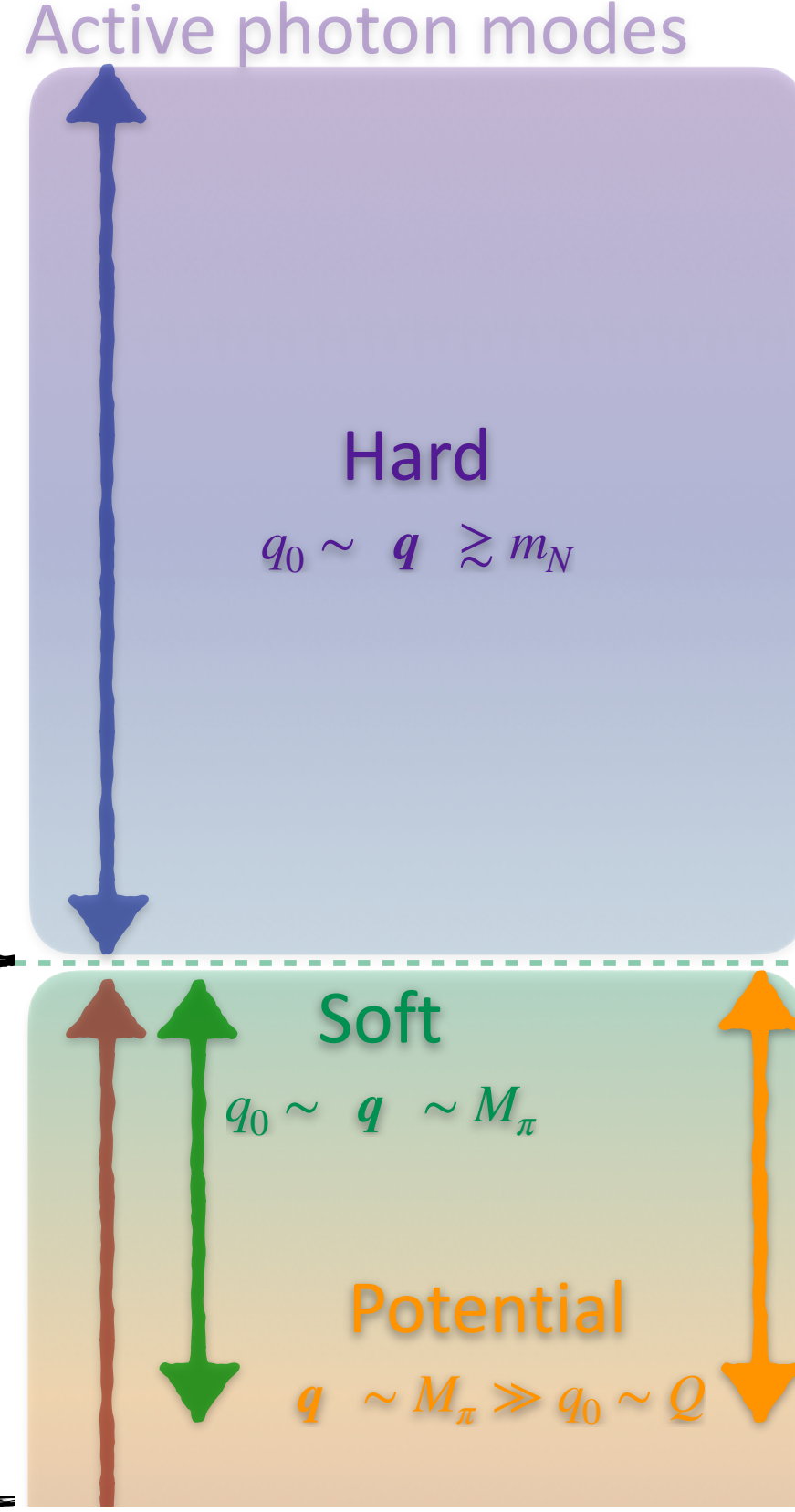
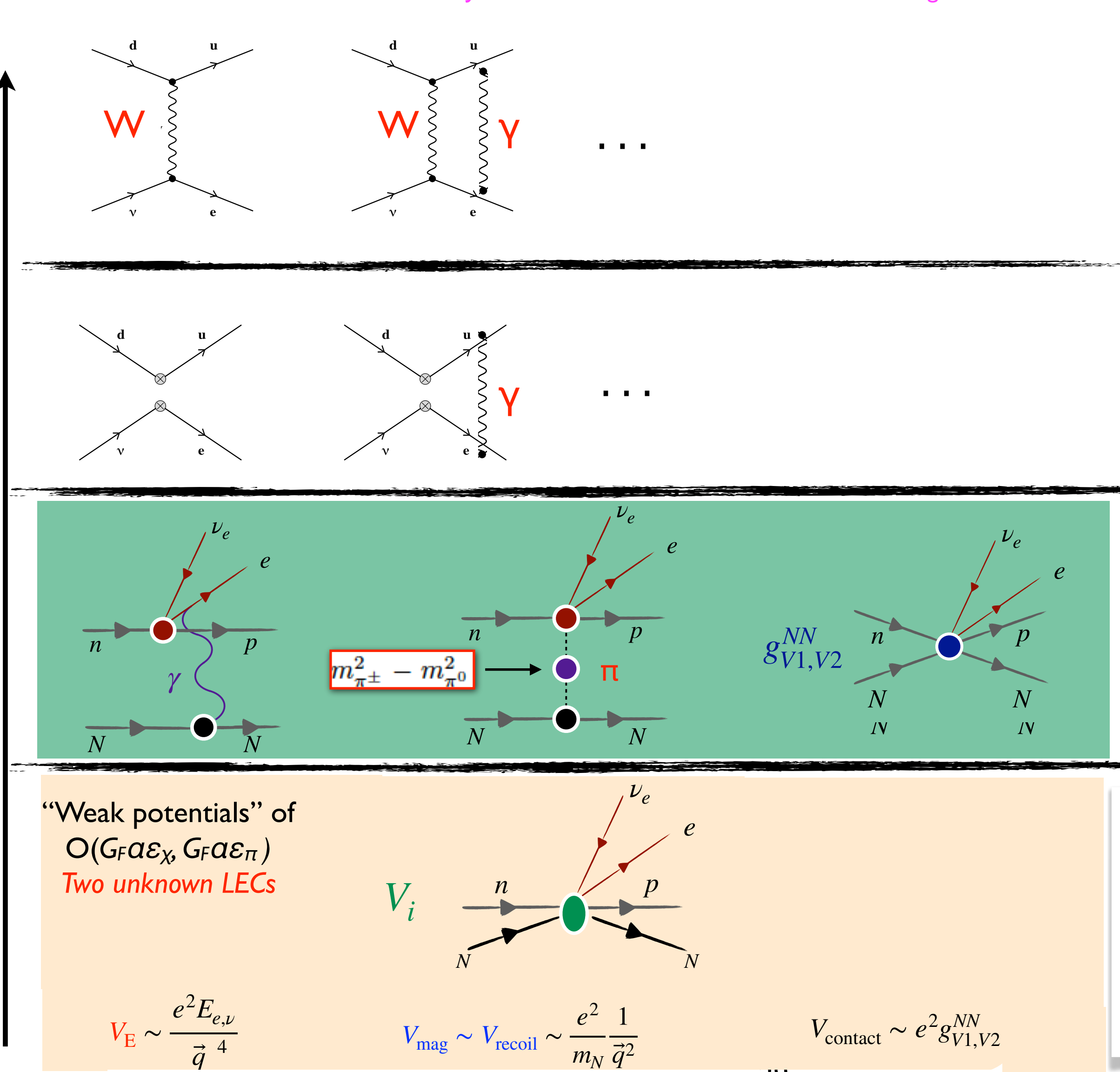


Non-perturbative matching
LQCD/Dispersive methods
Generate 1- and 2-nucleon
operators at $O(G_F)$

EFT for nuclear decays (I)

VC, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, E. Mereghetti, 2405.18469, 2405.18464

SM
 $M_{W,Z}$
LEFT
 Λ_χ
 (~GeV)
ChPT
 k_F, m_π
Chiral EFT
 $Q \sim q_{\text{ext}}, m_e$



Non-perturbative matching
 LQCD/Dispersive methods
 Generate 1- and 2-nucleon
 operators at $O(G_F \alpha)$

Integrate out soft &
 potential photons
 and pions

“Weak potentials” of
 $O(G_F \alpha \epsilon_\chi, G_F \alpha \epsilon_\pi)$
 Two unknown LECs

V_i

$V_E \sim \frac{e^2 E_{e,\nu}}{\vec{q}^4}$

$V_{\text{mag}} \sim V_{\text{recoil}} \sim \frac{e^2}{m_N} \frac{1}{\vec{q}^2}$

$V_{\text{contact}} \sim e^2 g_{V1,V2}^{NN}$

$$H_{EW} = \sqrt{2} G_F V_{ud} \bar{e}_L \gamma_\mu \nu_L \mathcal{J}_W^\mu,$$

$$\mathcal{J}_W^\mu = \sum_{n=1}^A \left(g_V \delta^{\mu 0} - g_A \delta^{\mu i} \sigma^{(n)i} \right) \tau^{(n)+} + (\mathcal{J}^{2b})^\mu + \dots$$

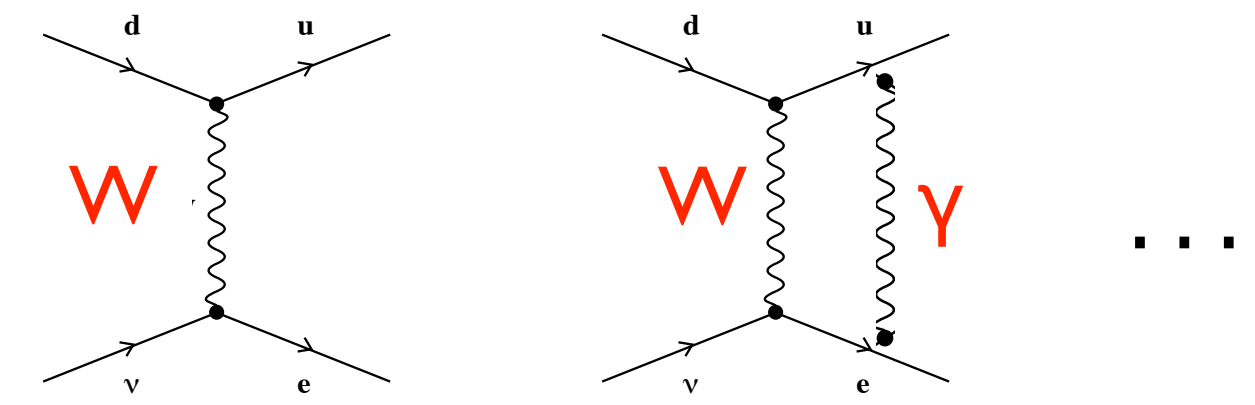
$$+ \delta^{\mu 0} (\mathcal{V}^0 + E_0 \mathcal{V}_E^0) + \delta^{\mu i} \mathcal{V}_i + p_e^\mu \mathcal{V}_{m_e} + \dots$$

EFT for nuclear decays (I)

VC, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, E. Mereghetti, 2405.18469, 2405.18464

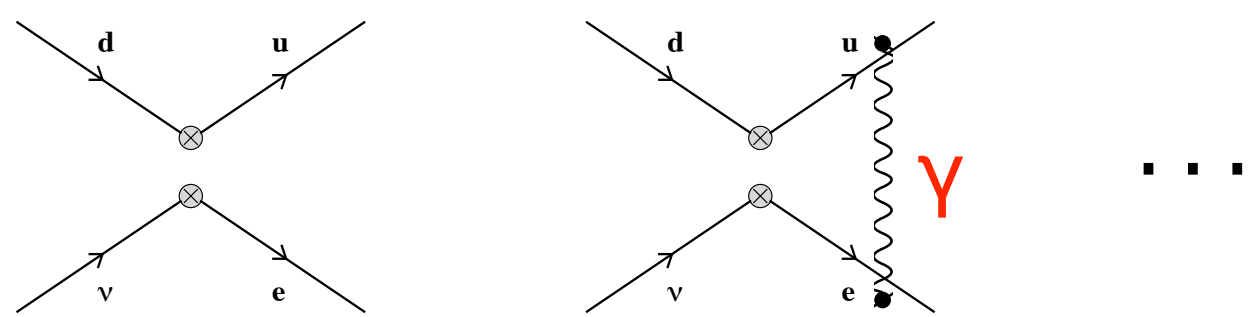
SM

E



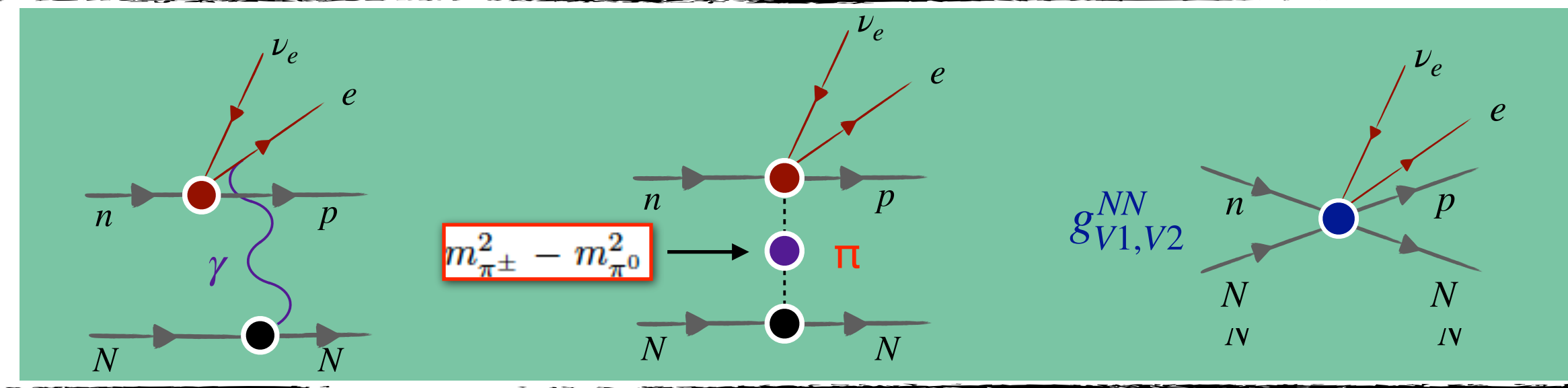
LEFT

$M_{W,Z}$



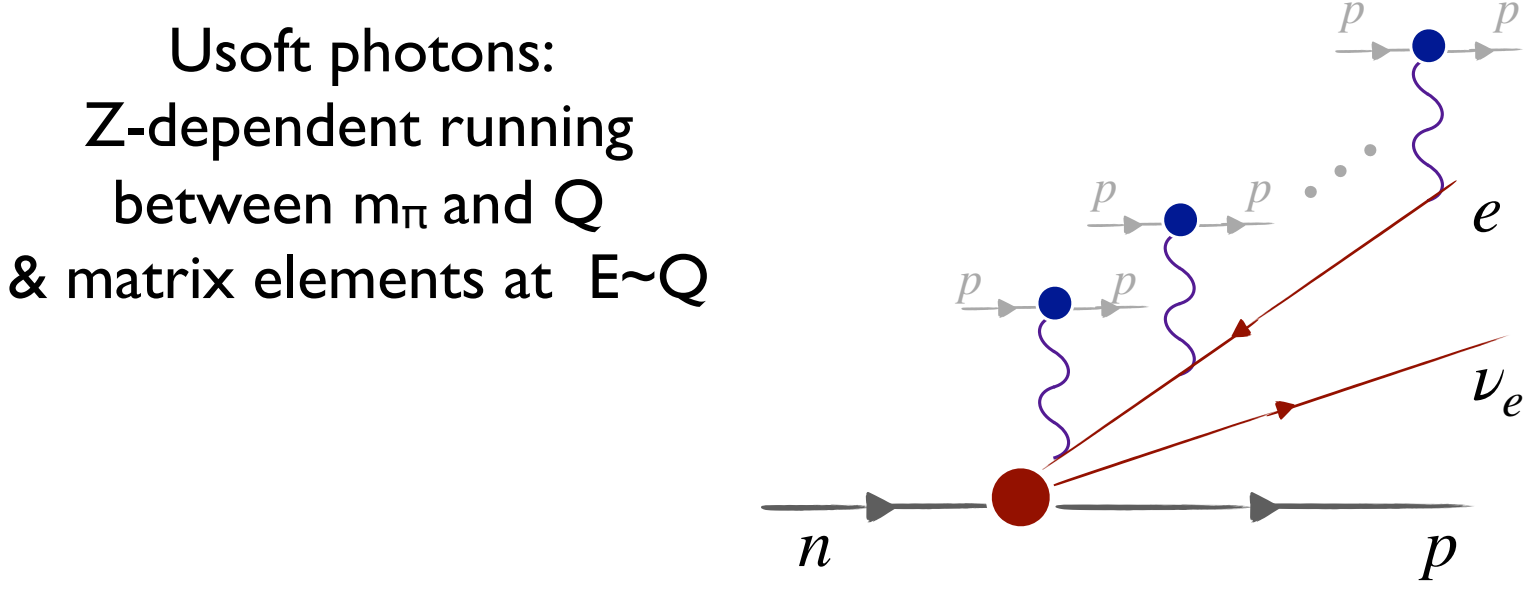
ChPT

**Λ_χ
(\sim GeV)**



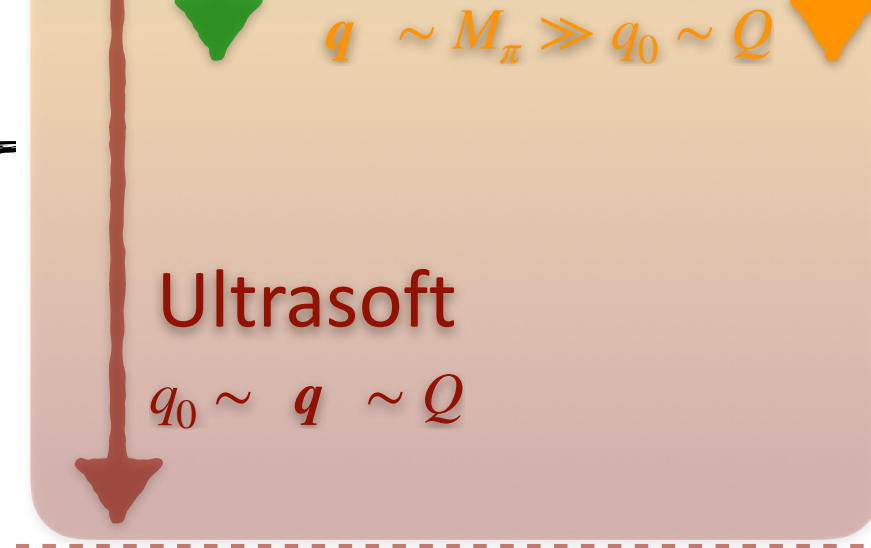
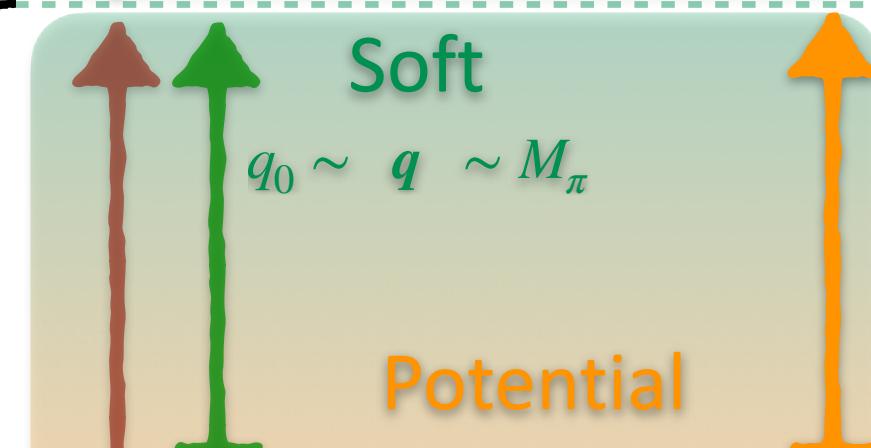
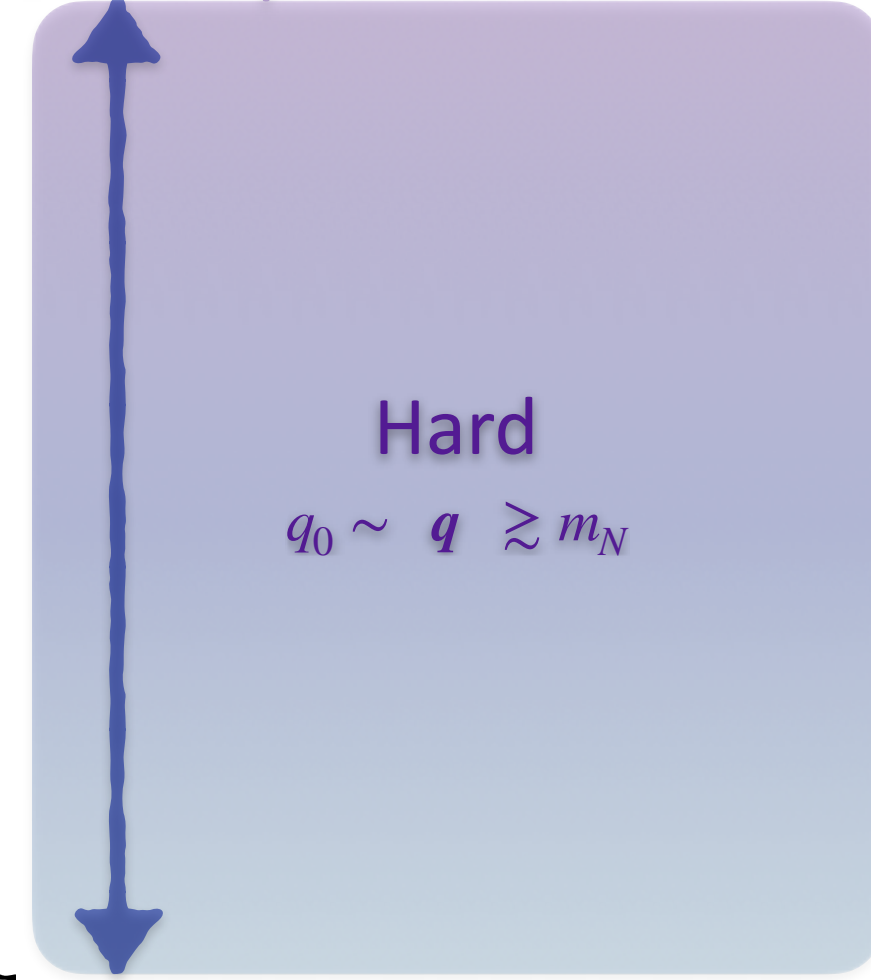
Chiral EFT

**k_F, m_π
 $Q \sim q_{\text{ext}}, m_e$**



Usoft photons:
Z-dependent running
between m_π and Q
& matrix elements at $E \sim Q$

Active photon modes



Non-perturbative matching
LQCD/Dispersive methods
Generate 1- and 2-nucleon
operators at $O(G_F)$

Integrate out soft &
potential photons
and pions

EFT for nuclear decays (2)

- EFT-based decay rate formula reorganizes ‘traditional’ corrections in terms of ‘matching and running’ (e.g. $C_{\text{eff}}^{(gV)}$)

Courtesy of Wouter Dekens

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{2(G_F V_{ud})^2}{(2\pi)^5} W(E_e, \mathbf{p}_e, \mathbf{p}_\nu) \tilde{C}(E_e) \bar{F}(\beta, \mu) [1 + \tilde{\delta}'_R(E_e, \mu)] (1 - \bar{\delta}_C) [1 + \tilde{\delta}_{\text{NS}}(E_e, \mu)] [C_{\text{eff}}^{(gV)}(\mu)]^2$$

The diagram illustrates the EFT-based decay rate formula with the following components and their associated labels:

- Fermi function** (π^2, Z enhanced): A red box above the $\tilde{C}(E_e)$ term.
- Isospin correction**: A green box above the $(1 - \bar{\delta}_C)$ term.
- RG/matching effects** ($\mu \gtrsim m_e$): A purple box above the $[C_{\text{eff}}^{(gV)}(\mu)]^2$ term.
- Shape/atomic/recoil corrections**: A black box below the $\tilde{C}(E_e)$ term.
- Outer correction** ($\mathcal{O}(\alpha/\pi)$): An orange box below the $[1 + \tilde{\delta}'_R(E_e, \mu)]$ term.
- Nuclear structure dependence**: A blue box below the $[1 + \tilde{\delta}_{\text{NS}}(E_e, \mu)]$ term.

- $\tilde{C}(E_e)$, $\tilde{\delta}_C$, $\tilde{\delta}_{\text{NS}}$ require nuclear structure input: good prospects of using ‘ab initio’ methods
- Significant new effect is in $\tilde{\delta}_{\text{NS}}$: short range potentials associated with currently unknown LECs

EFT for nuclear decays (3)

- Exploratory study in $^{14}\text{O} \rightarrow ^{14}\text{N}$ decay (Quantum Monte Carlo calculation of relevant matrix element)

$$V_{ud} = 0.97364(12)_{g_V}(10)_{\text{exp}}(22)_{\bar{f}}(13)_{\delta_{NS}^{\text{non-LEC}}(44)}_{\delta_{NS}^{\text{LEC}}(12)}_{\delta_c} [55]_{\text{total}}$$

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Largest uncertainty. Assumes $g_{V1,V2}^{NN} = 1/(4m_N F_\pi^2)$
LECs can be obtained by fitting data, once NME calculations for several isotopes become available.
Dispersive methods and lattice QCD can also be useful

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Residual scale dependence due to missing terms of $\mathcal{O}(\alpha^2 Z)$ in the Fermi function

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$$V_{ud} = 0.97364(12)_{g_V(10)} \exp(22)_{\bar{f}(13)} \delta_{NS}^{\text{non-LEC}}(44) \delta_{NS}^{\text{LEC}}(12) \delta_c [55]_{\text{total}}$$

Residual scale dependence due to missing terms of $\mathcal{O}(\alpha^2 Z)$ in the Fermi function

Largest uncertainty. Assumes $g_{V1,V2}^{NN} = 1/(4m_N F_\pi^2)$
 LECs can be obtained by fitting data, once NME calculations for several isotopes become available.
 Dispersive methods and lattice QCD can also be useful

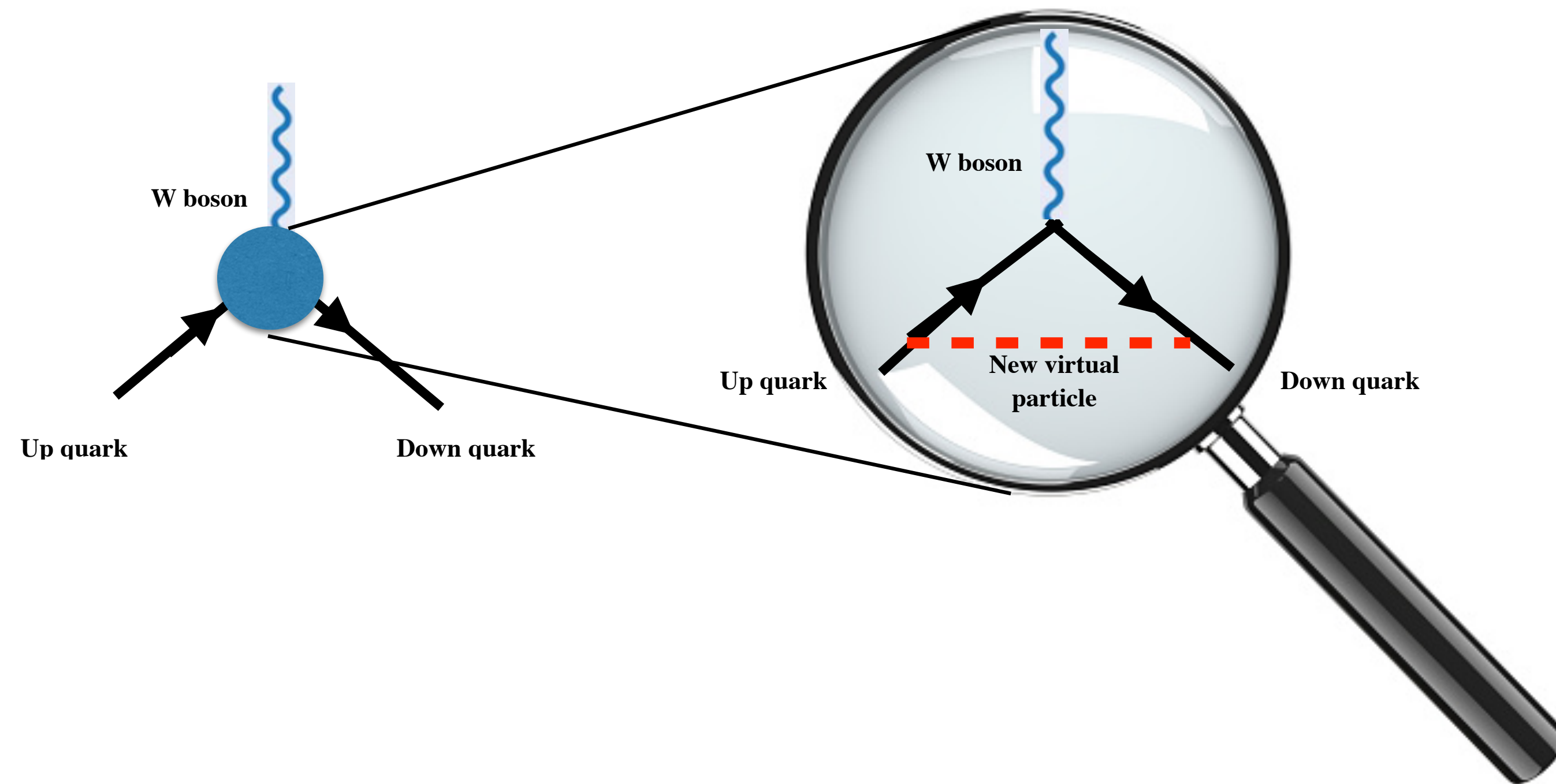
- To be compared with Towner-Hardy 2020 result (from $^{14}\text{O} \rightarrow ^{14}\text{N}$ decay alone):

$$V_{ud} = 0.97405(37)_{\text{total}}$$

(31) from δ_{NS}

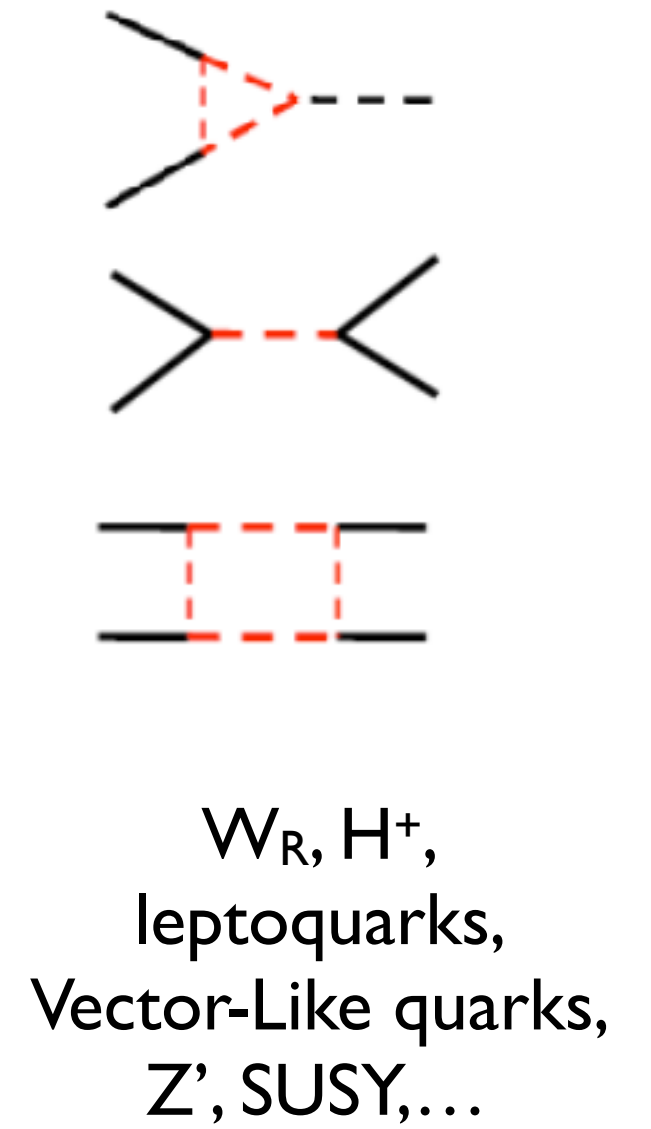
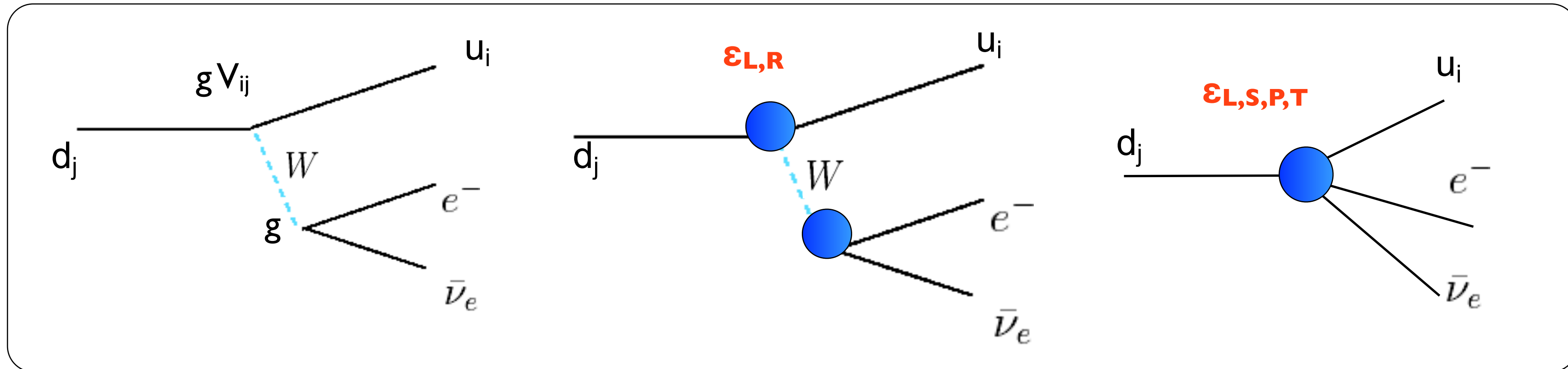
EFT has identified new correction and (temporarily) increased the uncertainty....
 But in the long run it's the only viable approach to quantify the uncertainties.

Implications for new physics



VC, A. Crivellin, M. Hoferichter, M. Moulson, 2208.11707, PLB
VC, W. Dekens, J. deVries, E. Mereghetti, T. Tong 2204.08440, PRD
VC, W. Dekens, J. de Vries, E. Mereghetti, T. Tong, 2311.00021, JHEP

Semileptonic processes beyond the SM



$E \sim \text{GeV (LEFT)}$



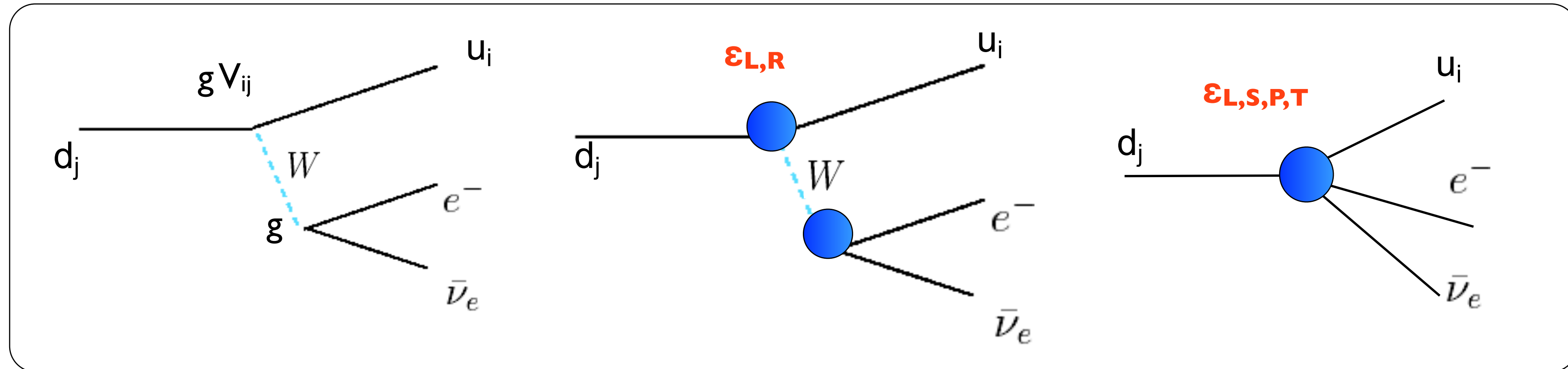
$\epsilon_\Gamma \sim \tilde{\epsilon}_\Gamma \sim (v/\Lambda)^2$

$$\mathcal{L}_{\text{SM}} - \frac{G_F V_{udj}}{\sqrt{2}} \sum_{\Gamma} \left[\epsilon_\Gamma^{(j)} \bar{\ell} \Gamma \nu_L \cdot \bar{u} \Gamma d_j + \tilde{\epsilon}_\Gamma^{(j)} \bar{\ell} \Gamma \nu_R \cdot \bar{u} \Gamma d \right]$$

$\Gamma = L, R, S, P, T$

BSM effects parameterized by 10(ud) + 10(us) effective couplings at $E \sim \text{GeV}$
They map into vertex corrections and 4-Fermion interactions above the EW scale

Semileptonic processes beyond the SM



$W_R, H^+,$
 leptoquarks,
 Vector-Like quarks,
 $Z',$ SUSY, ...

$E \sim \text{GeV (LEFT)}$



$\epsilon_\Gamma \sim \tilde{\epsilon}_\Gamma \sim (v/\Lambda)^2$

$$\mathcal{L}_{\text{SM}} - \frac{G_F V_{udj}}{\sqrt{2}} \sum_{\Gamma} \left[\epsilon_{\Gamma}^{(j)} \bar{\ell} \Gamma \nu_L \cdot \bar{u} \Gamma d_j + \tilde{\epsilon}_{\Gamma}^{(j)} \bar{\ell} \Gamma \nu_R \cdot \bar{u} \Gamma d \right]$$

$\Gamma = L, R, S, P, T$

Δ_{CKM} tension confirmed: points to specific new physics

Δ_{CKM} tension removed: strong constraints, complementary to traditional 'precision electroweak observables'

Corrections to V_{ud} and V_{us}

$$|\bar{V}_{ud}|_i^2 = |V_{ud}|^2 \left(1 + \sum_{\alpha} C_{i\alpha} \epsilon_{\alpha} \right)$$

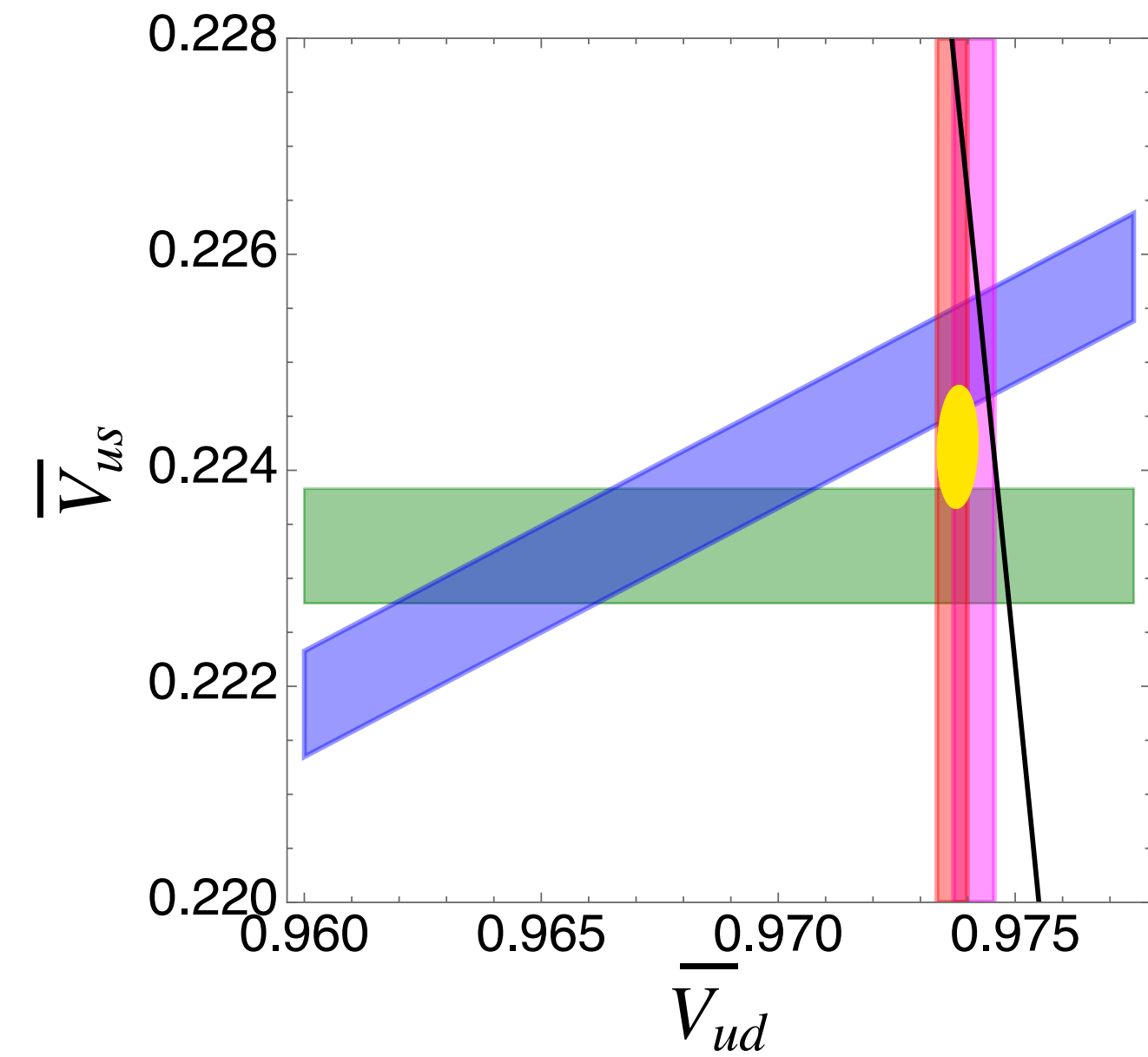
$$|\bar{V}_{us}|_j^2 = |V_{us}|^2 \left(1 + \sum_{\alpha} C_{j\alpha} \epsilon_{\alpha} \right)$$

Channel-dependent CKM elements extracted in the 'SM-like analysis'

Elements of the unitary CKM matrix

Known coefficients

BSM effective couplings



Find set of ϵ 's so that V_{ud} and V_{us} bands meet on the unitarity circle

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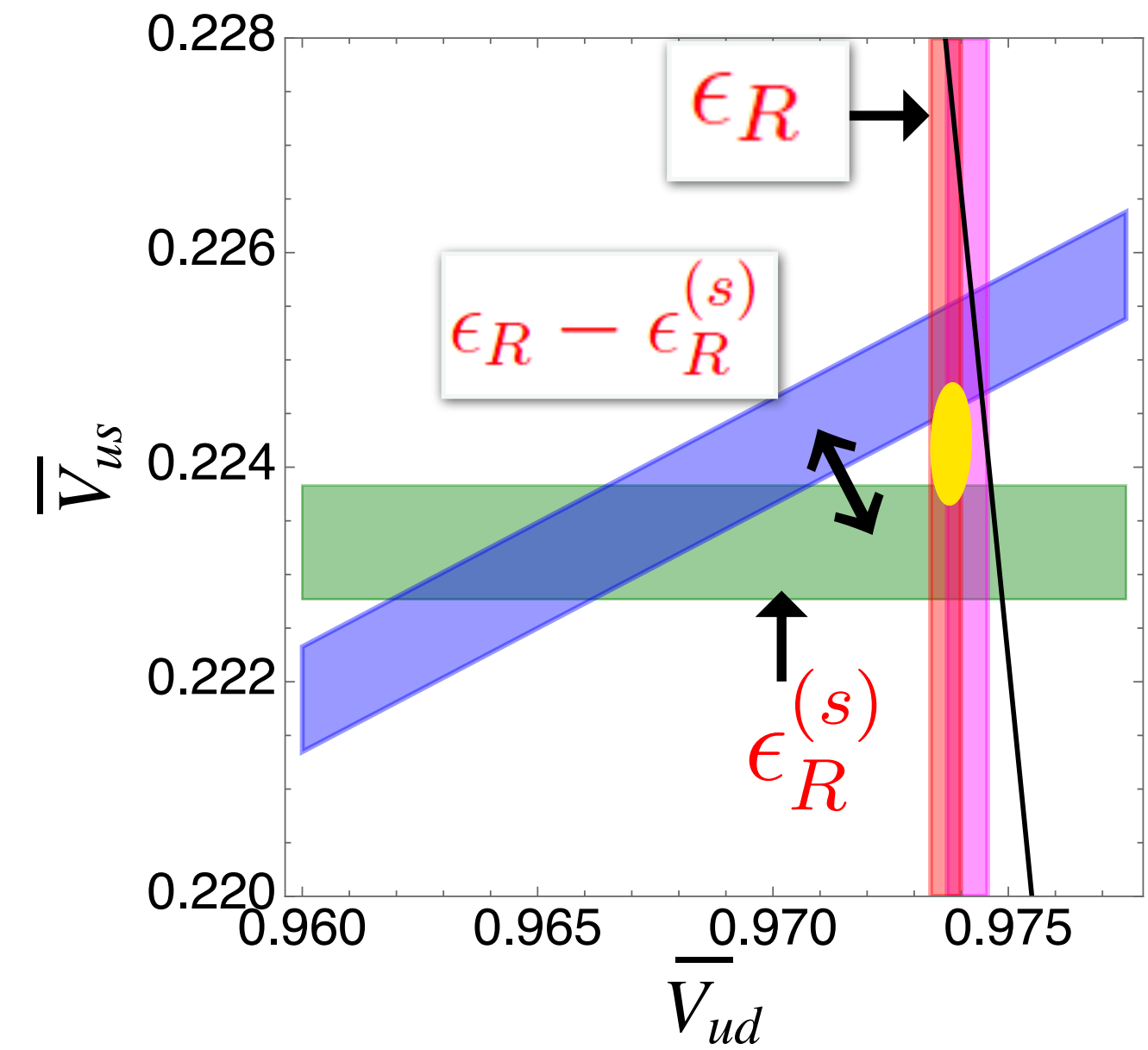
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Channel-dependent CKM elements extracted in the 'SM-like analysis'

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Find set of ϵ 's so that V_{ud} and V_{us} bands meet on the unitarity circle

Simplest 'solution': right-handed (V+A) quark currents

CKM elements from vector (axial) channels are shifted by $1+\epsilon_R$ ($1-\epsilon_R$).

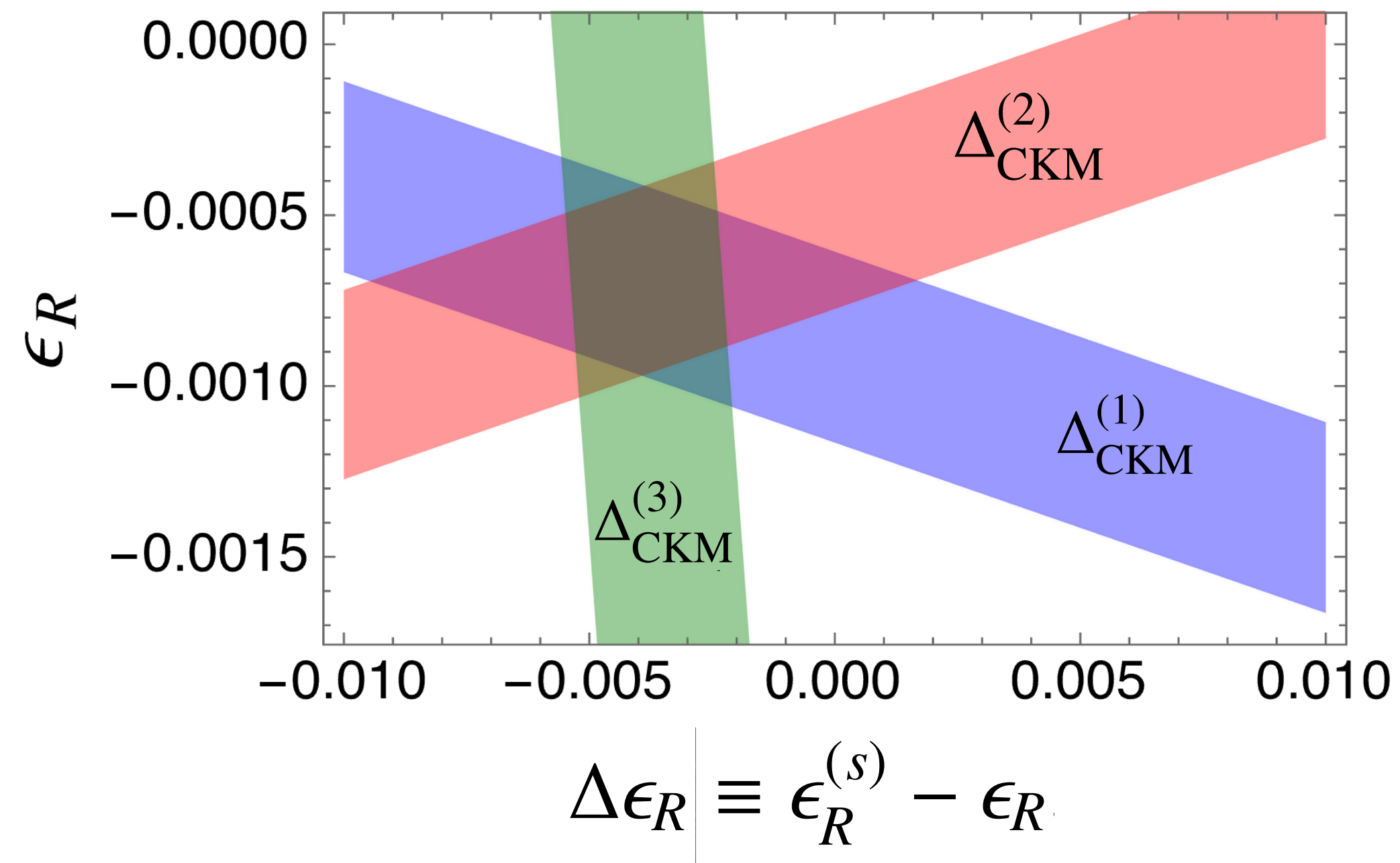
V_{us}/V_{ud} , V_{ud} and V_{us} shift in correlated way, can resolve all tensions!

Alioli et al 1703.04751
 Grossman-Passemar-Schacht 1911.07821
 VC-Crivellin-Hoferichter-Moulson 2208.11707
 VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, 2311.00021

For other BSM explanations, see A. Crivellin 2207.02507 and references therein

Unveiling R-handed quark currents?

VC-Crivellin-Hoferichter-Moulson 2208.11707



$$\begin{aligned}\Delta_{CKM}^{(1)} &= |V_{ud}^\beta|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1 \\ &= -1.76(56) \times 10^{-3} \\ \Delta_{CKM}^{(2)} &= |V_{ud}^\beta|^2 + |V_{us}^{K_{\ell 2}/\pi_{\ell 2}, \beta}|^2 - 1 \\ &= -0.98(58) \times 10^{-3} \\ \Delta_{CKM}^{(3)} &= |V_{ud}^{K_{\ell 2}/\pi_{\ell 2}, K_{\ell 3}}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1 \\ &= -1.64(63) \times 10^{-2}\end{aligned}$$



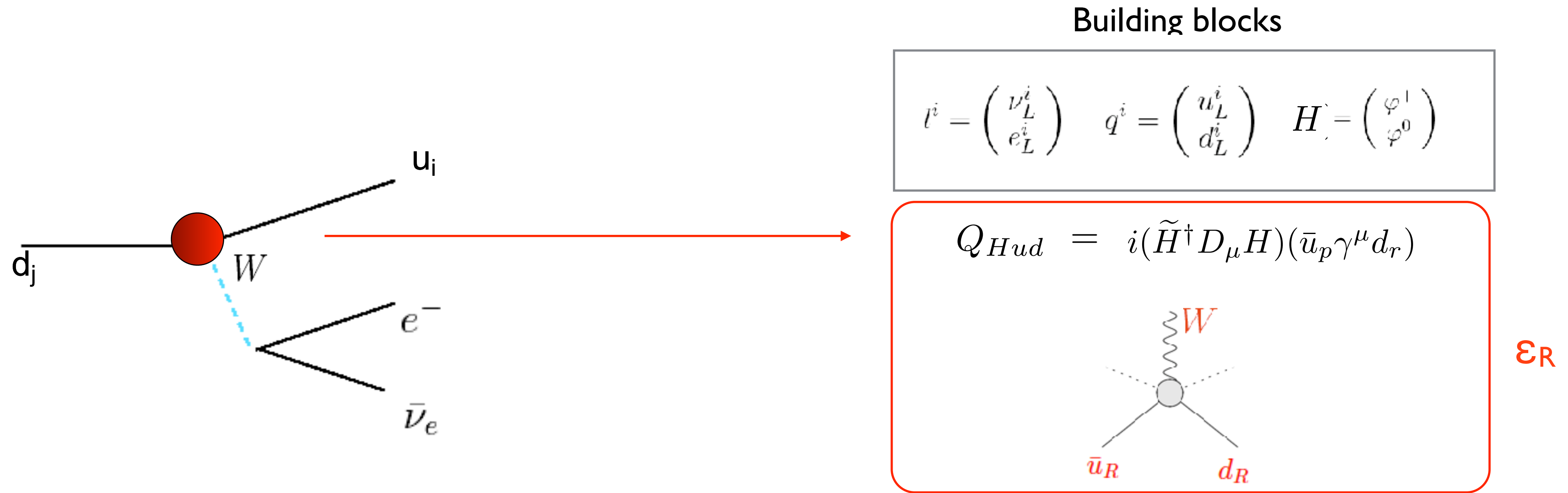
$$\begin{aligned}\epsilon_R &= -0.69(27) \times 10^{-3} \\ \Delta\epsilon_R &= -3.9(1.6) \times 10^{-3}\end{aligned}$$

$\Lambda_R \sim 5-10 \text{ TeV}$

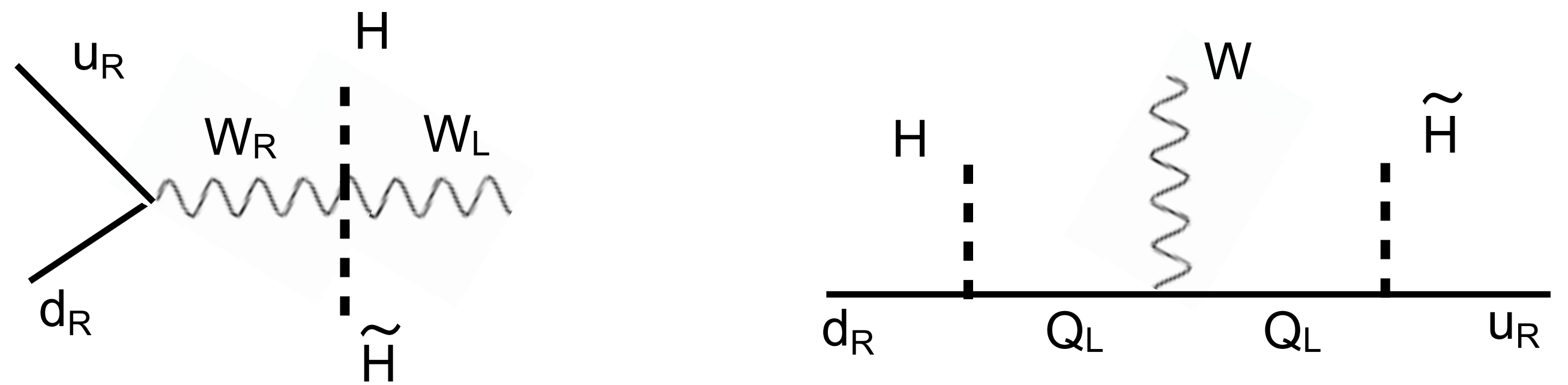
- Preferred ranges are not in conflict with constraints from other low-E probes
- Does the R-handed current explanation survive after taking into account high energy data?

High scale origin of ϵ_R

- ϵ_R originates from SU(2)xU(1) invariant vertex corrections in the SM-EFT

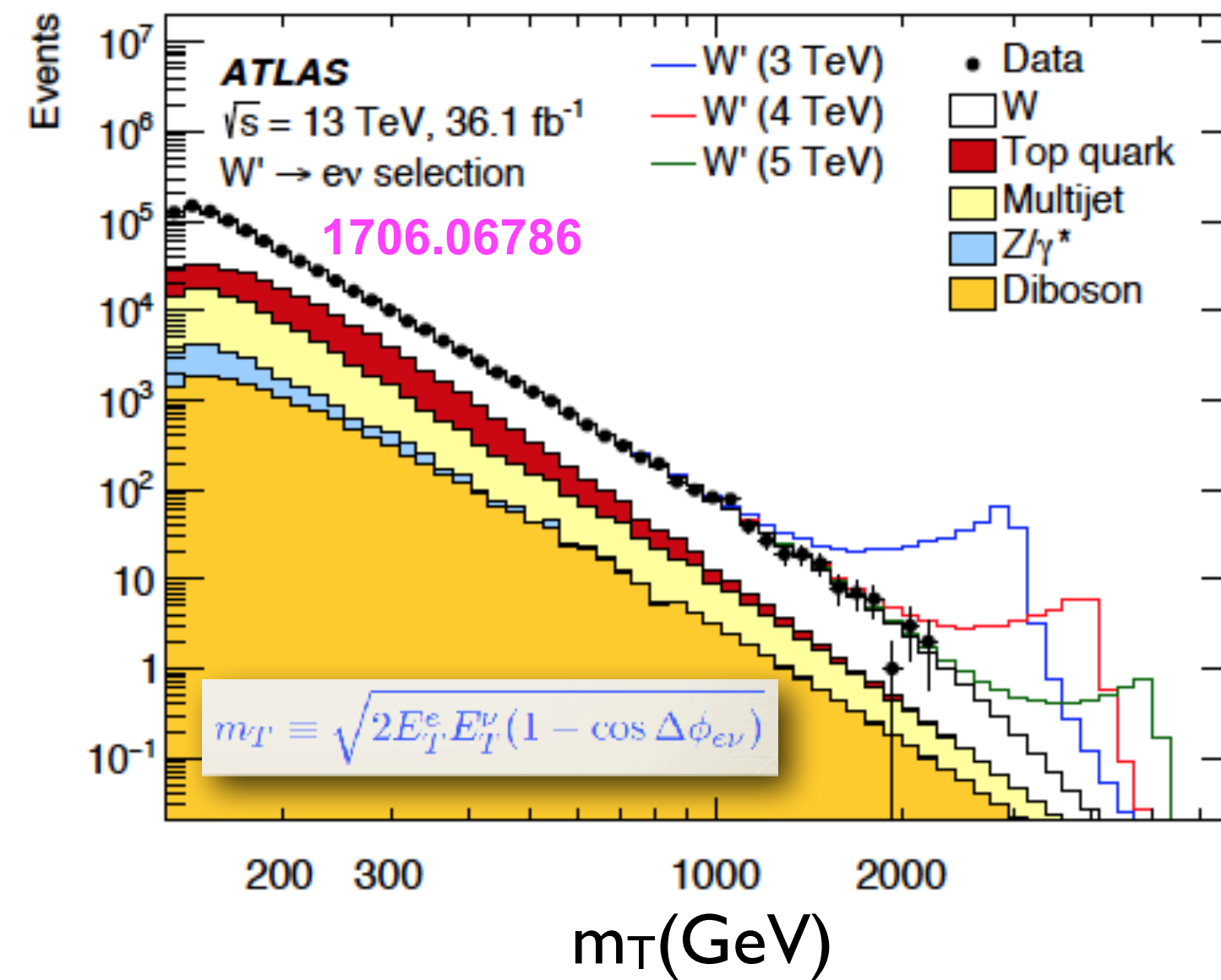
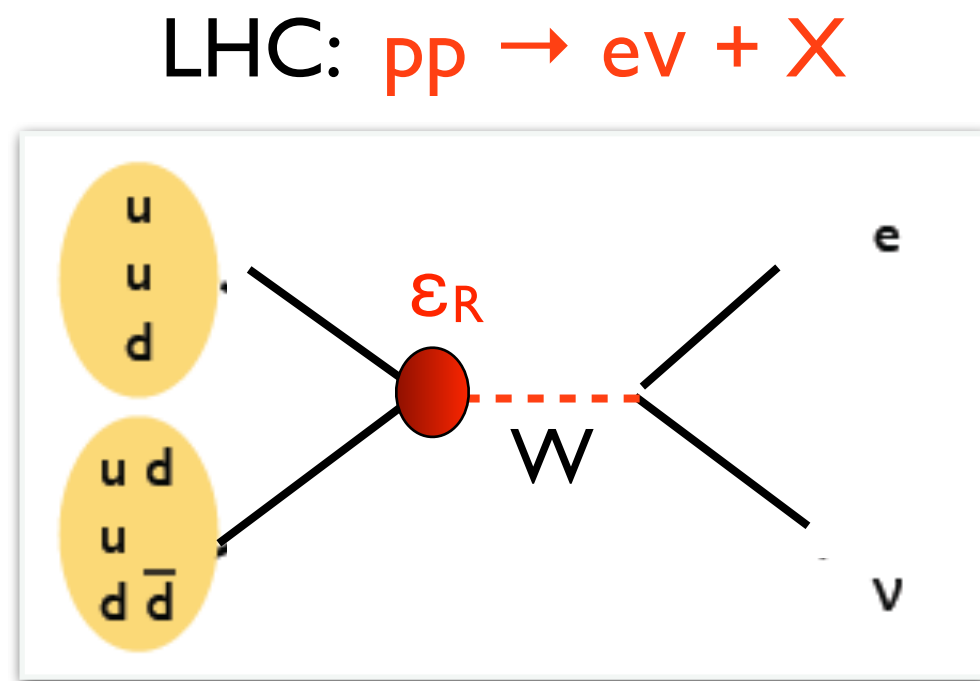


- Can be generated by W_L - W_R mixing in Left-Right symmetric models or by exchange of vector-like quarks



High Energy constraints on ϵ_R are weak

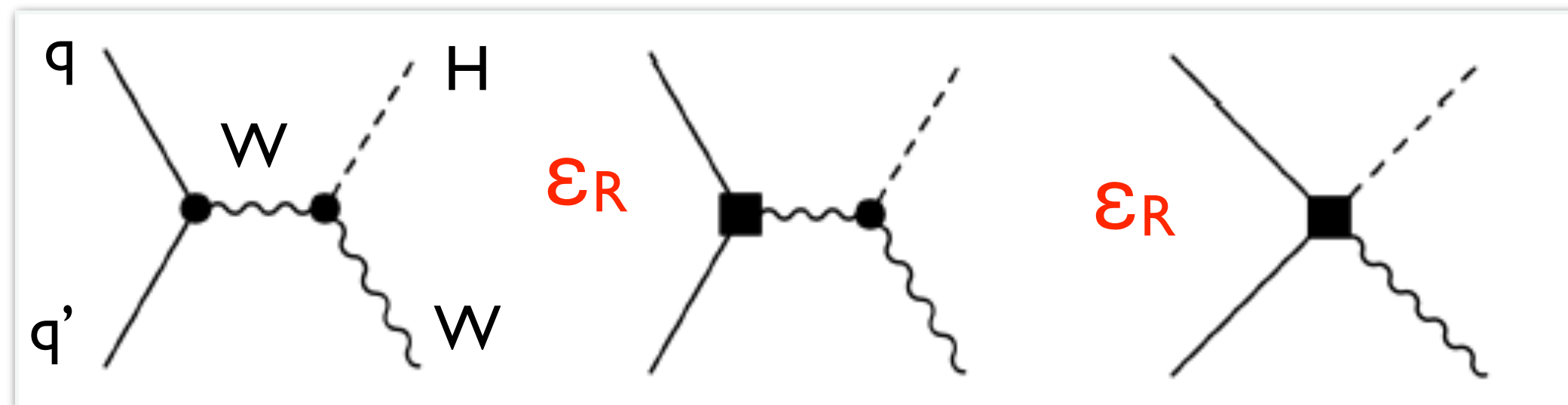
Contribute to $pp \rightarrow ev+X$ at the LHC



New contribution has same shape as the SM W exchange
 \rightarrow weak sensitivity

VC, Graesser, Gonzalez-Alonso 1210.4553
 Alioli-Dekens-Girard-Mereghetti 1804.07407
 Gupta et al. 1806.09006
 ...

Contributes to associated Higgs + W production at the LHC

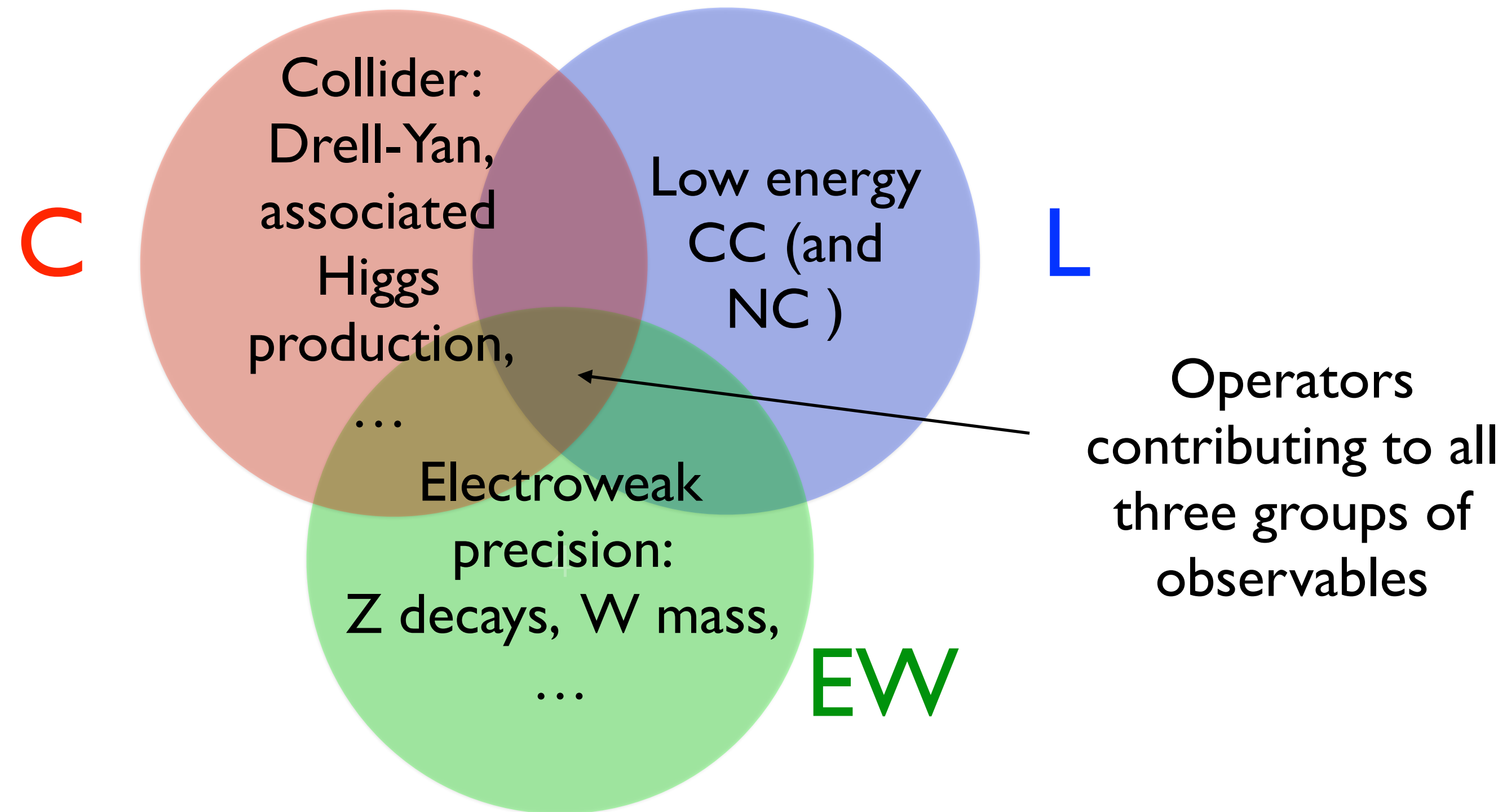


Current LHC results allow for $\epsilon_R \sim 5\%$

S. Alioli, VC, W. Dekens, J. de Vries, E. Mereghetti 1703.04751

Global analysis (I)

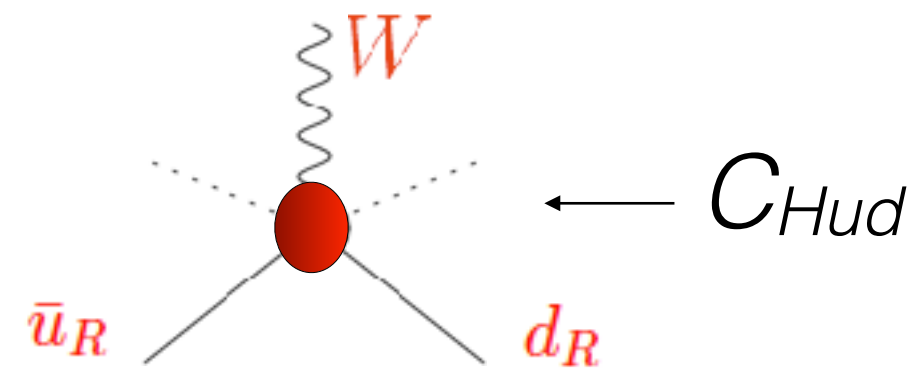
- A consistent analysis of beta decays in the SM-EFT requires including electroweak and collider data



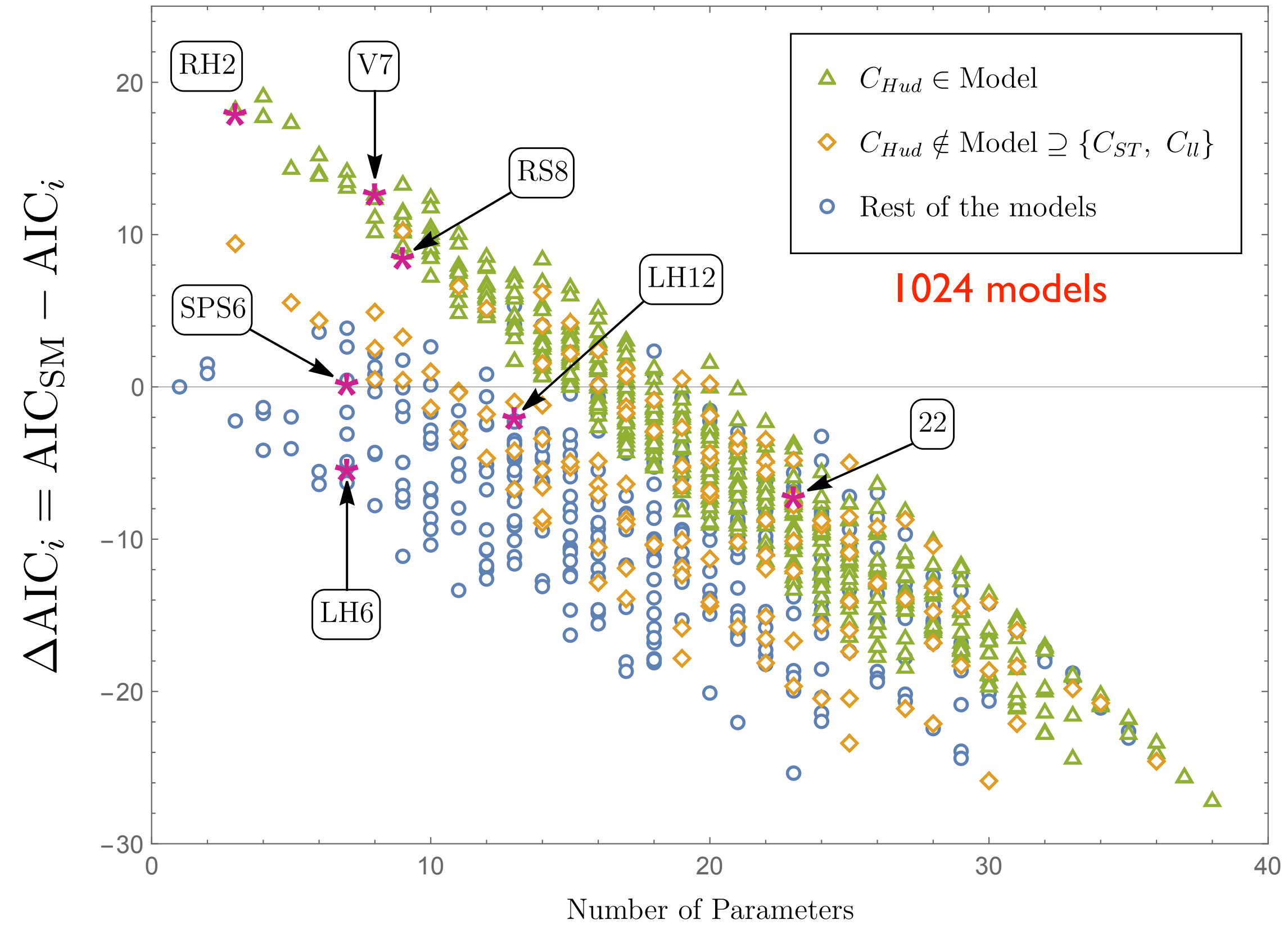
- CLEW analysis with no assumption about flavor symmetry requires 37 effective couplings

Global analysis (2)

- Performed ‘CLEWed’ analysis within SMEFT. Scanned model space by ‘turning on’ certain classes of effective couplings
- Akaike Information Criterion [$AIC = 2k - \ln(L)$] favors models with Right-Handed Charged Currents of quarks (V+A)



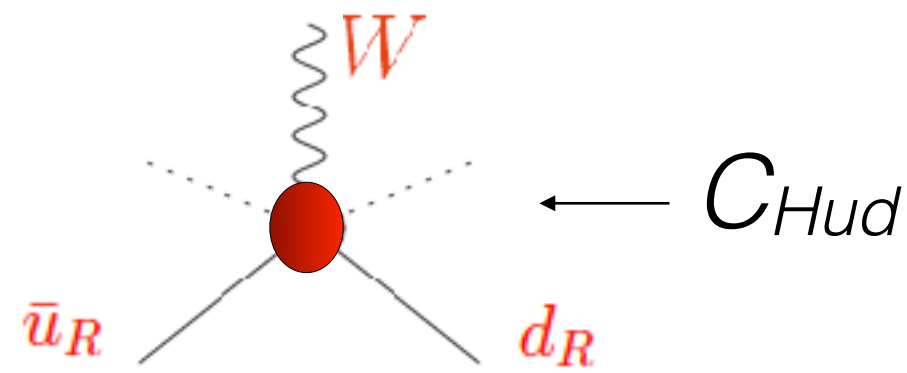
VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, JHEP 03 (24) 33, arXiv: 2311.00021



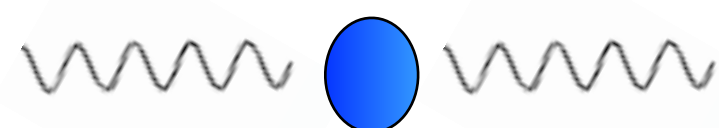
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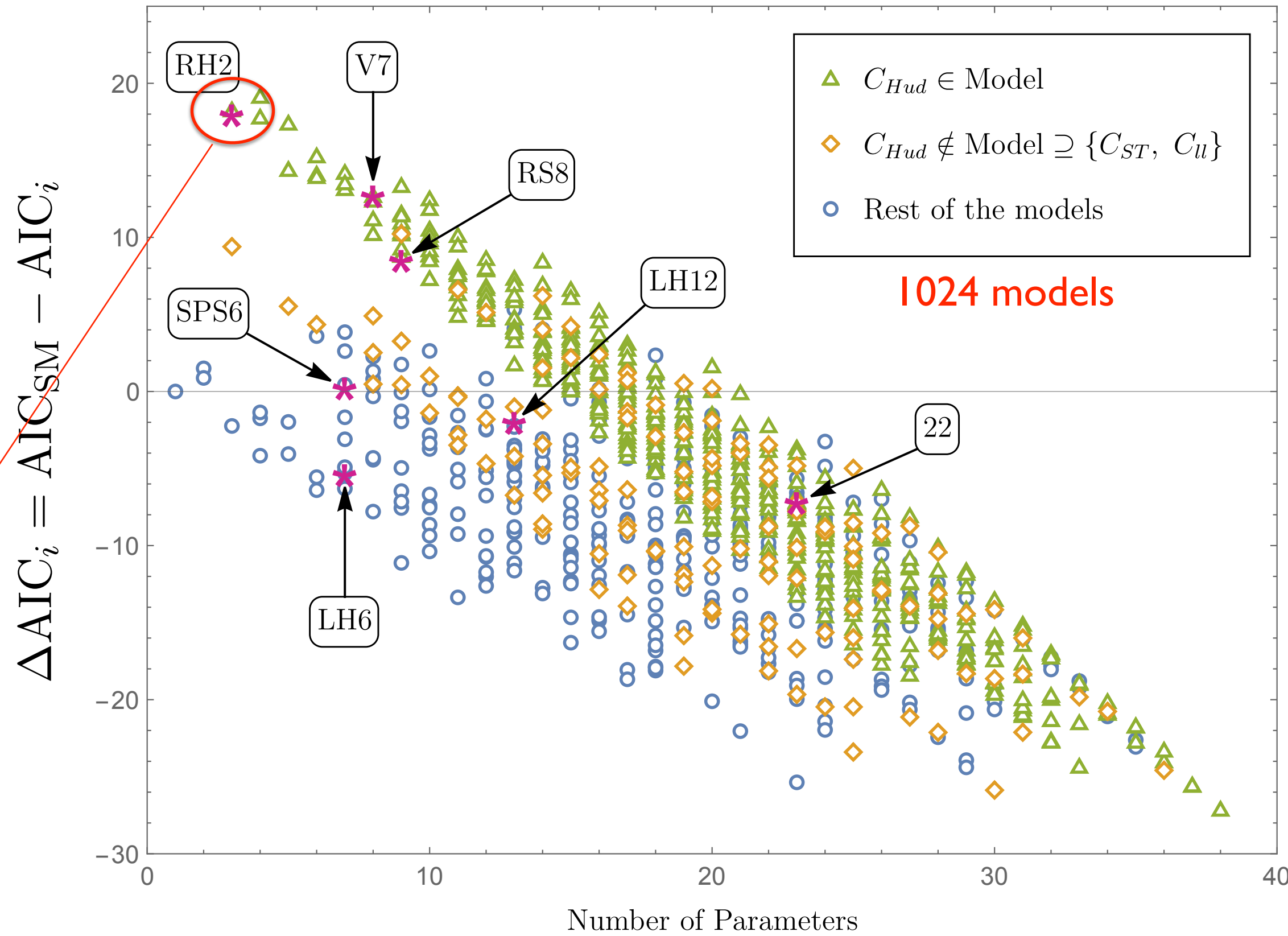
- Akaike Information Criterion [AIC = 2k - ln(L)] favors **models with Right-Handed Charged Currents of quarks (V+A)**



- Best fit to CLEW data: two RH CC vertex corrections and the S parameter



VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, JHEP 03 (24) 33, arXiv: 2311.00021



CKM “anomaly” not ruled out by other data.

Unitarity test provides relevant input to unravel possible new physics

Conclusions and outlook

The Cabibbo universality test is a precision tool to challenge the Standard Model.
and explore what may lie beyond it

- Current tensions in Cabibbo universality test could point to new physics at $\Lambda \sim \text{few TeV}$, with right-handed quark- W couplings a viable and testable culprit. However ...
- Both experimental and theoretical scrutiny is needed! Progress expected on several fronts:
 - **Experiment:** neutron, K , π , τ
 - **Theory:** lattice QCD+QED for neutron, K , π ; EFT+ ‘ab-initio’ methods for nuclei

Ongoing experimental and theoretical activities promise interesting developments

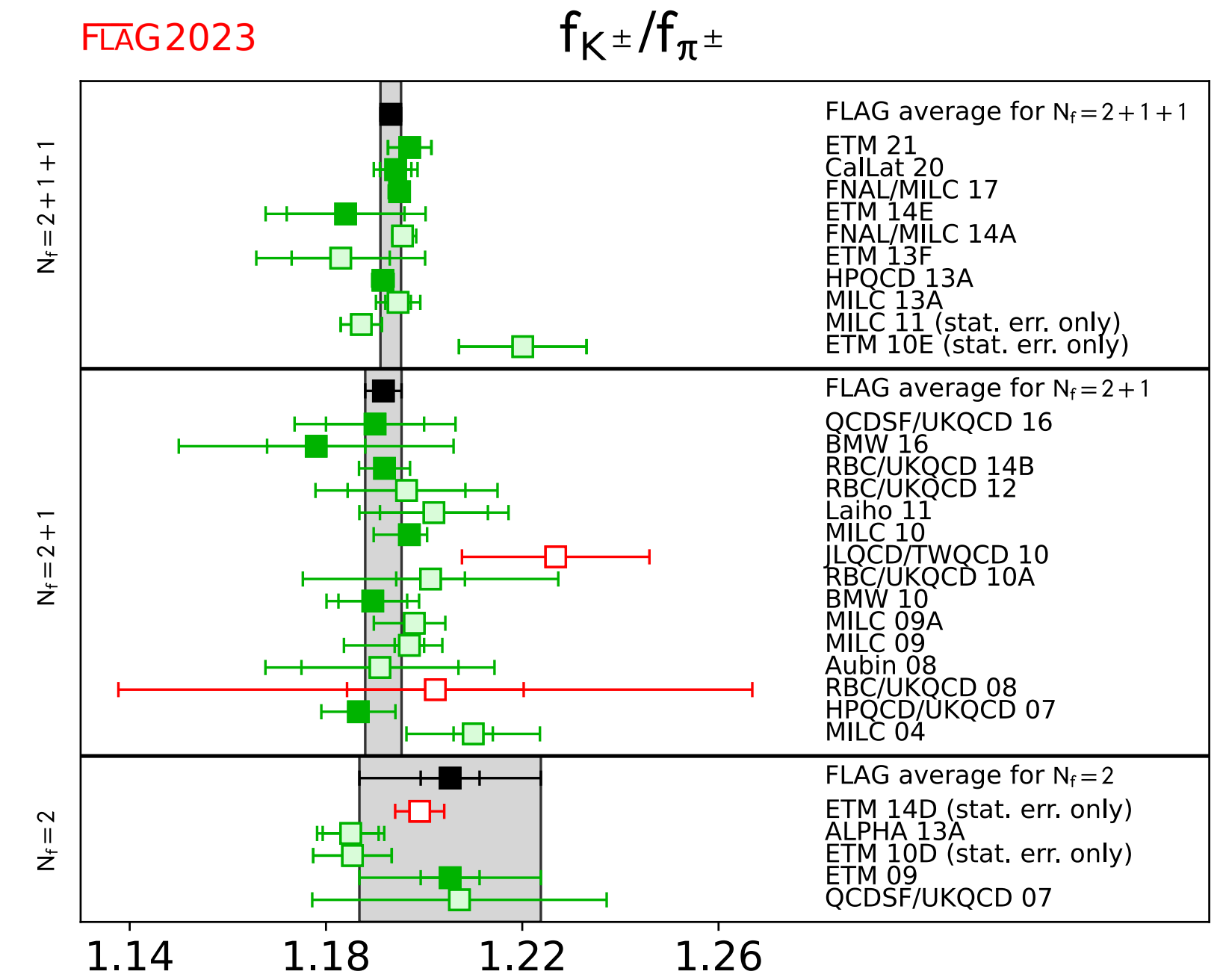
Backup

V_{us} from $K \rightarrow \mu \nu$ decays

$$\frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = \left(\frac{\Gamma_{K \rightarrow \mu \nu(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi \rightarrow \mu \nu(\gamma)} m_{K^\pm}} \right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{\Delta_{RC+IB}^{K\pi}}{2} \right)$$

- Lattice QCD calculations of f_K/f_π are at the 0.2% level
- First calculation of radiative and isospin-breaking corrections in LQCD.** Compatible with ChPT, factor of ~2 more precise

ChPT: VC-Neufeld, 1102.0563	** LQCD1: Di Carlo et al., 1904.08731	LQCD2: Boyle et al., 2211.12865
$\Delta_{RC+IB}^{K\pi} = -1.12(21)\%$	$\Delta_{RC+IB}^{K\pi} = -1.26(14)\%$	$\Delta_{RC+IB}^{K\pi} = -0.86(40)\%$



V_{us} from K → μν decays

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Potential issue (1):

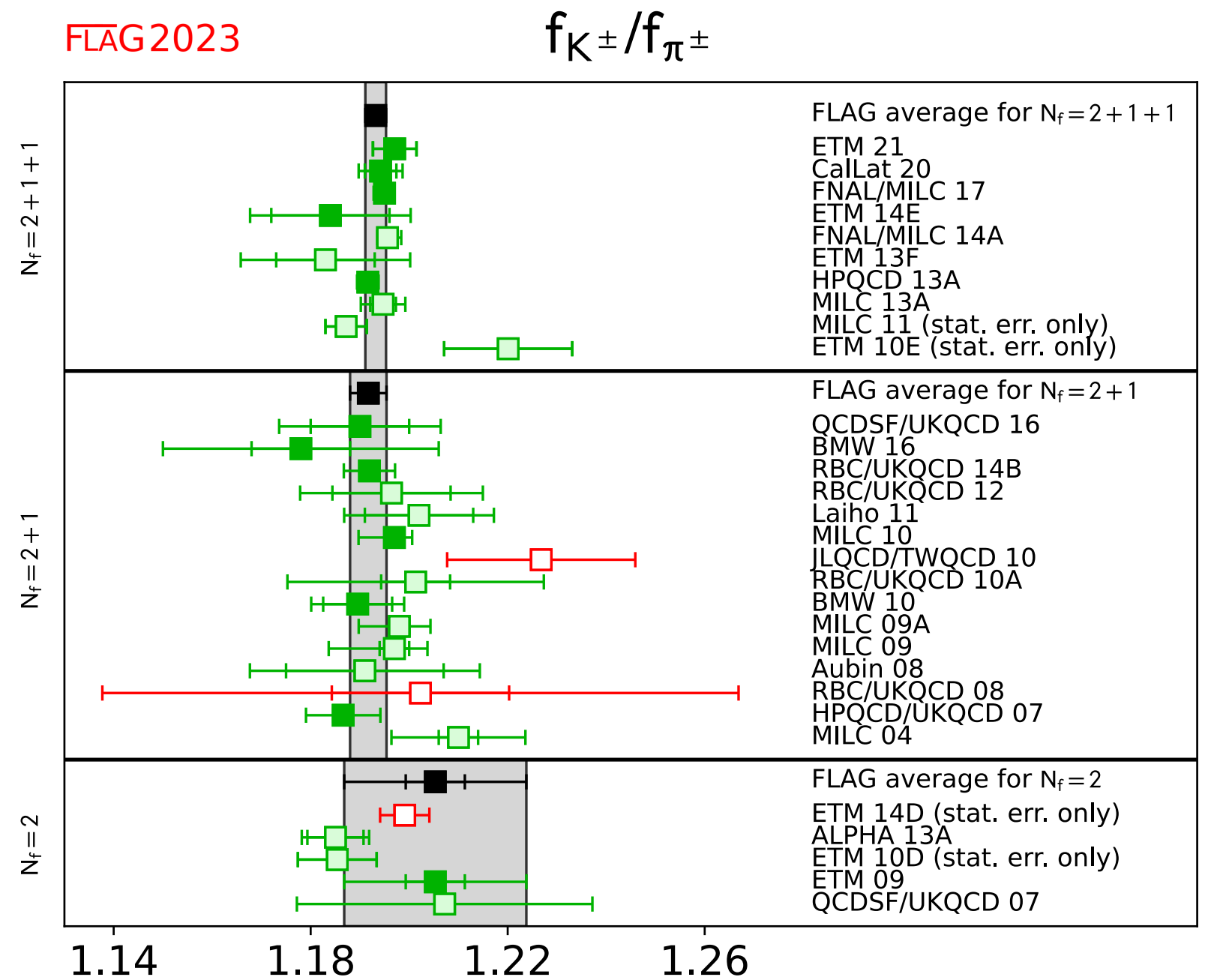
Kmu2 BR dominated by one measurement (KLOE)

Km3/Kmu2 BR measurement at 0.2% would have significant impact

$$\left. \frac{V_{us}}{V_{ud}} \right|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{RC+IB}[51]_{\text{total}}$$

Potential issue (2):

Isospin scheme dependence



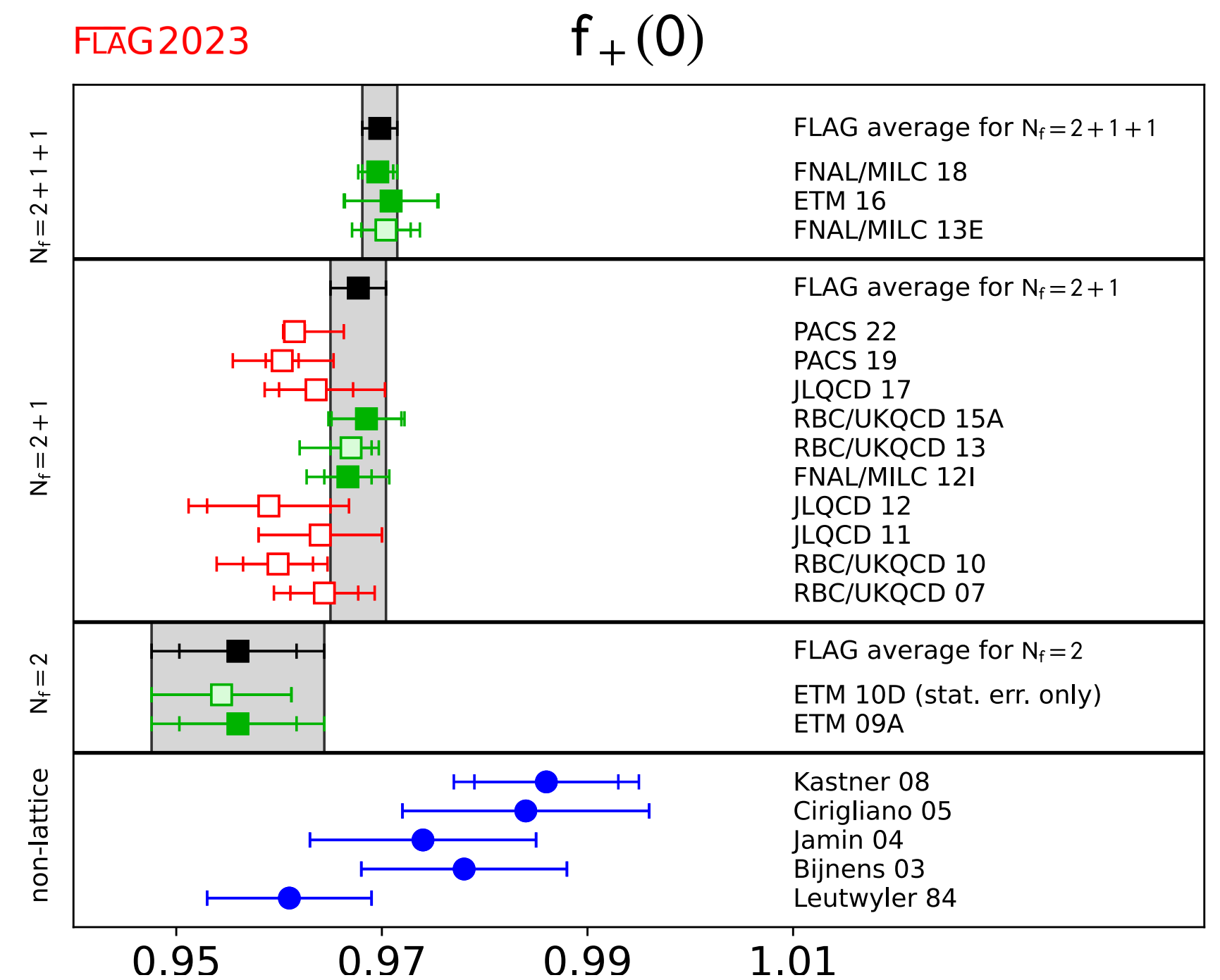
V_{us} from $K \rightarrow \pi l \nu$ decays

$$\Gamma_{K \rightarrow \pi l \nu(\gamma)} = \frac{C_K^2 G_F^2 S_{EW} |V_{us}|^2 M_K^5}{192\pi^3} |f_+^{K\pi}(0)|^2 I_{Kl} \left(1 + 2\Delta_{Kl}^{EM} + 2\Delta_K^{IB} \right)$$

- Lattice calculations of $\langle \pi | V | K \rangle$ @ 0.2%: $f_+^{K\pi}(0) = 0.9698(17)$
- New radiative corrections based on current algebra + lattice QCD. Consistent with ChPT, with much smaller uncertainties

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{EM}(K^0_{e3})$ [%]	0.50 ± 0.11	0.580 ± 0.016
$\Delta^{EM}(K^+_{e3})$ [%]	0.05 ± 0.12	0.105 ± 0.023
$\Delta^{EM}(K^+_{\mu3})$ [%]	0.70 ± 0.11	0.770 ± 0.019
$\Delta^{EM}(K^0_{\mu3})$ [%]	0.01 ± 0.12	0.025 ± 0.027

NEW: Seng et al, 1910.13209, 2103.00975, 2103.4843, 2107.14708, 2203.05217, Ma et al. 2102.12048
 OLD: VC, Giannotti, Neufeld 0807.4607



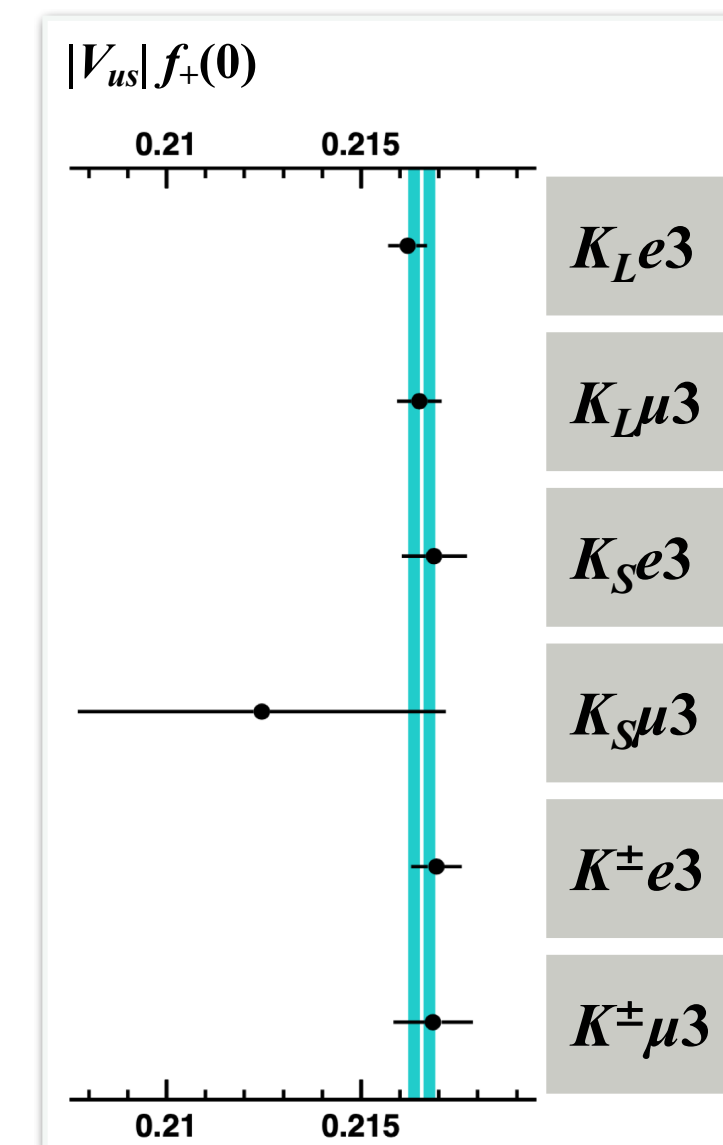
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- New radiative corrections based on current algebra + lattice QCD. Consistent with ChPT, with much smaller uncertainties
- Experimental input has received only small updates since 2010

Flavianet WG, 1005.2323

Moulson 1704.04104



$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{RC+IB}}[53]_{\text{total}}$$

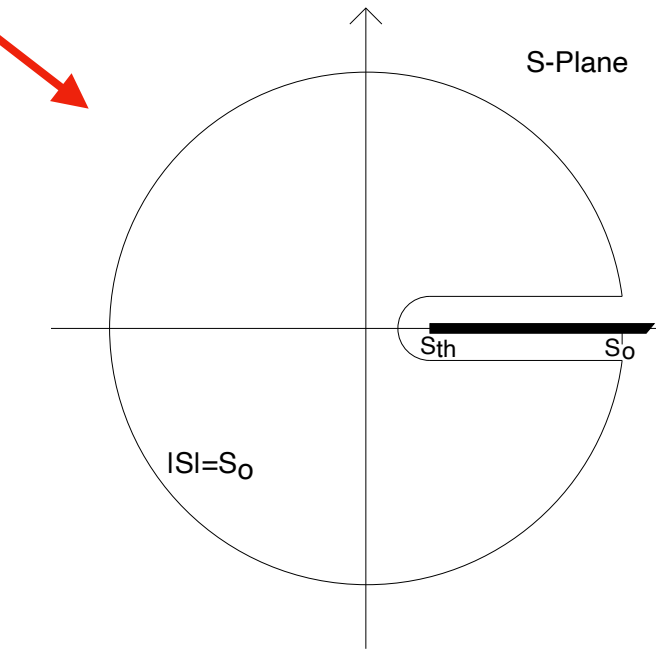
Potential issue: definition of 'isosymmetric QCD' in lattice ($f_+(0)$) vs calculations of $\Delta^{\text{EM,IB}}$

V_{us} from tau decays

- Inclusive ($\tau \rightarrow X_s \nu$): need integrated spectral functions + $\Delta\Pi_{jj}(s)$ on the $|s| = s_0 \sim m_\tau^2$ circle (OPE \rightarrow Lattice QCD)

$$R_\tau = \frac{\Gamma[\tau \rightarrow \text{hadrons } \nu_\tau]}{\Gamma[\tau \rightarrow \bar{\nu}_e e \nu_\tau]}$$

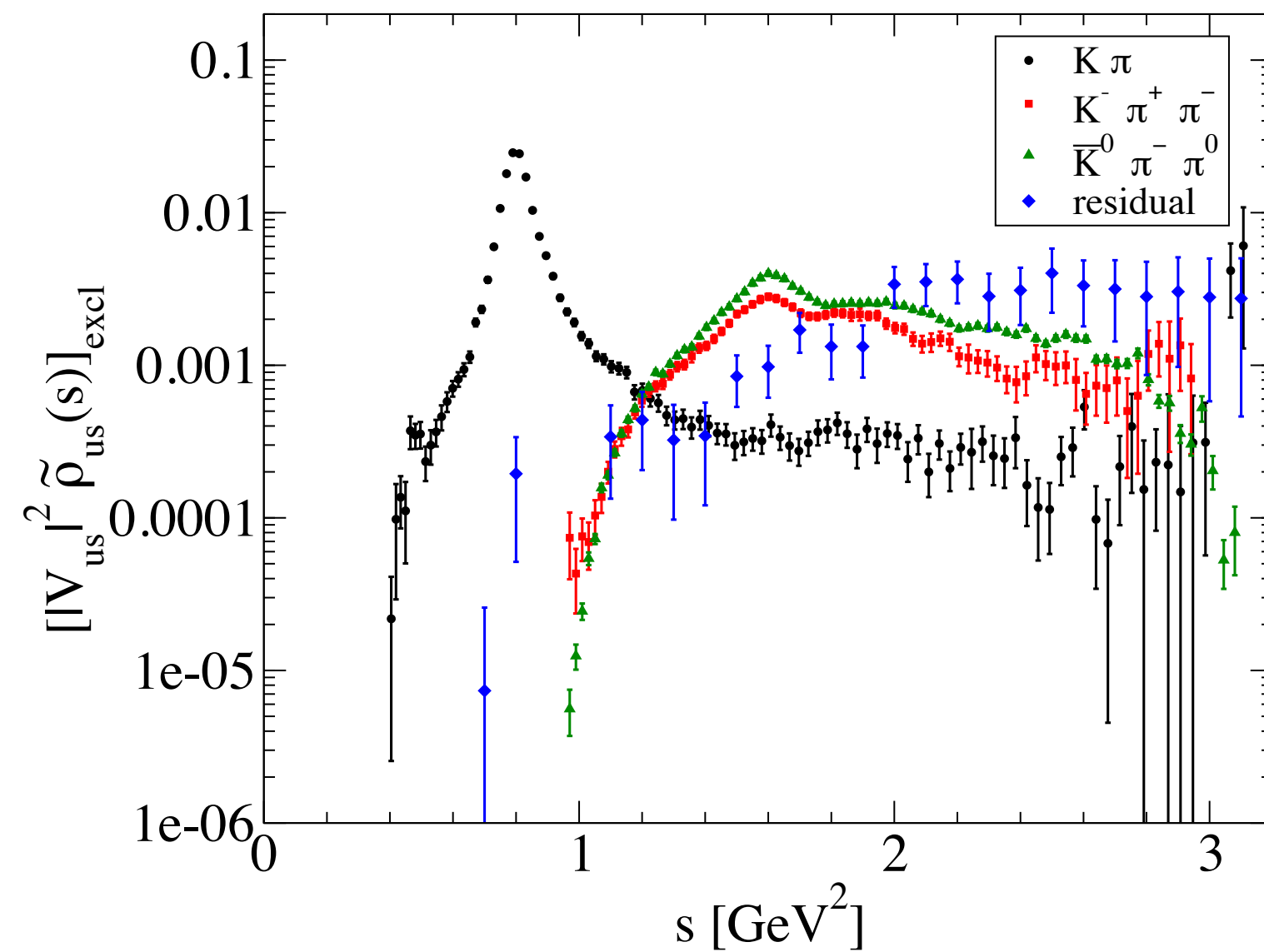
$$\frac{R_{\tau,ud}}{|V_{ud}|^2} - \frac{R_{\tau,us}}{|V_{us}|^2} = \delta R_{\tau,th}$$



Gamiz et al. hep-ph/0212230, hep-ph/0408044,

....

$$\int_0^{s_0} \omega(s) \Delta\rho(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} \omega(s) \Delta\Pi(-s) ds$$

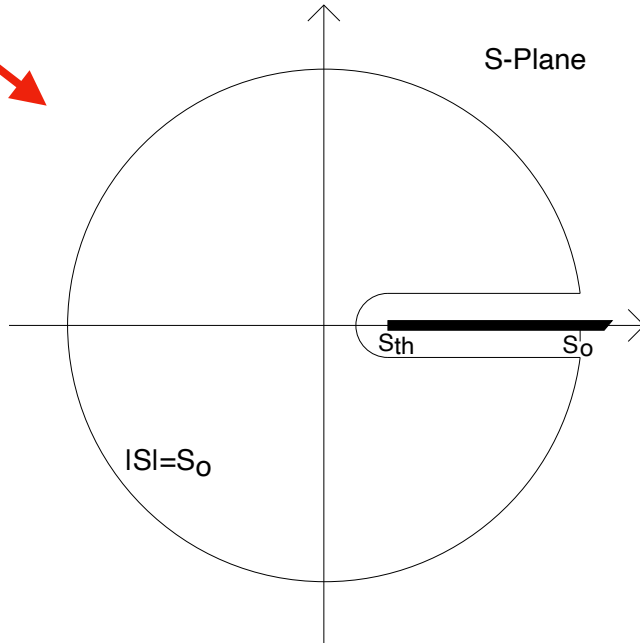


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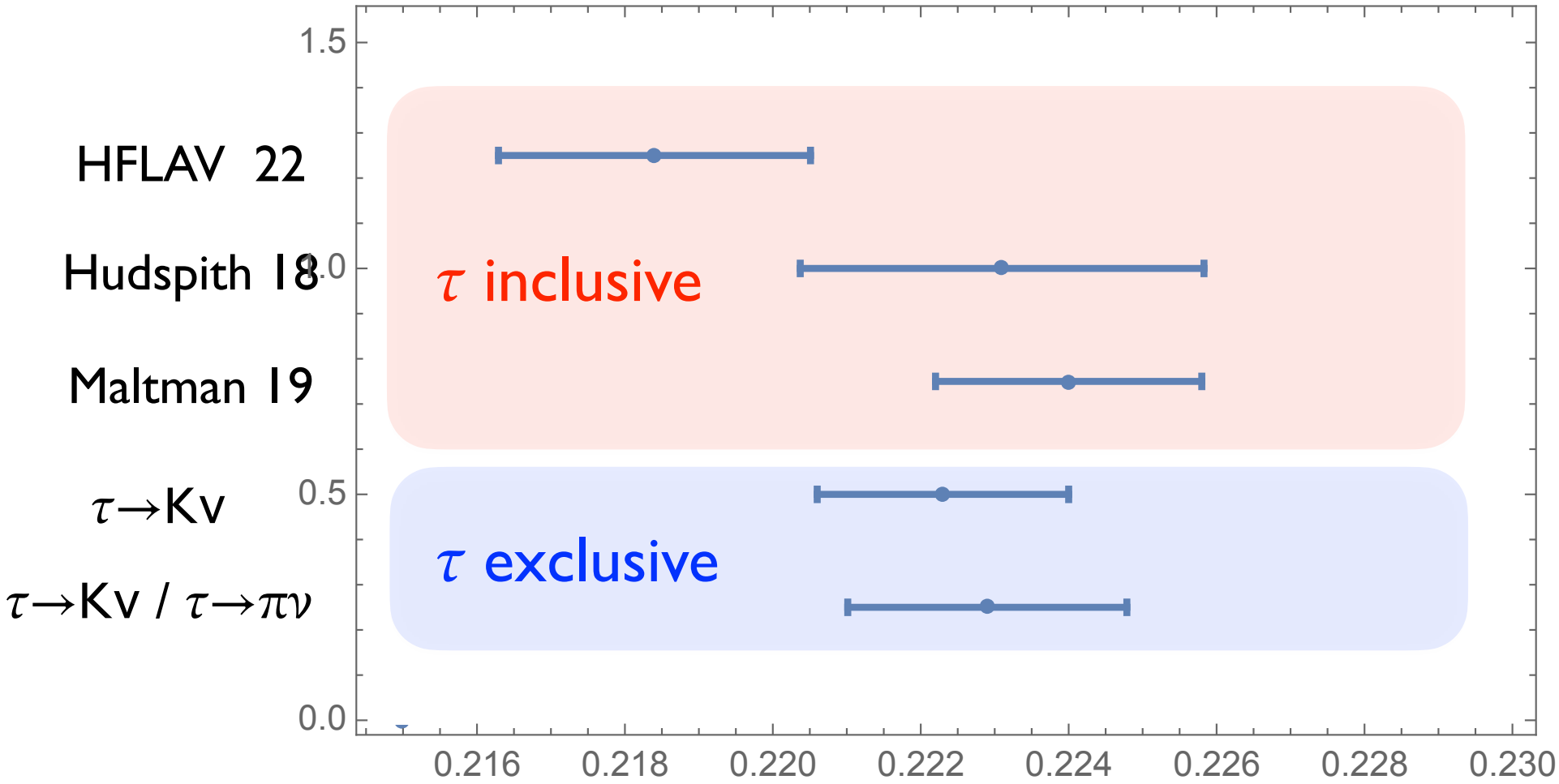
$$\frac{R_{\tau,ud}}{|V_{ud}|^2} - \frac{R_{\tau,us}}{|V_{us}|^2} = \delta R_{\tau,th}$$



Gamiz et al. hep-ph/0212230, hep-ph/0408044, ...

- Exclusive ($\tau \rightarrow K \nu$ / $\tau \rightarrow \pi \nu$): need partial widths, decay constants (LQCD) & radiative corrections

From A. Lusiani, Talk at MITP ELECTRO 2022



V_{us}

A. Lusiani, HFLAG WG (1909.12524)

method	experiment [%]	theory [%]	lattice QCD [%]	rad.corr. [%]
$\tau \rightarrow X_s \nu$	0.84	0.49		
$\tau \rightarrow K / \tau \rightarrow \pi$	0.72		0.18	0.40
$\tau \rightarrow K$	0.69		0.19	0.29

Experimental prospects:
Belle-II and possibly
tau-charm factory & FCC-ee

Theory prospects:

- (1) Radiative corrections are a bottleneck for exclusive modes;
- (2) lattice QCD will provide first-principles inclusive determination

RGEs in the LEFT

$$\mu \frac{dC_\beta^r(a, \mu)}{d\mu} = \gamma(\alpha, \alpha_s) C_\beta^r(a, \mu),$$

$$\gamma(\alpha, \alpha_s) = \gamma_0 \frac{\alpha}{\pi} + \gamma_1 \left(\frac{\alpha}{\pi}\right)^2 + \gamma_{se} \frac{\alpha}{\pi} \frac{\alpha_s}{4\pi} + \dots$$

$$\gamma_0 = -1$$

$$\gamma_1^{NDR}(a) = \frac{\tilde{n}}{18} (2a + 1),$$

$$\tilde{n} = \sum_f n_f Q_f^2$$

$$\gamma_{se} = +1$$

A. Sirlin 1982

Scheme-independent NLO Wilson Coefficient



$$C_\beta^r(a, \mu) = \left(1 + \frac{\alpha(\mu)}{\pi} B(a) \right) \times \bar{C}_\beta^r(\mu)$$

$$C_\beta^{LO}(m_c) = 1.01014,$$

$$C_\beta^{LL}(m_c) = 1.01043,$$

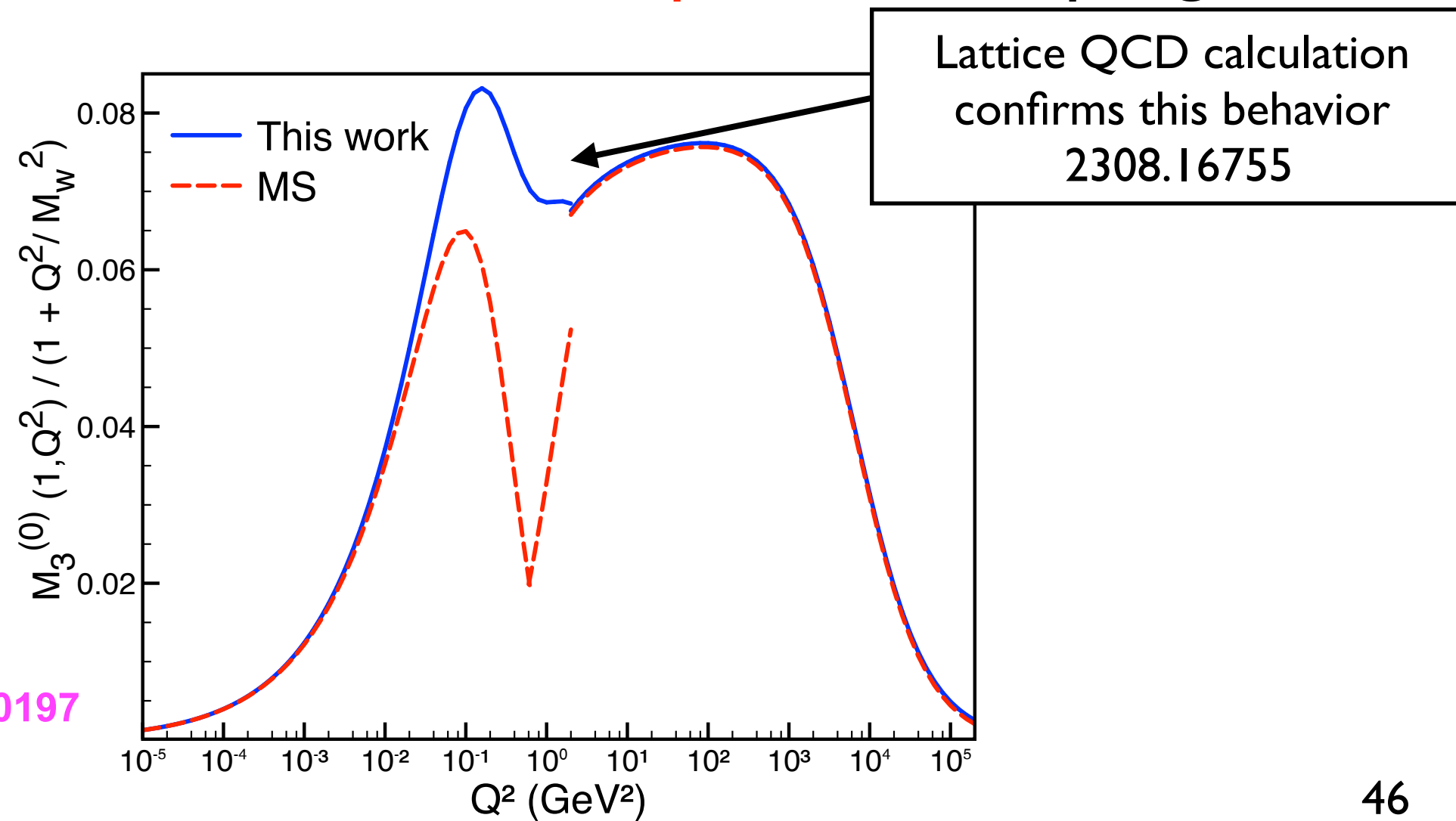
$$C_\beta^{NLL1}(m_c) = 1.01027,$$

$$\bar{C}_\beta^{NLL2}(m_c) = 1.01018.$$

NLL1 ($\alpha\alpha_s$) and NLL2 (α^2) RGEs essentially undo LL enhancement....

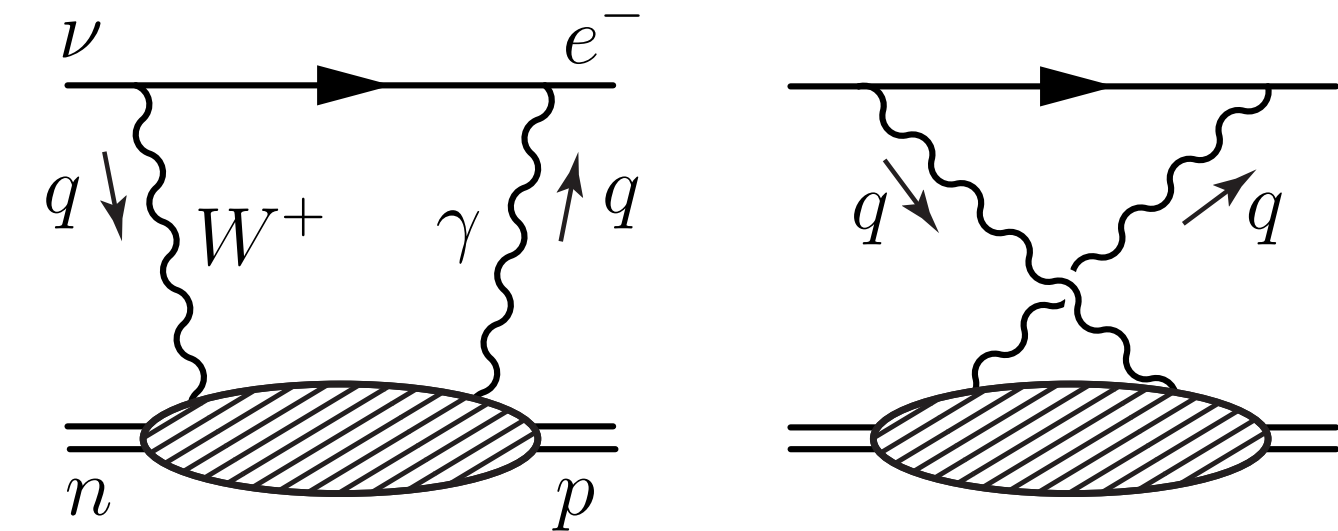
Radiative corrections: 'Sirlin's representation'

- Long history, starting in the 1950's. Modern approaches build upon Sirlin current algebra formulation from the '60 & '70s
- Recent new developments: **dispersive approach** to the non-perturbative input (γ - W box) for **neutron, pion, kaon and nuclear** decays & connection to LQCD and nuclear many-body first principles methods
- Example: EM correction to $n \rightarrow p$ vector coupling



Seng et al. 1807.10197

Seng et al. 1807.10197, Czarnecki et al, 1907.06737, Shiells et al. 2012.01580
Hayen 2010.07262, Gorchtein-Seng 2106.09185



Gorchtein, Feng, Jin, Seng, ...
2003.09798, 2003.11264, 2102.12048, 2308.16755
Gennari, Drissi, Gorchtein, Navratil, Seng, 2405.19281

Larger correction, smaller error.
It affects both neutron and nuclear decays

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02474(31)
Shiells, Blunden, Melnitchouk 2021	0.02472(18)
Combined [2208.11707]	0.02467(27)